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Part I

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ROTOR-BEARING DYNAMICS TECHNOLOGY DESIGN GUIDE
Part I Flexible Rotor Dynamics

SHAKER RESEARCH CORP.
BALLSTON LAKE, NEW YORK 12019

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TECHNICAL REPORT AFAPL-TR-78-6, Part I
Interim Report for Period April 1976 - November 1979

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This technical report has been reviewed and is approved for publication.

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This report describes an update of the Air Force Computer Programs concerned with the dynamics of high speed rotors. The new program, which replaces AFAPL/SFL Programs No. 100, 101, and 117, has the capability to perform a variety of rotordynamics analyses. These analyses include torsional vibration and bending vibration. The bending analyses consist of those for critical speeds, unbalance response, asynchronous response, and rotor stability. In addition, the program incorporates the following: a formulation based on whirl coordinates with an exact bending solution between rotor modelling

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20. ABSTRACT (Cont'd)

stations, the capability to utilize a small subset of these stations in much of the calculations, a set of bearing characteristics that depend on rotor speed and vibration frequency and that include anisotropy and damping, and an economical combination of a separate rotor analysis and the characteristics of the bearings. The report includes:an introduction to use of the program as well as a detailed user's manual, ~~The report~~ also ~~includes~~ both an overview of the mathematical basis of the program and a complete presentation of those mathematics.

FOREWORD

This report was prepared by Shaker Research Corporation under USAF Contract No. AF33615-76-C-2038. The contract was initiated under Project 304B, "Fuels, Lubrication, and Fire Protection," Task 304806, "Aerospace Lubrication," Work Unit 30480685, "Rotor-Bearing Dynamics Design."

The work reported herein was performed during the period 15 April 1976 to 15 November 1979, under the direction of John B. Schrand (AFWAL/POSL) and Dr. James F. Dill (AFWAL/POSL), Project Engineers. The report was released by the authors in November 1979.

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NOMENCLATURE

Symbol	Description
a	Denotes a constant
$\underline{a}$	Defined by Equations (5.9)
A	Denotes a constant
$\underline{\underline{A}}$	Mobility matrix
$\underline{\underline{B}}$	Damping matrix
C	Denotes a constant
c_α	Modal damping coefficient
$\underline{C}$	Defined by Equations (5.50)
$\underline{\underline{C}}$	Torsional and lateral connection matrices (Equations (5.17) and (5.56))
d	Mass diameter
D	Stiffness diameter
E	Young's modulus
F	External force on rotor
g	Gravitational constant
G	Shear modulus
$\vec{H}$	Angular momentum vector
i	$\sqrt{-1}$
$\vec{i}$	unit vector
$\underline{\underline{I}}$	Identity matrix
I	Area or mass moment of inertia
$\mathcal{I}$	Inertia tensor
$\underline{\underline{I}}$	Defined by Equation (A-3)

NOMENCLATURE (Continued)

Symbol	Description
$\vec{j}$	Unit vector
J	Area polar moment of inertia
$\mathcal{J}$	Transverse mass moment of inertia
$\vec{k}$	Unit vector
k	Lineal spring rate
$\underline{\underline{K}}$	Stiffness matrix or impedance matrix
K_n	Second derivative in cubic spline
ℓ	Length between supports of simple rigid rotor or length of a shaft segment
L	Length of simple rigid rotor
m	Mass
M	External moment on rotor, internal bending moment in rotor
$\mathcal{M}$	Rigid body inertia
$\underset{\sim}{P}$	Force vector
$\underset{\sim}{Q}$	Torsional or flexural primitive vector
$\vec{R}$	Position vector
$\underset{\sim}{R}$	Bearing reaction force vector
$\mathcal{R}$	Ellipse radius (major or minor)
s	Slope of deflected rotor
$\underline{\underline{S}}$	Defined by Equations (5.9) and (5.50)
$\underline{\underline{T}}$	Intersegment or intrasegment connection matrix
t	Time

NOMENCLATURE (Continued)

Symbol	Description
u	Real part of eigenvalue
v	Imaginary part of eigenvalue
V	Shear force in rotor
w	Displacement in whirl coordinates
W	Determinant (Defined by Equation 6.7).
$\underline{w}$	Defined by Equations (5.50)
$\underline{\underline{W}}$	Whirl coordinate transformation matrix
x	Vertical coordinate system axis
$\underline{x}$	Displacement vector
y	Horizontal coordinate system axis
z	Rotor spin axis
$\underline{\underline{Z}}$	Dynamic stiffness (impedance) matrix
α	Inclination angle of orbit ellipse, modal subscript
β	Defined by Equation (5.42)
γ	Defined by Equation (5.7)
δ	Displacement of rotor center, distance from rotor center to mass center, denotes variation of a quantity
Δ	Axial offset of rigid body inertia
ε	Magnitude of error
ζ	Defined by Equation (5.48)
θ	Euler angle, shaft slope, first derivative in cubic spline
$\underline{\underline{\Theta}}$	Coordinate transformation matrix
κ	Eigenvalue or constant in eigenvalue problem

NOMENCLATURE (Continued)

Symbol	Description
ν	Frequency of vibration
ρ	Density of rotor material
$\underline{\underline{\sigma}}$	Defined by Equations (5.11) and (5.52)
ϕ	Shaft twist about z axis, Euler angle
$\underline{\Phi}$	Defined by Equations (5.9)
$\underline{\underline{\phi}}$	Coordinate transformation matrix
ψ	Euler angle
$\underline{\underline{\Psi}}$	Coordinate transformation matrix
ω	Angular velocity component, rotor speed
$\vec{\omega}$	Angular velocity vector

Subscripts

1, 2, etc.	Denotes elements of a vector, distinguishes among constants
1, 2	Denotes major and minor axes
b	Denotes bearing
f	Denotes flexure
i	Inner diameter
i, j	Denotes elements of a matrix
J	Denotes bearing station number
K	Denotes segment
M	Total number of segments used to model rotor
n	Denotes arbitrary interval in cubic spline or second derivative
N	Denotes bearing station number and number of intervals in cubic spline

NOMENCLATURE (Continued)

Subscripts	Description
o	Outer diameter, equilibrium (unperturbed) condition
p	Denotes pedestal or polar
t	Denotes torsion or transverse
x, y	Denotes z-x and z-y planes respectively
x, y, z	Denote Cartesian components of a vector
xy, xx, yx, yy	Denote elements of stiffness and damping matrices
$\|\|, \perp$	Defined in Equation (5.96)
α	Denotes a modal quantity

Superscripts

$(\ )'$	Denotes perturbation, denotes estimate, refers quantity to an associated coordinate system, denotes derivative; or refers to right end of a segment
$(\ )''$, $(\ )'''$	Refers quantity to associated coordinate systems, or denotes derivative
$(\acute{\ })$	Forward (corotational whirl)
$(\grave{\ })$	Backward (counter rotational whril)
$(\hat{\ })$	Either forward or backward whirl
$(\ )^{-1}$	Denotes inverse of matrix
$(\dot{\ })$	Differentiation with respect to time

SECTION I

INTRODUCTION

The practice of employing numerical computation to study vibrations of machinery has become a reality since Holzer's classical work in 1921 [1]. Understandably, during the earlier days when electronic digital computation equipment was not available, only the relatively simple torsional problem was considered. Two decades passed, when Myklestad's treatment of the lateral vibrations of a beam-like structure [2] was first formulated to study airplane wings. This important event took place when many of the finest minds in the history of modern technology were devoted to the engineering of the earthbound flying machine. It was rather remarkable that this development took place at all when digital computing equipment was of the electro-mechanical type and its memory capacity was mainly limited to a single register. Although the fundamental matrix methods were already available, their execution was done literally by hand, including the process of recording intermediate numbers. Adaptations of Myklestad's method to calculate critical speeds of a flexible rotor was soon proposed by Prohl [3]. By this time, most of the important elements in the computation methodology of machinery vibration were fully established. In the following years, the technological impacts of modern, high speed, digital, electronic computers were felt in all phases of engineering activities. Use of such computers in rotor dynamics studies is now routine. Engineers of the present generation are unburdened of the drudgery of computation chores.

The peculiar role of fluid-film bearings in the dynamics of a high speed rotor was first expounded by Newkirk in the mid 20's [4, 5]. Since then, journal bearing related instability of a rotor received attention from many investigators [6, 7, 8]. Designation of elastic and damping properties to study oil-film journal bearing effects on rotor vibrations was formally proposed by Hagg and Sankey [9]. In this respect, due to cross-coupling and anisotropic effects, dynamics of a rotor-bearing system assumed a higher degree of complexity than other problems of linear vibration in both physical concepts and computation procedures. Since relatively small lateral motion may represent a large fraction of the

radial clearance of a fluid-film bearing, interests in non-linear bearing effects soon developed. Unfortunately, only specialized topics in non-linear mechanics can be treated with any degree of thoroughness [10, 11, 12, 13]. Many attempted to utilize direct integration to study transient problems of rotor dynamics. Effectiveness of the latter approach is still a matter of much controversy. Lack of total success in its advocacy may be attributed to the following:

- o Numerical algorithms for direct integration require attention to control truncation error to avoid numerical instability.

  Proper implementation of this method to a realistic problem is generally quite costly.

- o Unless addressed to solve a problem for which some non-linear feature of the system is particularly prominent, the results of integration are disappointingly inconsequential. It is actually quite difficult to identify a worthy non-linear problem.

- o Uncertainties in the operating conditions of the fluid-film bearing (e.g. bearing clearance, lubricant temperature, and effects of feeding) often over-shadow improvement of accuracy which may be related to non-linear effects.

While earlier studies of dynamic fluid-film properties were restricted to rigid body motions of the rotor, a major step forward in the technology of rotor-bearing dynamics was advanced when a series of technical reports and associated computer software were published under the sponsorship of the Air Force Aero-Propulsion Laboratory, Wright-Patterson Air Force Base. This set of documents, known as Rotor-Bearing Dynamics Design Technology AFAPL-TR-65-45, consists of ten technical reports and ten computer programs, which are respectively listed in Tables 1 and 2. Within the realm of a linear characterization of the support bearings, the flexible rotor can thus be analyzed not only to obtain the critical speed characteristics, but also to predict unbalance response, stability threshold, and parametric excitation with allowances for cross-coupling

TABLE 1

TECHNICAL REPORTS

CURRENT ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY
SERIES AFAPL-TR-65-45

PART NO.	PART SUB-TITLE	AD NO.
I	State-of-the-Art	466390
II	Rotor Stability Theory	466391
III	Design Handbook for Fluid Film Bearings	466392
IV	Ball Bearing Design Manual	466393
V	Computer Program Manual for Rotor Response and Stability	470315
VI	The Influence of Electromagnetic Forces on the Stability and Response of an Alternator Rotor	822401
VII	The Three Lobe Bearing and Floating Ring Bearing	829895
VIII	Spiral Grooved Floating Ring Journal Bearing	852998
IX	Thrust Bearing Effects on Rotor Stability	872980
X	Feasibility Study of Electromagnetic Means to Improve the Stability of Rotor Bearing Systems	869324

NOTE: A typical example identifying a part is:

AFAPL-TR-65-45, Part I, "Rotor Bearing Dynamics Design Technology, State-of-the-Art", AD No. 466390.

TABLE 2

COMPUTER PROGRAMS

CURRENT ROTOR-BEARING DYNAMICS DESIGN TECHNOLOGY SERIES
AFAPL-TR-65-45

PART NO.	PROGRAM USE	AFAPL/SFL PROGRAM NO.
V	Rotor Stability	100
V	Rotor Unbalance Response	101
IV	Static Ball Bearing	102
VI	Rotor Response with EMF	103
VI	Rotor Stability with EMF	104
VII	Hybrid Journal Bearing Performance	105
VII	Three Lobe Bearing	106
VIII	Floating Ring with Herringbone	106
VIII	Herringbone Journal Bearing	108
IX	Rotor Stability with Thrust Bearing	117

and anisctropy. Throughout the passing years, this approach for rotor dynamics analysis has gained widespread acceptance. In the meantime, some new trends of technological development evolved; an improved analytical treatment of non-conservative (damped) rotor-bearing systems became known [14], and understanding of users' interests (of Rotor-Bearing Dynamics Design Technology) increased through experience.

As a part of an ongoing contract with the Air Force Aero-Propulsion Laboratory to update the Rotor-Bearing Dynamics Design Technology, this report covers the topic of Flexible Rotor-Bearing Dynamics. It will supersede the reports AFAPL-TR-65-45, Parts V and IX (AD No. 470315 and 872980), and the computer program described in Section III will replace AFAPL-SFL Programs No. 100, 101, and 117.

Essential improvements over the original version include the following:

- Distributed analysis is used. User is not required to subdivide a long uniform shaft segment to control accuracy.
- Local inclination (shaft slope) may be designated as an independent degree of freedom (in addition to deflection) in the lateral shaft motion.
- Both angular and lineal stiffness and damping coefficients may be assigned to each bearing.
- Vibration frequency may be specified by the user to be different from the shaft rotational rate.
- Effects of bearing modification and/or relocation can be studied with a single input setup.
- Condition of stability is established for <u>all</u> natural modes in a user specified frequency range. Quantitative indication of the stability condition is expressed in terms of the critical damping ratio of each natural mode.

- Forcing function can be designated as a force system and/or a moment system at each external-excitation station.
- The forcing function can be either stationary or rotating.
- In a damped response analysis, the damped resonance is automatically searched for within the specified frequency range.

Section II contains a description of the capabilities of the new rotor dynamics computer program. Section III is the User's Manual, including instructions and examples. An overview of the theory upon which the program is based is given in Section IV. The theory itself is presented in Section V, which contains complete derivations of the analytical basis of the program. Section VI gives attention to the special eigenvalue analyses employed in the new program. A derivation of the small inclination gyroscopic d'Alembert effect is included as Appendix A because there is no other convenient reference for this information. Appendix B covers the various uses of the spline curve-fitting scheme, which is employed in the real and the complex eigenvalue algorithms. Appendix C contains the listing o the source program in FORTRAN.

SECTION II

ROTORDYNAMIC ANALYSIS AND CAPABILITIES OF THE COMPUTER PROGRAM

The primary objective in the performance of rotordynamic analysis is to establish whether or not conditions of hazardous vibrations can develop to threaten the safe operation of the machine. The analysis presented in this report and the accompanying software cover synchronous and asynchronous passive responses as well as self-excited shaft whirls.

Figure 1 illustrates the role of rotordynamic analysis in a typical rotor engineering effort. In this diagram, the total analysis effort is divided into three steps. Step I analysis deals with the determination of potential resonances with obvious excitation sources; e.g., mass unbalance and gear mesh compliance. This is the minimum step which should be carried out in the evaluation of every new design. Generous variations in bearing support stiffness estimation should be allowed since the resonance frequencies of the lower modes can be quite sensitive to the precise values of the support stiffnesses. If little likelihood of resonance is revealed, subsequent steps of analysis may not be necessary. If undesirable resonances are uncovered, consideration should be given to alterations of shafting construction, bearing sizing, and/or bearing relocation to separate the applicable natural frequencies and speed-related excitations. Step II analysis would be performed when near-resonances appear to be unavoidable. Damped response amplitudes are sought. Complete characterization of the bearing supports should be incorporated, including damping coefficients and pedestal compliance where applicable. Balancing limits should be accordingly established to ensure ample safety margins. Step III analysis deals in part with self-excited shaft whirls. Various "obscure" factors can cause self-excited shaft whirls; the most commonly known one is the cross-coupling effect of fluid-film bearings. The need to perform Step III analysis largely depends on past experience with similar designs. It is necessary to be able to characterize with sufficient realism the relevant excitation sources to make this analysis meaningful. One should particularly pay attention to induced forces

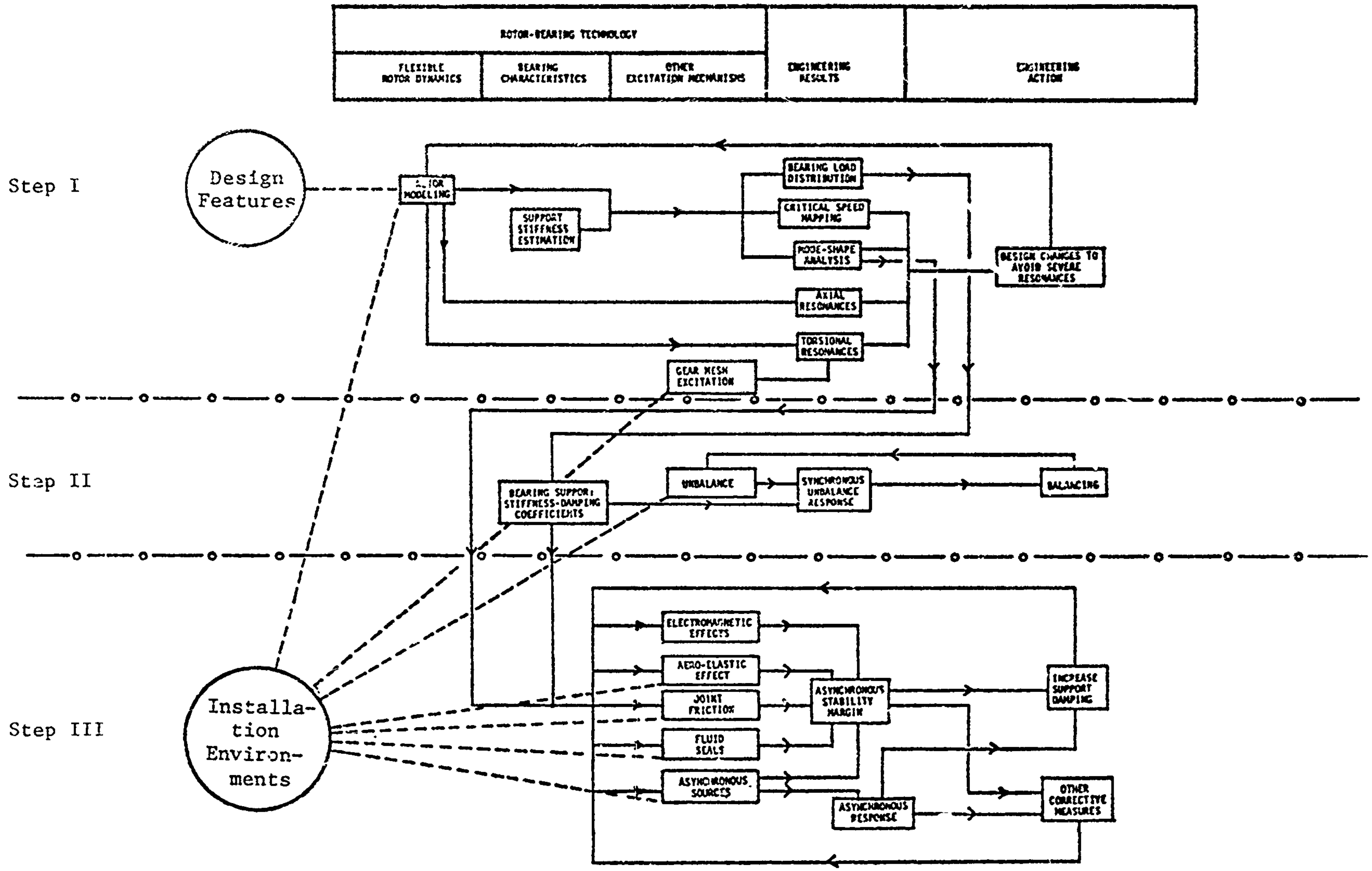

Figure 1 Steps in a Complete Rotordynamics Analysis Procedure

associated with small clearance impellers, flow leakage seals, and conditions of magnetic gap.

The computer program described in this report enables the user to perform all the rotordynamics analyses mentioned above. In addition, the program has unique capabilities that allow the user to "customize" his analysis to address the topics of greatest interest. In the remainder of this section, a general description of the capabilities of the program are given. The material in this section, plus that in Section III (which is the "User's Manual"), should be sufficient for use of the program. The reader is referred to Section IV for a more detailed discussion of the topics covered in the present section. The mathematical foundations for both sections are given in Section V, VI and in Appendices A and B.

2.1 Modelling of the Rotor

The rotor is modelled for the dynamic analyses by dividing it into numerous segments. Typically, the segments will lie between cross-sections where changes in shaft diameter or shaft material occur. In addition, segment length will be determined by the location and size of impeller/turbine-discs, or be terminated at places along the rotor length where excitation forces can be imposed or vibration displacements are of interest. It should be noted that the analytical basis of the program employs, within each segment, the exact bending solution for a constant cross-section beam. As a result, the user need not subdivide such portions of the rotor for increased accuracy. This characteristic is one of the features of the program.

Stations are cross-sections along the rotor where segments join. Stations not only locate all segment connections, but also locate bearings, excitation points, points for displacement results, and points of reference for representing rigid body inertias. Although the user must define sufficient stations to describe his rotor, the analysis procedure does not have to work continuously with all of the stations employed. The

user will typically select a subset of these stations when his program is run. This subset (called the "action" or "excitation" stations) will include only those stations at which

- o bearing damping or bearing anisotropy are to be considered
- o bearings are to be added, removed, or have stiffness values change with speed or frequency
- o excitation forces or moments are to be applied
- o the results obtained for the motions of the rotor are desired.

The size of this subset of rotor stations influences the computer time required for completing the analysis of the rotordynamics. Because the size of the subset typically will be much smaller than that used in representing the rotor, significant computer economy can be achieved. The ability to employ only a few such stations is another of the features of the program.

2.2 Representing the Operational Speed and Frequency Range

The computer program employs a frequency domain, forced response rotor dynamics analysis. In this, the results obtained for critical speeds, response, stability, etc., are based on separate calculations made at discrete vibration frequencies and associated rotor rotational speeds. The user is required to specify these speeds and frequencies. Most generally, he can direct the program to employ a number of rotor speeds and, at each speed, to employ a number of frequencies. (For critical speed and unbalance response calculations, the situation is simple since each speed is associated with an identical frequency). The results obtained will be dependant on the speed-frequency choice. The rotor speeds must be those at which the rotor will run or must cover the range in which vibrational problems occur. The frequencies must likewise cover the range in which vibrational problems occur. For example, in a critical speed calculation, the frequency range (and associated rotor speed range) defines the frequency limits within which the criticals will be found. In a stability analysis calculation, the frequency range (at a given rotor speed) defines the frequency limits within which an unstable mode can be found.

Because the method of "cubic spline" (see Appendix B) is used to determine the root(s) of the characteristic determinant, accuracy of the calculated natural frequency and the mode shape is somewhat sensitive to the choice of the frequency range and the number of frequency points within the frequency range. Experience has shown that a range of 20 percent of the lowest natural frequency should be satisfactory. The number of frequency points should be between four and seven. In many cases, four points are quite adequate. A large number is desirable if two modes are expected to be contained in the designated frequency range. Very little gain in accuracy can be achieved with more than seven points while computation time would be significantly increased. Accuracy of the results can be verified by repeating the calculation with the same number of frequency points along with a smaller frequency range and noting the magnitudes of any changes in the results.

2.3 Representing the Characteristics of the Bearings

Since bearings can influence significantly the dynamics of the rotor, the computer program has been formulated to treat the rotor bearings in a convenient and economical manner. The program first calculates the dynamic characteristics of the rotor using nominal bearings*. This calculation is termed Level I. The program then accepts bearing characteristics that can be anisotropic and that include damping. These characteristics can vary with frequency and with rotor speed. The associated calculations are termed Level II. In effect, in Level II the dynamic properties of the rotor-bearing system are developed from those of Level I for the rotor itself. The economics associated with this technique can be significant, since the Level I calculation is not done repeatedly in an analysis. This technique is, consequently, another feature of the computer program.

In use, the user provides to the program, for each bearing, the value for its nominal stiffness and the values for those actual characteristics which are appropriate in the situation under consideration. For the actual bearing characteristics, each bearing can be characterized at each speed and frequency by as many as 16 quantities. The detailed description of these quantities is given in Section 4.2.4 of this report.

*These nominal bearings are isotropic and contain no damping.

2.4 Producing the Rotordynamics Analyses

Depending on the input setup beyond Level I, the computer program allows the user to produce five different analyses for lateral (bending) vibrations of the rotor-bearing system. Each analysis or run mode is described separately below.

2.4.1 Critical Speeds

Critical speed analysis concerns resonance with mass unbalance. Three restrictive conditions are imposed internally in the computer program when this run mode is selected. Firstly, the excitation frequency is set to be the same as the rotational speed in consistent units. Secondly, anisotropy in the rotor system is ignored. Finally, non-conservative aspects (i.e. damping) in the rotor system are ignored.

Upon finding each critical speed, the normalized mode shape is calculated and printed out. The critical speeds can be determined with the input set up for an idealized rotor model, for which bearing characteristics are assigned as a set of isotropic springs. Since it is desirable to know how the critical speeds vary with bearing stiffness, the calculations can be repeated for new sets of bearing stiffness data with the aid of impedance alteration.

2.4.2 Unbalance Response

To obtain damped unbalance response, the user must provide bearing characteristics at each speed according to the most general format and forcing functions in terms of mass moments. The computer program automatically checks whether or not anisotropy is present then makes allowance for response ellipticity where appropriate. The relative phasing of mass moments among various planes can be assigned by the user. The computer will internally set the forcing function to be of the rotating type regardless of the input entry. Calculated response will be presented as orbit parameters at each of the active stations at speed points specified in the input.

The computer will automatically search for one or more damped critical speed(s) within the specified speed range and compute the corresponding response(s).

2.4.3 Asynchronous Resonances

To determine asynchronous resonances, the user specifies the range of asynchronous frequencies of interest at each rotor speed. He also has the option of specifying bearing support characteristics at each speed-frequency combination. As in the critical speed calculation, non-conservative aspects of the rotor are neglected; however, isotropy is not presumed. The asynchronous resonance mode shapes are calculated along with the natural frequencies. Where isotropy is not preserved, the mode shape is described in terms of orbit parameters.

2.4.4 Asynchronous Response

This run mode is to Asynchronous Resonances as Unbalance Response is to Critical Speeds. There is, however, an additional parameter available to the user to specify the character of the forcing function. Asynchronous excitation may be encountered either in the form of a space fixed system or a rotating system. Gear mesh excitation, partial inlet nozzle, and flow blockage all represent space fixed asynchronous forcing systems. Rotating forcing systems are less common; one example of the latter is an unbalanced rotating stall of a compressor stage. For this reason, the user is allowed to identify whether the forcing function is space fixed or rotating. Again, asynchronous damped resonance is automatically searched within the user-specified frequency range and the response at the corresponding condition is calculated in addition to those at user-specified frequencies.

2.4.4 Stability Analyses

Stability analysis is performed at each user-specified speed by

searching for non-conservative natural modes in user-specified asynchronous frequency ranges. The critical damping ratio is determined along with the natural frequency. A negative critical damping ratio indicates instability. The normalized mode shape of each mode is printed out in terms of orbit parameters at the active stations.

SECTION III

DESCRIPTION OF COMPUTER PROGRAM

This section constitutes a manual for use of the rotordynamics program. It first describes, in detail, the input data and then discusses the output provided by the program. Examples of both input and output are also given.

3.1 Input Data

Input data consist of job title, rotor geometry, material properties, bearing characteristics, dynamic characteristics such as shaft speeds, vibration frequencies and types of excitation, and kinds of dynamic analyses to be performed.

Depending on the particular type of analysis required, the input information may vary. Data cards in sequence and their specifications are listed in the following. Each integer must be right-justified in its field.

In Level I input (Cards 1 - 13)* specify shaft geometry and speed frequency combinations at which subsequent calculations will be carried out. (Cards 2 - 9) specify geometrical, physical and material modelling parameters of the shaft, (Cards 1, 10 - 13) specify the bearings and speed frequency combinations. Card 14 contains control parameters which relate to the type of analysis desired. Cards 15 and 16 deal with additional data (Level II) which, together with structural characterization furnished by Level I input, complete the dynamic specification of the total rotor system. Card 17 allows entry of user-specified conditions to initiate the iterative computations required of a non-conservative rotor system.

*The term "Card" as used for the input data description refers, in general, to a <u>set</u> of input cards. For example, Card 4 is a set of individual cards, one for each shaft segment.

Card 1 - Number of Speed Groups and Execution Level Control

Column 1 - 5: NCASE, FORMAT (I5) - A positive integer that indicates the number of speed groups to be run for a single set of data (Cards 2 - 9). Cards 10 - 13, 15 - 17 are to be repeated for each speed group to specify further speed-frequency combinations and associated Level II bearing revisions and forcing function designations, within each speed group. If NCASE is set to zero, omit Cards 2 - 13; execution will skip to card 14 using the user-furnished data file on Tape (10), which should contain previously generated Level I interim output (See KRUN).

Column 6 - 10: KRUN, FORMAT (I5) - Execution level control.

0 - Data (Cards 1 - 13) are stored on Tape (10) for further runs with Level II alterations, execution then stops. (Run Level I only).

1 - Level I data are stored on Tape (10) as above and execution proceeds normally with (Card 14) and any other Level II alterations (Card 15 - 17). (Run Level II).

Card 2 - Title Card

Column 1 - 16: IDENT, FORMAT (4A4) — 15 character job identification code. Last character in the allowed field (Column 16) should be a blank for visual separation from TITLE.

Column 17 - 80: TITLE, FORMAT (16A4) — 64 character job title.

Card 3 — Rotor Model Parameters, Action Stations, and Printout Control (FORMAT (16I5))

Column 1 - 5: LSEG -- Number of shaft segments (maximum 75).

Column 6 - 10: LMAT — Number of different materials of which the rotor is composed.

Column 11 - 15: LMAS — Number of lumped mass stations; e.g., disks, collars and impellers.

Column 16 - 20: LBEA — Number of bearing stations (maximum 4).

Column 21 - 25: LTYP — Type of vibration of the rotor

1 torsion vibration

2 bending vibration

3 both

Column 26 - 30: LEXI — Number of action stations (maximum 20). A geometric station may represent two action stations, see Card 9.

Column 31 - 35: LPRI — Level I bearing printout control.

0 - default, printout bearing data

-1 - supress bearing data output

Card 4 – Geometry of Shaft Segments, FORMAT (I5, 5X, 5F10.0). One Card 4 must be included for each shaft segment; therefore, the value of LSEG on Card 3 equals the number of Card 4's to be included in the input deck.

Column 1 - 5: L – Shaft segment index number (1 through LSEG).

Column 6 - 10: Blank.

Column 11 - 20: X(L) – Length of the L-th segment (inches).

Column 21 - 30: D1 – Stiffness I.D. (inches).

Column 31 - 40: D2 – Stiffness O.D. (inches).

Column 41 - 50: D3 – Mass I.D. (inches).

Column 51 - 60: D4 – Mass O.D. (inches).

NOTE: D3 and D4 cannot be equal.

Card 5 — Material Properties, FORMAT (2I5, 3E10.4). One card must be provided for each different shaft material; therefore LMAT cards must be provided.

Column 1 - 5: NMAT — Material index number (1 through LMAT).

Column 6 - 10: N — Station number of new material (N must equal 1 on the first Card 5)

Column 11 - 20: DO — Density (lb./cu.-in.).

Column 21 - 30: E — Young's modulus (lb./sq.-in.)

Column 31 - 40: G — Shear modulus (lb./sq.-in.)

Card 6 -- Physical Properties of Lumped Mass Stations, FORMAT (2I5, 4F10.0)
LMAS cards must be provided (Card 3)

Column 1 - 5: L - Lumped mass index number (1 through LMAS).

Column 6 - 10: N3(L) - Station number of the L-th lumped mass.

Column 11 - 20: W(L) - Weight of the L-th lumped mass (lb.)

Column 21 - 30: P(L) - Polar inertia of the L-th lumped mass (lb.- in.^{2}).

Column 31 - 40: T(L) - Transverse inertia of the L-th lumped mass (lb.- in.^{2}).

Column 41 - 50: Y(L) - Offset of C.G. of the L-th lumped mass, measured from station N3(L) (inches).

Card 7 — Physical Properties of Bearing Stations, FORMAT (2I5, 6F10.0), LBEA cards must be provided (Card 3)

Column 1 - 5: L — Bearing index number (1 through LBEA).

Column 6 - 10: N5(L) — Station number of the L-th bearing.

Column 11 - 20: D5(L,1) — Vertical (along x) misalignment of the L-th bearing (inches).

Column 21 - 30: D5(L,2) — Horizontal (along y) misalignment at the L-th bearing (inches)

Column 31 - 40: S5(L,1) — Vertical static lineal stiffness of the L-th bearing (lb./in.)

Column 41 - 50: S5(L,2) — Horizontal static lineal stiffness (lb./in.)

Column 51 - 60: T5(L,1) — Angular stiffness of the L-th bearing, in the vertical plane (in.-lb./rad.).

Column 61 - 70: T5(L,2) — Angular stiffness in the horizontal plane (in.-lb./rad.).

The above bearing data are primarily for determining the load distribution among bearings due to gravity and due to bearing misalignment. They are not used for critical speeds or response.

Card 8 – Action Stations, FORMAT (16I5)

Maximum 2 cards (20 stations); LEXI data values must be provided (Card 3). A geometric station may be repeated to designate both lineal and angular actions, see Card 9.

Column 1 - 80: N8(I), I=1 - LEXI – Station number of excitation point.

Since the computer program is based on the distributed analysis of shaft segments, only stations which are important and significant for investigating mode shapes and for assigning mass unbalance or external loads need to be assigned as action stations. Bearing stations, heavy lumped inertia stations, stations with external excitation, and stations where vibration motions are of interest usually fall into this category. The number of action stations should be as small as possible but should include bearings and end stations. Generally 4 to 7 stations are used. If floating number exponent overflow occurs (this often happens on a single precision computer) reduce the number of action stations.

Card 9 – Action Station Capability. FORMAT (16I5)

Maximum 2 cards (20 stations); LEXI data values must be provided (Card 3)

Column 1 - 80: K8(I), I=1, LEXI – Excitation type associated with each action station specified on Card 8.

Action station capability characterizes both input and output at the station; e.g., lateral force type denotes force input and/or deflection output. Lateral moment type denotes moment input and/or slope output.

The type of excitation is designated as follows:

K8(I)	Torsional Moment	Lateral Force	Lateral Moment
1	-	-	-
2	X	-	-
3	-	X	-
4	-	-	X
5	X	X	-
6	X	-	X
7	-	X	X
8	X	X	X

Note: Total degrees of freedom must not exceed 40. Each lateral force type or a lateral moment type action station is assigned two degrees of freedom, while each torsional moment type action station is given a single degree of freedom. For example, if K8(L) = 6, the L-th action station has a total of 1 + 0 + 2 = 3 degrees of freedom.

The number of sets of cards 10 - 13 must equal the value of NCASE on Card 1.

<u>Card 10</u> – <u>Run Mode Control</u>, FORMAT (I5)

Column 1 - 5: <u>LRUN</u> – Synchronism indicator

1 synchronous frequency

0 asynchronous frequency

LRUN in card 14 of input must be consistent with the value of LRUN entered here.

Card 11 — Speed Loop Range, FORMAT (2I5, 2F10.0)

For IRUN = 1 or 2 in card 14, LRUN in card 10 must be set to 1, and it is necessary to select a speed range over which analysis is to be conducted and the number of speed points (maximum of 11) of which the range is comprised. A number of speed groups may be used as designated by NCASE in card 1 to cover a large speed range by several speed groups. The speed ranges of various speed groups may overlap or be consecutive or be separated. IFRE would be defaulted internally to a value of 1.

For IRUN = 3, 4, or 5, in card 14, LRUN in card 10 must be set to 0; the rotor speed is constant and therefore the number of speed points is 1 (within each speed group). Under this condition a number of different frequency ranges (other than running) each consisting of a number of frequency points (maximum of 11) may be selected to investigate the asynchronous frequencies. Selection must be made, therefore, of speed groups, speed points (always 1), frequency groups, and frequency points.

Column 1 - 5: I7 - Number of speed points to be run in each speed group. This determines the number of times the set of cards 12 - 13 are to be repeated as a pair within each speed group (cards 10 - 13).

Column 6 - 10: IFRE — Number of frequency groups at each speed point specified by I7 above (cannot exceed 5).

Column 11 - 20: SPE1 — Lower bound of the speed range.

Column 21 - 30: SPE2 — Upper bound of the speed range.

Card 12 — Nominal Dynamic Bearing Stiffnesses, FORMAT (2I5, 2F10.0),
Total number of cards must equal LBEA×I7.

Column 1 - 5: L — Bearing stations index number (1 through LBEA).

Column 6 - 10: I6 — Input control of dynamic bearing data;
1 continue data entry
0 denotes last data card for current speed group and allows subsequent omission of further repetition of card 12 if bearing data do not vary within a speed group.

Column 11 - 20: XS(L, 1) — Lineal bearing stiffness (lb./in.) for Level I

Column 21 - 30: XS(L, 2) — Angular bearing stiffness (in.-lb./rad.) for Level I only.

The core of computations lies in the speed (outer) and the frequency (inner) loops. The shaft-speed dependent bearing data are entered within the speed loop.

Card 13 — Frequency-Loop Range, FORMAT (2I5, 2F10.0)

Total number of cards must equal IFRE×I7 for each speed group (card is omitted if LRUN = 1).

Column 1 - 5: J — Frequency group index number (1 through IFRE).

Column 6 - 10: I8 — Number of frequency points of the frequency loop.

Column 11 - 20: FRQ1 — Lower bound of the frequency range.

Column 21 - 30: FRQ2 — Upper bound of the frequency range.

Card 14 – Run Mode Control Parameters for Level II, FORMAT (16I5)

One card for each input setup.

Column 1 - 5: IRUN – Run mode indicator

1 compute critical speeds

2 determine unbalance response

3 compute natural modes of asynchronous resonance

4 determine asynchronous response

5 perform stability analysis

The allowable value for IRUN must be compatible with the value of LRUN in card 10 as indicated by check marks in the following table.

	IRUN				
LRUN	1	2	3	4	5
0			✓	✓	✓
1	✓	✓			

The program automatically solves the real eigenvalue problem in the specified frequency/speed range for IRUN = 1 and 3. For IRUN = 2, 4 and 5, the complex eigenvalue problem may be treated according to the value assigned to IGEN (column 26 - 30 of this card).

Column 6 - 10: ITYPE – Type of shaft motion due to excitation.

0 whirling

1 space fixed

(Internally defaulted to 0 for IRUN = 1, 2, 3)

Column 11 - 15: IBRG – Number of bearings to be revised.

The program allows the bearing data used in Level I to be altered. This permits the modelling of anisotropic bearings and bearings having damping, and also permits bearing replacement and the interchange of bearing stations.

Column 16 - 20: IPRI — Controls output of Level II bearing data and excitation force.

0 prints out bearing data and excitation force at each speed point

-1 suppresses printing of bearing data and excitation force

Column 21 - 25: IDIAG — Diagnostic output control

0 no diagnostics

2 complete diagnostics

Column 26 - 30: IGEN — Controls number of damped natural modes to be found in the complex eigenvalue computations.

0 the complex eigenvalue vector subroutine will be bypassed and no complex roots will be sought. (IRUN = 2, 4, 5)

<0 |IGEN| damped natural modes will be sought and iteration diagnostics will be furnished.

>0 IGEN damped natural modes will be sought without diagnostics output.

Column 31 - 35: ISLO - Controls iteration criterion and speed.

0 yields normal iteration

>0 convergence criterion is multiplied by 10^{ISLO}.

<0 convergence criterion is multiplied by 10^{-ISLO} and iteration stepping for damping is decreased by 2^{ISLO}

Normally set to zero. If iteration oscillates or does not converge, yet a damped natural mode is expected within speed group, try ISLO equal to 1 or -1. Absolute value of ISLO should not exceed 3 and rarely be as high as 2.

Column 36 - 40: IBEG — Allows user input of starting values for complex eigenvalue iterations.

0 use internal logic to determine starting values. Omit card 17.

-1 read iteration starting values from card 17.

Card 15 — Revision of Bearing Coefficients Control, FORMAT (4I5)

IBRG cards must be provided for each speed frequency combination.

Column 1 - 5: L — Bearing index number (1 through IBEG).

Column 6 - 10: NBRG — Station number of bearing to be revised.

Column 11 - 15: KBRG — New bearing type

1 radial (lineal)

2 angular

Column 16 - 20: IREV — Revision data control

0 use previous data, bypass Card 15A

1 read card 15A

Card 15A — Revision of Bearing Coefficients, FORMAT (8E10.4)

IBRG cards must be provided for each speed-frequency combination (card is omitted if IREV (Card 15) = 0).

Column 1 - 80: KXX, BXX, KXY, BXY, KYX, BYX, KYY, BYY — Bearing coefficients; their identifications are:

First Letter: K, stiffness coefficient

Unit: lb./in. (lineal bearing),

in.-lb./rad. (angular bearings)

B, damping coefficient.

Unit: lb.-sec./in. (lineal bearing)

in.-lb.-sec./rad. (angular bearing)

Second Letter: Reaction coordinate.

Third Letter: Displacement coordinate.

Card 16 — External Excitation Sources, Control FORMAT (2I5)

One card must be provided for each speed-frequency combination (this card and cards 16A, 16B and 16C are omitted if IRUN=1, 3, or 5 on Card 14).

Column 1 - 5: IFORCE — Number of excitation planes excited.

Column 6 - 10: IFDATA — Force data input control
0 use previous data and omit Cards 16A - 16C.
1 read Cards 16A - 16C.

Cards 16A - 16C — External Excitation Sources
One of each card must be provided for each speed-frequency combination (cards are omitted if IRUN = 1, 3 or 5 or if IFDATA = 0)

Card 16A — FORMAT (16I5)

Column 1 - 80: NFORCE(I), I = 1, IFORCE — Station number of the I-th excited station.

Card 16B — FORMAT (16I5)

Column 1 - 80: KFORCE(I), I = 1, IFORCE — Excitation type
1 lateral force
2 lateral moment

For IRUN = 2 KFORCE must be = 1

For IRUN = 4 KFORCE may be 1 or 2

Card 16C — FORMAT (4E10.4)

Column 1 - 40: F1, G1, F2, G2 — Excitation components

First letter = F component in-phase with time reference
G component leading time reference by 90 degrees

Second letter = 1 forward rotation (whirl)
2 backward rotation (whirl) (ITYPE = 0; i.e., rotating excitation)
(backward rotation would be applicable to counter rotating shafts)

Card 17 — User Furnished Starting Values for Complex Eigenvalue Iterations, FORMAT (2E20.4)

These cards are required if IBEG (card 14) = -1 and if IGEN (card 14) ≠ 0. |IGEN| cards must be supplied for each iteration set. An iteration set is a speed group for IRUN = 2 or a frequency group for IRUN = 4 or 5.

Column 1 - 10: FBRG — Iteration starting frequency.

Unit: Hz

CBRG — Iteration starting damping coefficient.

Unit: lb.-sec./in.

3.2 Input Samples

The input samples are based on a model rotor which was previously analyzed and tested by Tonnesen and Lund [15]. The model rotor is 46.85 inch (1190 mm) long and its total weight is 413.32 lbs. The shaft is divided into 32 segments with 18 inertial stations and 2 bearing stations. A scaled outline of the model rotor is shown in Figure 2. In the dynamic analysis, 4 action stations are chosen; they include 2 bearing stations and 2 inertia stations.

The original rotor is supported by two identical fluid-film bearings of the plain journal type. Each bearing has a radial clearance of 0.00181 inch, a bore diameter of 2.468 inch and a length of 0.74 inch. The pedestal supports are considered to be rigid. The dimensionless dynamic coefficients of this bearing have been kindly furnished by Prof. Lund and are listed in Table 3. Fluid viscosity of 3.349 cp is used in the sample calculations.

The present section will describe the setup of the input cards for solving various rotor dynamics problems. Emphasis here will be placed on illustrating the contents of the setups. In some studies, it is necessary to run the program several times. The input data of the next run will necessarily depend on the results of the previous run. Thus, in order to give the rationale for selecting the particular content of input data, occasional reference will be made to relevant portions of output data which is to be presented later in Section 3.4.

3.2.1 Critical Speeds (IRUN = 1)

As ball bearings are commonly used in many rotor systems, the critical speed calculation will be illustrated with the original fluid-film bearings replaced by a pair of deep-groove ball bearings. The stiffness coefficients were computed by a separate computer program prepared for the USAF under the same contract [16].

A complete set of input cards for the determination of a critical speed of the ball bearing rotor is shown in Table 4. Due to preload,

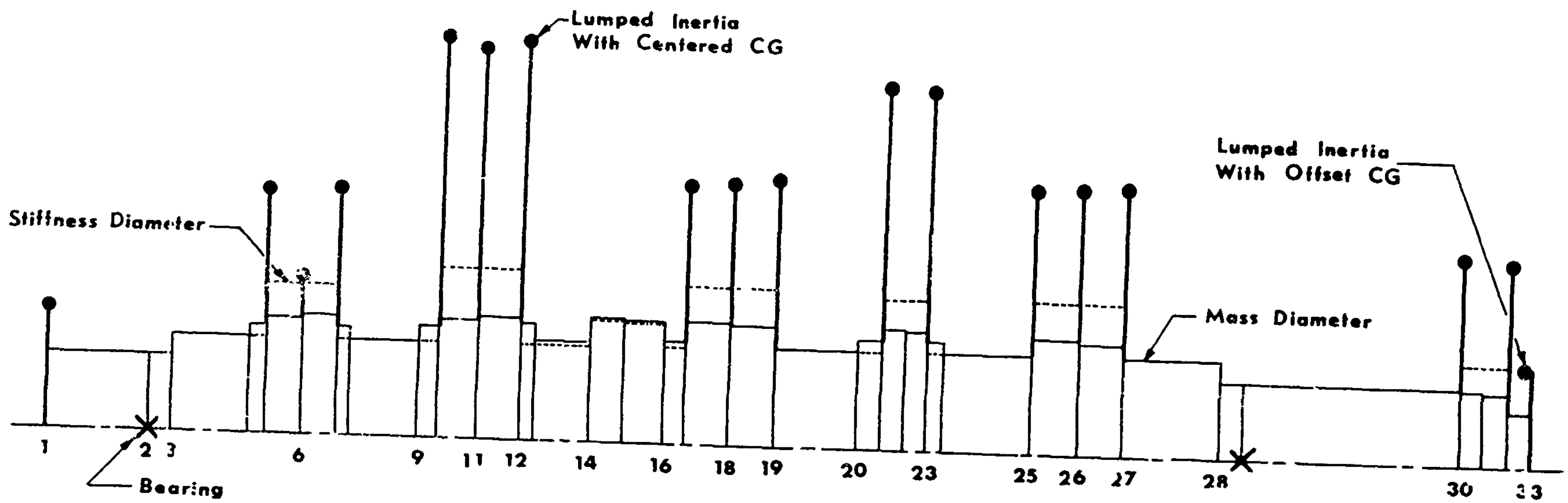

Figure 2 Outline of Sample Rotor

TABLE [illegible]

DIMENSIONLESS DYNAMIC COEFFICIENTS OF JOURNAL BEARING
USED IN SAMPLE ANALYSIS

BEARING TYPE: PLAIN CYLINDRICAL JOURNAL (L/D = 0.3)

(Courtesy of Prof. J. W. Lund)

1/S	CK_{xx}/W	CK_{yy}/W	CK_{xy}/W	CK_{yx}/W	$\sigma CB_{xx}/W$	$\sigma CB_{yy}/W$	$\sigma CB_{xy}/W$	$\sigma CB_{yx}/W$
3.97100D-01	2.11750D 00	1.99290D 00	4.15120D 00	-1.53160D 00	7.76580D 00	3.73570D 00	2.13760D 00	2.13760D 00
4.51900D-01	2.25070D 00	1.96480D 00	4.02760D 00	-1.34010D 00	7.47140D 00	3.41220D 00	2.10520D 00	2.10520D 00
5.74800D-01	2.53300D 00	1.91680D 00	3.87730D 00	-1.03330D 00	7.08110D 00	2.91310D 00	2.05150D 00	2.05150D 00
7.21400D-01	2.84270D 00	1.87840D 00	3.81210D 00	-7.91800D-01	6.86570D 00	2.54260D 00	2.01100D 00	2.01100D 00
9.00400D-01	3.19160D 00	1.84320D 00	3.80070D 00	-5.86700D-01	6.76130D 00	2.24490D 00	1.97360D 00	1.97360D 00
1.12370D 00	3.58920D 00	1.81160D 00	3.83120D 00	-4.06100D-01	6.74200D 00	1.99900D 00	1.94100D 00	1.94100D 00
1.40860D 00	4.04930D 00	1.78290D 00	3.89720D 00	-2.41300D-01	6.79380D 00	1.78990D 00	1.91230D 00	1.91230D 00
1.78030D 00	4.59080D 00	1.75670D 00	3.99650D 00	-8.58000D-02	6.91130D 00	1.60890D 00	1.88710D 00	1.88710D 00
2.27890D 00	5.24090D 00	1.73290D 00	4.13070D 00	6.57000D-02	7.09590D 00	1.44640D 00	1.86500D 00	1.86500D 00
2.96820D 00	6.03880D 00	1.71100D 00	4.30430D 00	2.18100D-01	7.35550D 00	1.30040D 00	1.84600D 00	1.84600D 00
3.22680D 00	6.31540D 00	1.70490D 00	4.36510D 00	2.64800D-01	7.44980D 00	1.25900D 00	1.84090D 00	1.84090D 00
3.72860D 00	6.82400D 00	1.69510D 00	4.47700D 00	3.44500D-01	7.62690D 00	1.19220D 00	1.83310D 00	1.83310D 00
5.44960D 00	8.36460D 00	1.67370D 00	4.81070D 00	5.50200D-01	8.17230D 00	1.04050D 00	1.81800D 00	1.81800D 00
1.20330D 01	1.28286D 01	1.64580D 00	5.69510D 00	9.92400D-01	9.68620D 00	8.01700D-01	1.80340D 00	1.80340D 00
4.13220D 01	2.54861D 01	1.63100D 00	7.67880D 00	1.84810D 00	1.31878D 01	5.49100D-01	1.79880D 00	1.79880D 00

Nomenclature:

S	Sommerfeld Number
C	Radial Bearing Clearance
K_{xx}, K_{yy}, etc.	Stiffness Coefficients
W	Bearing Load
ω	Shaft Rotational Speed (radians/sec.)
B_{xx}, B_{yy}, etc.	Damping Coefficients

TABLE 4

EXAMPLE OF INPUT DATA FOR CRITICAL SPEED ANALYSIS

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
1 1								1	1 Speed Group
TEST TRUN	I LUNG	ROTOR WITH	OVERHUNG	DISK AND	RIGID SUPP	ORT ON BAL	BRGS	2	
32 1	18 2	2 4	0					3	
1	3.1496	0.0	2.4724	0.0	2.4724				
2	0.7874	0.0	2.4724	0.0	2.4724				
3	2.4016	0.0	3.1496	0.0	3.1496				
4	0.42913	0.0	3.1496	0.0	3.4803				
5	1.14173	0.0	4.9016	0.0	3.8189				
6	1.14173	0.0	4.9016	0.0	3.8386				
7	0.39764	0.0	3.1496	0.0	3.500				
8	2.224444	0.0	3.1496	0.0	3.1496				
9	0.52756	0.0	3.1496	0.0	3.7087				
10	1.33856	0.0	5.5118	0.0	3.8976				
11	1.33966	0.0	5.5118	0.0	3.9764				
12	0.49606	0.0	3.0866	0.0	3.7795				
13	1.79136	0.0	3.0866	0.0	3.2283				
14	1.00396	0.0	3.9764	0.0	4.0551				
15	1.37406	0.0	3.8583	0.0	3.9370				
16	0.57874	0.0	3.0866	0.0	3.3465			4	32 Segments
17	1.44494	0.0	5.1969	0.0	4.0157				
18	1.44494	0.0	5.1969	0.0	3.9094				
19	2.68504	0.0	3.1496	0.0	3.1496				
20	0.5983	0.0	3.1496	0.0	3.5039				
21	0.74016	0.0	4.8819	0.0	3.8386				
22	0.74016	0.0	4.8819	0.0	3.8189				
23	0.56693	0.0	3.1496	0.0	3.4646				
24	2.76773	0.0	3.1496	0.0	3.1496				
25	1.44493	0.0	4.8819	0.0	3.7008				
26	1.44493	0.0	4.8819	0.0	3.5433				
27	3.06693	0.0	3.1496	0.0	3.1496				
28	0.78740	0.0	2.4724	0.0	2.4724				
29	6.94880	0.0	2.4724	0.0	2.4724				
30	0.22835	0.0	3.3071	0.0	2.440[illegible]				
31	0.68898	0.0	3.3071	0.0	2.4016				
32	0.68898	0.0	1.5748	0.0	1.2323				
1 1	2.8300E-01	2.8508E+[illegible]	3.9520E+04					5	1 Material
1 1	0.6600	1.3640	0.6820	0.0					
2 5	10.208	99.231	48.422	0.0					
3 6	10.780	0.98549	48.081	0.0					
4 7	10.01	98.549	48.081	0.0					
5 10	34.758	698.71	467.17	0.0					
6 11	34.804	654.04	448.42	0.0					
7 13	34.650	697.35	467.17	0.0					
8 17	12.606	126.85	66.154	0.0					
9 18	13.464	127.19	66.154	0.0					
10 19	12.980	128.56	66.836	0.0				6	18 Lumped Masses
11 21	29.238	473.38	244.16	0.0					
12 23	29.414	477.06	244.50	0.0					
13 25	13.476	130.26	67.518	0.0					
14 26	14.234	130.26	67.859	0.0					
15 27	13.728	131.29	68.20	0.0					
16 30	13.882	46.135	23.87	0.0					
17 32	13.948	46.035	23.87	0.0					
18 33	0.66	1.3640	0.682	-0.1575					
1 3	0.0	0.0	0.0	0.0	0.0	0.0		7	2 Static Bearings Default Null Entry
2 29	0.0	0.0	0.0	0.0	0.0	0.0			
2 11	10 22							8	
3 3	3 3							9	
1								10	Synchronous
5 1	4500.0	6500.0						11	
1 1	779870.0	0.0							
2 1	826180.0	0.0							
1 1	782460.0	0.0							
2 1	828520.0	0.0							
1 1	785490.0	0.0						12	2 Bearings Each Speed 5 Speed Points
2 1	831280.0	0.0							
1 1	789020.0	0.0							
2 1	834500.0	0.0							
1 1	793100.0	0.0							
2 0	838260.0	0.0							
1 0	0 0	0 0	0 0					14	
1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789		

the ball bearing is essentially isotropic. Damping is usually negligible. Body forces on the rolling elements alter the contact compliance between each rolling element and the races, therefore the stiffness of the ball bearing is somewhat speed dependent. Thus a ball bearing support can be modelled as an isotropic spring with a speed dependent stiffness and can be completely specified at Level 1 (card 12). Subsequently, at Level 2, it is not necessary to revise dynamic bearing data (IBRG = 0 in card 14). It may also be noted that the static bearing stiffness values are left as zero because they are used only to calculate the bearing load distribution, which is independent of the stiffness values in a two bearing rotor system.

3.2.2 Unbalance Response (IRUN = 2)

To allow safe operation above one or more critical speeds, a popular trend is to damp-mount a ball bearing rotor. A damper, in parallel with a centering spring, is installed around the outer race of the ball bearing. The centering spring should be soft enough to allow the damper to function effectively; at the same time it should be stiff enough to limit eccentricity of the rotor from becoming excessive. The unbalance response analysis will illustrate the use of damp mounting of the ball bearing rotor considered above. The stiffness of the centering spring is chosen to be about 1/4 of that of the ball bearing. For the present purpose, 800,000 lb./in. and 200,000 lb./in. are respectively assumed to be the stiffness values of each ball bearing and of each centering spring.

Before proceeding to calculate unbalance response of the damp-mounted rotor, it is first necessary to determine the critical speeds of the soft-mounted rotor. A Level I analysis is performed for this purpose. Ground-to-rotor stiffness value at each support point is 160,000 lb./in., which is arrived at by adding the compliance values (reciprocals of the stiffness values) of the ball bearing and of the centering spring to get the overall compliance. The input set-up for the Level I analysis of the soft-mounted rotor

is given in Table 5. Those portions of this input deck which are the same as those in Table 4 are omitted to save space. A single speed range of 2000-9000 rpm is used for this study.

An examination of locations of sign reversals of "co-rotational end determinants," which have been determined by the Level I analysis, revealed critical speeds to be near 4350 rpm and 7000 rpm. Unbalance response analysis is therefore set up for two speed ranges; namely 4000-5000 rpm and 6000-8000 rpm. The damper is assumed to be the viscous type and its damping coefficient is assumed to render a rigid rotor critical damping ratio of 0.2 at 4500 rpm. The effective values of ground-to-rotor stiffness and damping coefficient should both be speed dependent according to Section 4.2.4. The degree of speed dependence of the effective stiffness coefficient turns out to be quite small; that of the effective damping coefficient is somewhat more significant but can still be neglected within each speed range. Therefore, the dynamic bearing data for Level II calculation is given a constant isotropic stiffness coefficient and a constant isotropic damping coefficient in each of the two speed ranges. Such an input set-up is given in Table 6; again, only those data lines different from those in the previous examples are shown. A single unit unbalance (1 in.-oz.) is assigned to a different axial location in order to maximize the relevant modal response in each speed range. By setting NCASE = 2 in card 1, a single input set-up is used to study both speed ranges.

3.2.3 Asynchronous Resonance (IRUN = 3)

The asynchronous resonance analysis is quite similar to the critical speed analysis. However, the (asynchronous) resonance condition is searched through for a frequency range at constant shaft speed. Also, backward whirls and/or anisotropy can be considered. A Level I example, as shown in Table 7, will demonstrate the possibility of incorporating the approximate behavior of the fluid film bearing at non-synchronous conditions and an estimation of the torsional resonance frequency of the rotor. In card 1, NCASE is set to 2, even though in both speed groups rotor speed is 8000 rpm. This is done so that the dynamic bearing stiffness

TABLE 5

EXAMPLE OF INPUT DATA FOR LEVEL I ANALYSIS

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
1 0								1	Level I Only
TEST IRUN=	2A LUND	ROTOR UNB	ALANCE RES	PONSE WITH	SOFT-MOUN	TED DAMPER		2	
32 1	18 2	2 4	0					3	
								4-7	Same as Table 4
2 11	18 29							8	
3 3	3 3							9	
1								10	Synchronous
11 1	2000.0	9000.0						11	
1 1 2 0	160003.0 160000.0	0.0 0.0						12	2 Bearing Set Use for All Speeds
1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789		

TABLE 6

EXAMPLE OF INPUT DATA FOR DAMPED UNBALANCED RESPONSE

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
2 1								1	2 Speed Groups
TEST IRUN=	2H LUND	ROTOR UNB	ALANCE RES	PONSE WITH	SOFT-MOUN	TED DAMPER		2	
32 1	18 2	2 4	0					3	
								4-7	Same as Table 4
2 11	18 29							8	
3 3	3 3							9	
1								10	Speed Group No. 1
9 1	4000.0	5000.0						11	9 Speed Points
1 1	160000.0	0.0						12	
2 0	160000.0	0.0							
1								10	Speed Group No. 2
9 1	6000.0	8000.0						11	9 Speed Points
1 1	160000.0	0.0						12	
2 0	160000.0	0.0							
2 0	2 0	0 0	0 0					14	2 Level II Brgs.
1 2	1 1							15	Speed Group No. 1
1.5282D+05	1.0056D+02	0.0	0.0	0.0	0.0	1.5282D+05	1.0056D+02	15A	
2 29	1 1							15	
1.5282D+05	1.0056D+02	0.0	0.0	0.0	0.0	1.5282D+05	1.0056D+02	15A	
1 1								16	
18								16A	
1								16B	
1.0	0.0	0.0	0.0					16C	
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 1							15	Speed Group No. 2
1.4545D+05	9.1544D+01	0.0	0.0	0.0	0.0	1.4545D+05	9.1544D+02	15A	
2 29	1 1							15	
1.4515D+05	9.1544D+01	0.0	0.0	0.0	0.0	1.4545D+05	9.1544D+02	15A	
1								16	
29								16A	
1								16B	
1.0	0.0	0.0	0.0					16C	
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 2	1 0							15	Bearing Default
2 29	1 0								
1 0								16	Excit'n Default
1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789		

TABLE 7

EXAMPLE OF INPUT DATA FOR ASYNCHRONOUS BENDING AND TORSIONAL RESONANCE STUDIES

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
2 0								1	Level I Only
TEST IRUN=	3A LUND	ROTOR WIT	H SIMULATE	D FLUID FI	LM BRGS AN	D TORS ANA	LYSIS	2	
32 1	18 2	3 4	0					3	Torsion & Bending
								4-7	Same As Table 4
2 11	18 29							8	
3 3	3 3							9	
0								10	Speed Group #1 (Asynchr)
1 1	8000.0	8000.0						11	1 Freq. Group
1 1	129500.0	0.0						12	
2 0	153500.0	0.0							
1 7	63.0	73.5						13	7 Freq. Points
0								10	Speed Group #2
1 2	8000.0	8000.0						11	2 Freq. Groups
1 1	984400.0	0.0						12	
2 0	1272000.0	0.0							
1 9	92.8	113.1						13	Freq. Group #1,9 Points
2 5	300.0	350.0						13	Freq. Group #2,5 Points
1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789		

values can be varied according to the center frequency of each frequency range. The approximate non-synchronous stiffness value of the fluid film bearing is determined by a procedure described in a separate report of the present contract [17]. LTYP = 3 in card 3 designates that both bending and torsional analyses are to be performed. The first speed group contains a frequency range to explore the "rigid mode" with a reduced stiffness value which corresponds to the "half speed" behavior of a fluid film bearing. The second speed group contains two frequency ranges; the first of these concerns the relatively stiff near-synchronous behavior of the fluid film bearing, and the second frequency range is intended to locate the torsional resonance frequency.

Bending asynchronous behavior of the rotor is further studied by invoking Level II calculations to establish asynchronous resonance frequencies and the associated mode shapes. LTYP is changed to 2 in card 3 to dedicate the study to bending motions exclusively. The second frequency range in the second speed group is accordingly removed. Because the isotropy approximation is retained, it is not necessary to introduce new dynamic bearing data in Level II. Table 8 shows the input data for calculating asynchronous bending resonances.

3.2.4 Asynchronous Response (IRUN = 4)

Damped asynchronous response is considered with fluid-film bearing data derived from Table 3. Dimensions and lubricant properties are indicated in the beginning of Section 3.2. Load levels of each bearing are based on Level I calculation (see Item 5.0 in Table 13).

This calculation is performed with the same speed-frequency combinations used in the previous analysis (Table 8), which has generated an intermediate data file that is reusable. Therefore, the present calculation begins with Level II by accessing the pre-stored intermediate data file (NCASE = 0 in card 1). Table 9 contains the complete input set-up for this run.

TABLE 8

EXAMPLE OF INPUT DATA FOR ASYNCHRONOUS BENDING RESONANCES

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
2 1								1	Both Levels
TEST IRUN=3,4,5 LUND ROTOR ASYNCHRONUUS BENDING ANALYSIS								2	
32 1	18 2	2 4	0					3	
								4-7	Same as Table 4
2 11	18 29							8	
3 3	3 3							9	
0								10	Freq. Group No. 1
1 1	8000.0	8000.0						11	
1 1 2 0	129500.0 153500.0	0.0 0.0						12	
1 7	63.0	73.5						13	
0								10	Freq. Group No. 2
1 1	8000.0	8000.0						11	
1 1 2 0	984400.0 1272000.0	0.0 0.0						12	
1 9	92.8	113.1						13	
3 1	0 0	0 0	0 0					14	
1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789		

TABLE 9

EXAMPLE OF INPUT DATA FOR DAMPED ASYNCHRONOUS RESPONSE
TWO NATURAL MODES SOUGHT WITH USER-SPECIFIED TRIAL CONDITIONS

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
0								1	Level II Only
4 1	2 0	0 -2	0 -1					14	2 M.Iter. Supply SV
1 2	1 1							15	Freq.Gr.1; 7 Freq Pts
6.6956E+05	9.2412E+02	4.5395E+05	2.2607E+02	3.0312E+04	2.2607E+03	1.7531E+05	1.5177E+02	15A	Data-Bearing
2 29	1 1							15	
8.9993E+05	1.1572E+03	5.6968E+05	2.7264E+02	4.9946E+04	2.7264E+02	2.1097E+05	1.7124E+02	15A	Data-Bearing No. 2
1 1								16	
18								16A	
1								16B	Ex. at St. No. 18
1.0	0.0	0.0	0.0					16C	
1 2	1 0							15	6 Sets - Data Default
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
6.8114D+01		1.3569D-00						17	SV Mode No. 1 (Conv.)
6.8114D+01		1.3569D-00							SV Mode No. 2 (Repeat)
1 2	1 1							15	Freq.Gr.2; 9 Freq.Pts
6.6956E+05	9.2412E+02	4.5395E+05	2.2607E+03	3.0312E+04	2.2607E+03	1.7531E+05	1.5177E+02	15A	
2 29	1 1							15	
8.9993E+05	1.1572E+03	5.6968E+05	2.7264E+02	4.9946E+04	2.7264E+03	2.1097E+05	1.7124E+02	15A	
1 1								16	
29								16A	
1								16B	Ex. at St. #29
1.0	0.0	0.0	0.0					16C	
1 2	1 0							15	8 Sets - Data Default
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
1 2	1 0							15	
2 29	1 0								
1 0								16	
9.75726D+01		2.25614D-00						17	SV Mode No. 1 (Conv.)
1.02660D+02		4.00150D-00							SV Mode No. 2 (Conv.)
1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789		

Two natural modes are sought in each frequency group and a record of the iteration progression is requested by setting IGEN = -2 in card 14. Starting conditions for seeking the natural modes are furnished by the user as indicated by IBEG = -1. The starting conditions are based on results of the stability analysis which was performed first (see Section 3.2.5). That analysis established that there is one natural mode in the first frequency group; however, because two sets of starting conditions are needed, the same set of starting conditions is repeated for a second time. There are two natural modes in the second frequency group.

Excitation in the present example is spatially fixed (ITYPE = 1 in card 14). It is represented by a unit force (1 lb.) at station 18 for the first frequency group but shifted to station 29 for the second frequency group (see card 16A).

3.2.5 Stability Analysis (IRUN = 5)

Example for the stability analysis deals with precisely the same rotor system as the above example except that the excitation system is not needed in the stability analysis. The stability analysis actually is included in the damped asynchronous response analysis if the option to do so is specified (IGEN $\neq$ 0 in card 14). The stability analysis is a shorter run because response calculation to a specific excitation is not performed. In the particular example, a relatively highly damped mode in the second frequency group required repeated trials before a fully converged solution is established; therefore, the stability analysis was actually completed first, then the damped asynchronous response was executed with starting conditions which are the results of the stability analysis.

Again, because the same speed-frequency combinations as for the asynchronous resonance calculation are used, Level I calculations are bypassed by accessing the pre-stored intermediate data file.

The first trial for the stability analysis was run with the input set-up listed in Table 10. Printing of Level II bearing data is suppressed with IPRI = -1. Two natural modes are sought in each frequency group (which is associated with one of two speed groups) and a record of the iteration is requested by setting IGEN = -2 in card 14. Since no specific clue is available for starting the iterative processes, the default for the internal starting logic is accepted with IBEG = 0 in card 14. The rest of the input list concerns Level II bearing data for each speed group.

The first trial yielded one damped natural mode in each frequency group. According to the pattern of the iteration progression, it was surmised that there is only one mode in the first frequency group and that the second frequency group may have two natural modes but the internal limits of the iteration process had prevented convergence to the second mode. Therefore, a second trial was carried out with the input listing of Table 11. User's prerogative to specify the starting values is requested with IBEG = -1 in card 14. The starting values of frequency and damping are furnished as card 17 for each mode in each frequency group. In the first frequency group, because a second mode is not expected, the same starting values are repeated. In the second frequency group, the starting values for the second mode (of the second frequency group) are those which would be continued by the iterative process if it had not been terminated. A full discussion on the record of iteration is provided in Section 3.4.5. The second trial still did not reach full convergence; however, the general trend previously noted was reaffirmed and the need for an "accelerated" change in the damping value was quite apparent. Therefore, a third trial with the starting damping value somewhat "accelerated" beyond the last step of iteration is used as shown in Table 12. Full convergence was indeed realized in the third trial.

TABLE 5

EXAMPLE OF INPUT DATA FOR STABILITY ANALYSIS

FIRST TRIAL WITH INTERNALLY DETERMINED STARTING VALUES

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
0								1	Level II Only
5 1	2 -1	0 2	0 0					14	No Print Brg. Data; SV
1 2	1 1							15	Freq.Group #1; 7 Freq. Points
6.6956E+05	9.2412E+02	4.5395E+05	2.2607E+04	3.0312E+04	2.2607E+02	1.7531E+05	1.5177E+02	15A	Data-Bearing #1
2 29	1 1							15	
8.9993E+05	1.1572E+03	5.6968E+05	2.7264E+04	4.9946E+04	2.7264E+02	2.1097E+05	1.7124E+02	15A	Data-Bearing #2
1 2	1 0							15	6 Sets of Data Default
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 1							15	Freq.Group #2; 9 Freq. Points
6.6956E+05	9.2412E+02	4.5395E+05	2.2607E+04	3.0312E+04	2.2607E+02	1.7531E+05	1.5177E+02	15A	
2 29	1 1							15	
8.9993E+05	1.1572E+03	5.6968E+05	2.7264E+04	4.9946E+04	2.7264E+02	2.1097E+05	1.7124E+02	15A	
1 2	1 0							15	8 Sets of Data Default
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	1234567890	123456789		

TABLE 11

EXAMPLE OF INPUT DATA FOR STABILITY ANALYSIS

SECOND TRIAL WITH USER-FURNISHED STARTING VALUES

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
0								1	Level II Only
5 1	2 -1	0 -2	0 -1					14	User to Furnish St. Values
1 2	1 1							15	Freq. Group #1
6.6956E+05	9.2412E+02	4.5395E+05	2.2607E+02	3.0312E+04	2.2607E+02	1.7531E+05	1.5177E+02	15A	
2 29	1 1							15	
8.9993E+05	1.1572E+03	5.6968E+05	2.7264E+02	4.9946E+04	2.7264E+02	2.1097E+05	1.7124E+02	15A	
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
6.8114D+01		1.3569D-00						17	St.Val.Mode #1 (Conv.)
6.8114D+01		1.3569D-00						17	St.Val.Mode #2 (Repeat)
1 2	1 1							15	Freq. Group #2
6.6956E+05	9.2412E+02	4.5395E+05	2.2607E+02	3.0312E+04	2.2607E+02	1.7531E+05	1.5177E+02	15A	
2 29	1 1							15	
8.9993E+05	1.1572E+03	5.6968E+05	2.7264E+02	4.9946E+04	2.7264E+02	2.1097E+05	1.7124E+02	15A	
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
9.7573D+01		2.2559D-00						17	St.Val.Mode #1 (Conv.)
1.0240D+02		7.6460D-01						17	St.Val.Mode #2 (Extrap.)
1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789		

TABLE 12

EXAMPLE OF INPUT DATA FOR STABILITY ANALYSIS
THIRD TRIAL WITH USER-FURNISHED STARTING VALUES

1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789	Card Number	Remarks
0								1	Level II Only
5 1	2 -1	0 -2	0 -1					14	User to Furnish St. Values
1 2	1 1							15	Freq. Group No. 1
6.6956E+05	9.2412E+02	4.5395E+05	2.2607E+02	3.0312E+04	2.2607E+02	1.7531E+05	1.5177E+02	15A	
2 29	1 1							15	
8.9993E+05	1.1572E+03	5.6968E+05	2.7264E+02	4.9946E+04	2.7264E+02	2.1097E+05	1.7124E+02	15A	
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
6.8114D+01		1.3569D-00						17	St.Val.Mode No. 1 (Conv.)
6.8114D+01		1.3569D-00						17	St.Val.Mode No. 2 (Repeat)
1 2	1 1							15	Freq. Group No. 2
6.6956E+05	9.. 412E+02	4.5395E+05	2.2607E+02	3.0312E+04	2.2607E+02	1.7531E+05	1.5177E+02		
2 29	1 1							15	
8.9993E+05	1.1572E+03	5.6968E+05	2.7264E+02	4.9946E+04	2.7264E+02	2.1097E+05	1.7124E+02	15A	
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
1 2	1 0							15	
2 29	1 0								
9.7573D+01		2.2559D-00						17	St.Val.Mode No. 1 (Conv.)
1.0240D+02		4.0000D-00						17	St.Val.Mode No. 2 (Conv.)
1 1234567890	2 1234567890	3 1234567890	4 1234567890	5 1234567890	6 1234567890	7 1234567890	123456789		

3.3 Output Description

The output of the computer program consists of printouts of the input data with appropriate headings and of dynamic responses. Intermediate results of computations can also be printed out -- these printouts are controlled by the diagnostic parameters specified in the input setup. In the following, descriptions of the output emphasize its overall organization and contain supplementary definitions for the terminology appearing in the headings. Item numbers on the following pages refer to the "Item Number" column on the computer printouts given in Tables 13 through 20.

A. SUMMARY OF ROTOR MODEL

Items 1.0 through 4.0 summarize the rotor model input data. They are formulated under self-explanatory headings to allow convenient visual checking. The program checks logical aspects of the corresponding input setup. If an error is detected, an error message is written out and further execution is halted.

Item 5.0 gives static journal bearing load distributions, rotor weight, and the axial location of its center of gravity.

Item 1.0 Title and Summary

The title is printed out in the first line. It is followed by the summary of the input data for the rotor model; namely,

Shaft Segment:

Number of consecutive shaft segments of uniform cross section in the rotor model. (The numbering of nodal stations starts from the beginning of the first segment to the end of the last segment).

Shaft Materials:

Number of different sets of material properties, including weight density, Young's modulus, and shear modulus.

Lumped Inertia Stations:

Number of stations where concentrated inertia properties (in addition to the distributed mass in the shaft elements) have been assigned.

Bearings:

Number of stations where radial and/or angular stiffness may be assigned.

Item 2.0 Shaft Dimensions

They are tabulated in six columns:

Element No.: index number in the tabulation loop.

Length (in.): length of the individual shaft segment.

I.D. (in.): inner diameter for calculating flexural rigidity.

O.D. (in.): outer diameter for calculating flexural rigidity.

M.I.D. (in.): "mass inner diameter" used for calculating mass distribution.

M.O.D. (in.): "mass outer diameter" used for calculating mass distribution.

Item 3.0 Shaft Materials

There are four columns in the table:

Starting Node: segment number at which the present material data set begins to apply until the next data set appears.

Density (lbs./cu.-in.): weight density.

Young's Mod (psi): Young's modulus.

Shear Mod (psi): shear modulus.

3.1 Error Message

If the first line in the above tabulation does not refer to the first shaft element, an error message will be written out and further execution will be halted.

Item 4.0 Lumped Inertias

Following the heading, there are five columns:

Nodal Station: nodal station number at which the present set of lumped inertias is assigned.

Weight (lb.): weight of an additional rigid body attached to the designated nodal station.

Polar Inertia: ($lb.-in.^2$) weight polar moment of inertia of the attached rigid body.

Trans. Inertia: ($lb.-in.^2$) weight transverse moment of inertia of the attached rigid body.

CG Offset (in.): axial location of the center of gravity of the attached rigid body relative to the nodal station.

4.1 Error Message

If a nodal station number in the above tabulation exceeds the available range, an error message will be written out and further execution will be halted.

Item 5.0 Bearing Load Distribution

Static bearing loads due to gravity and misalignment are calculated according to static bearing data entered. This group of output is bypassed if no bearing is indicated (LBEA = 0 in the input). If only one bearing is detected, static equilibrium will not be possible, thus the program will write an error message and stop execution. If zero radial bearing stiffness is assigned to a designated bearing station in either direction, a default value of 0.1 lbs./in. will be assigned and an "attention message" will be printed out.

Numerical tabulation contains:

Brg. No.:	bearing index number.
St. No.:	station number assigned to each bearing.
Location (in.):	axial location of the bearing.
Vertical Plane:	misalignment, load, and moment in the vertical plane.
Horizontal Plane:	misalignment, load, and moment in the horizontal plane.

The tabulation is followed by the total weight and the axial location of the center of gravity.

B. SUMMARY OF DYNAMIC DATA (LEVEL I)

Item 6.0 Number of Speed Groups

This statement gives the number of speed groups treated in the particular calculations as specified by the parameter NCASE of input card 1. It also indicates the number of times Items 8.0 through 13.0 will appear.

Item 7.0 Type of Vibration

This statement identifies whether the computation pertains to torsional vibration, bending vibration, or both as determined by the control parameter, LTYP, given by input card 3.

Item 8.0 Speed Group Number

This heading applies to Items 9.0 through 13.0 to follow.

Item 9.0 Excitation Frequencies

Frequency group number is indicated and the frequencies of the group in Hz are printed out. This item is omitted for synchronous cases as dictated by LRUN = 1 in input card 10.

C. LEVEL I DYNAMIC RESULTS

Item 10.0 Torsional Response Data

If the value of 1 is assigned to the parameter LTYP in input card 3, calculations pertaining to torsional vibrations only will be performed. The following group of output will be printed for each of the excitation frequencies one at a time. This group of output will be suppressed if the value of 3 is assigned to LTYP to treat both torsional and bending vibrations; instead, the torsional results will then be described in terms of the "Holzer" stiffness (see Item 12.0).

10.1

The dynamic torsional response distribution for all nodal stations due to a unit twist moment applied at the end of the rotor is listed. A new page is used for listing these data. In the event that a no-twist (null torsional displacement) condition is calculated, an alert message is written, saying

"END NODE IS STATIONARY AT #.####E ## HZ
TORS STIFF BELOW IS END TORQUE PER UNIT TWIST OF FIRST NODE."

Captions preceding the tabulation contain:

Frequency (Hz):	Frequency of torsional excitation.
Node No.:	The station number at which the exciting torque is applied. The last station number is printed here.
Location (in.):	Axial location of the above node.
Tors. Stiff: (in.-lb./rad.)	The dynamic torsional stiffness at the excitation point. In the event that the above-described alert message had been written out, the definition given there applies.

Response at each node is tabulated as follows:

Node:	Station number at which the response is indicated.
Location (in.):	Axial location of station.

Relative Twist:	Normalized torsional displacement. In this listing the peak torsional displacement is used as the scale for normalization.

The actual twist per unit excitation torque is:

$$\frac{\text{RELATIVE TWIST (present station)}}{\text{RELATIVE TWIST (end)} \cdot \text{TORS. STIFF.}}$$

If the rotor end is a stationary point as indicated by the alert message then the above formula is replaced by:

$$\frac{\text{RELATIVE TWIST (present station)}}{\text{RELATIVE TWIST (first station)} \cdot \text{TORS. STIFF.}}$$

Unit Twist Rate: (rad./in.-lb.)	Local twist rate per unit excitation torque (applied at the end of the rotor).

10.2

The dynamic torsional response distribution at selected response stations due to a unit twist moment applied at each selected excitation station is listed.

The selected stations are all those identified in input card 8. Torsional excitation stations are those with the corresponding value of "Excitation Type Index", K8(I) = 2, 5, 6 or 8 in input card 9. A new page is used for each excitation station. Three special conditions can exist to cause a corresponding alert message to be written. If the excitation station is found to be stationary, either rotor half (aft or fore) must also be stationary. This condition is recognized as (1) fore-segment resonance or (2) aft-segment resonance. The alert message is written out, including the required special definition for "Tors. Stiff." For condition (3), torsional resonance of the system, the alert message is also written out.

Appropriate captions precede the listing of response distribution. A four-column response tabulation is the same as previously described except that only the selected response stations are included. The relative twist is normalized at the excitation station except for the three special cases discussed above. For fore-segment resonance, the twist at the first station is taken to be unity. For aft-segment resonance, the twist at the rotor end is taken to be unity. For torsional resonance, the peak twist is taken to be unity.

Item 11.0 Bending End Determinants

"Myklestad" determinants or end determinants of the bending problem are listed under appropriate headings of the frequency range in concern. For the synchronous case as designated by LRUN = 1 in input card 10, each frequency (Hz) should be precisely 1/60 times speed (RPM). Co-rotational and counter-rotational determinants are separately listed. This listing is omitted if LTYP = 1 in input card 3; i.e., if torsional response only is calculated.

Examination of this listing for a change of sign of the end determinant allows one to locate the critical speed (for the synchronous problem) or the resonant frequency (for the asynchronous problem) approximately without performing Level II computations.

Item 12.0 Torsional End Stiffness

"Holzer" stiffness or end stiffness of the torsional problem is listed alongside the frequency of excitation. This output is activated only when the parameter LTYP in input card 3 is given the value 3 to designate both torsional and bending calculations and the parameter LRUN in input card 10 is set to 0 for asynchronous calculations.

Examinations of this listing for a change of sign of the torsional end stiffness allows one to locate the torsional resonance frequency. No other provision is made to determine the torsional resonance frequency in this program.

Item 13.0 Level I Dynamic Bearing Data

Dynamic bearing data, as used in Level I (isotropic representation), are tabulated under headings of bearing number, station number, location measured from left end of rotor (axial distance), radial stiffness and angular stiffness. One set is given for each speed. This output is omitted if only torsional calculations are performed (LTYP = 1 in input card 3) or if so designated by the user (LPRI = -1 in input card 3).

D. LEVEL II DATA

Item 14.0 Calculation Summary

The following messages are printed out:

- an alphanumeric string giving the job identification and job title as entered in input card 2;
- Level II run mode according to IRUN (see descriptions under input card 14);
- type of excitation (for IRUN = 4 only) according to ITYPE (input card 14); and
- under respective headings, station numbers at which torsional and/or bending excitations are applied according to input cards 8 and 9 are listed.

These messages together with Items 6.0 through 9.0 describe the overall content of the dynamic results.

Item 15.0 Tabulation of Level I Matrices

Matrices are printed out for each speed-frequency combination if the parameter IDIAG of input card 14 is set to the value 2.

15.1

If torsional calculation is performed (LTYP = 1 or 3 in input card 3) the torsional mobility and impedance matrices are printed out in succession. The torsional matrices are speed independent and are thus printed out for distinct frequencies only.

15.2

If bending calculations are performed (LTYP = 2 or 3 in input card 3), the following bending matrices are listed under their corresponding headings in sequence.

- Level I bending mobility matrix in the Cartesian representation.
- Level I bending impedance matrix in the Cartesian representation (omitted if torsional calculation is not performed because Item 15.3 would contain the bending impedance only).

15.3

Level I impedance summary contains the following information:

- job identification
- job title
- speed in revolutions per second
- excitation frequency in Hz, and
- the total number of degrees of freedom

The consolidated bending-torsional impedance matrix is printed out on a new page. The bending impedance matrix is in the Cartesian representation and is in the upper left corner. The torsional impedance matrix is in the lower right corner.

Item 16.0 Level II Bearing Data

This set of output is omitted if both LBEA (input card 3) and IBRG (input card 14) are zero.

Under captions of speed (revolutions per minute) and excitation frequency (Hz), bearing data are tabulated under appropriate headings. A separate line is written for each bearing. If the bearing is capable of rendering both angular and translational restraints, the angular characteristics of the bearing are written after its translational characteristics. The headings of the table are self-explanatory. Each line contains the following information:

- bearing index number (line number in table).
- station number of bearing in rotor model.
- radial or angular bearing.
- eight matrix coefficients of the bearing,

  K = stiffness coefficient

  B = damping cofffficient

The units of K are in (lb./in.) for a radial bearing and in units of (in.-lb./rad.) for an angular bearing. The units of B are those of K times seconds.

Item 17.0 Level II Bending Impedance Data

Impedance matrices including Level II bearing data are printed out if the parameter IDIAG is set to 2 in card 14.

17.1

Bending impedance matrix with Level II bearing data is printed in the Cartesian representation provided IBRG (input card 14) is not zero.

17.2

Bending impedance matrix with Level II bearing data is printed in rotating coordinates.

E. LEVEL II OUTPUTS -- RESULTS OF DYNAMIC BENDING ANALYSIS

This set of output covers 5 different modes of dynamic analysis corresponding to IRUN = 1 to 5 (input card 14). The 5 modes are:

- (i) Critical speed (IRUN = 1)
- (ii) Unbalance response (IRUN = 2)
- (iii) Asynchronous resonance (IRUN = 3).
- (iv) Asynchronous response (IRUN = 4).
- (v) Stability analysis (IRUN = 5).

(i) and (iii) are for undamped systems while (ii), (iv), and (v) are for damped systems. A description of each case is given in the following.

Item 18.0 Critical Speed Calculation

Following the heading "Critical Speed Determinants," the determinants of the impedance matrices are tabulated versus the corresponding frequencies. The changes of sign signify the existence of natural frequencies.

For critical speed calculation, only co-rotational (forward whirl) is considered. The number of co-rotational roots found is indicated.

The tabulation of mode shape consists of four columns. The first column gives the natural frequency. The second column shows the axial location of each excitation station. The third and fourth columns give the corresponding deflection and slope (as appropriate for the station, see Section 3.1). These are normalized with respect to the maximum deflection and slope.

Item 19.0 Unbalance Response

19.1

The excitation data and the corresponding response orbit are tabulated at each rotor speed. Since this is a synchronous case, speed (rpm) = 60 x frequency (Hz). The excitation data include the index number, the station number, the type of excitation (force or moment), and the orientation. The unbalance excitation is always a forward whirl; however, at each excitation point, there may be a distinct phase angle based on a reference point on the shaft which can be chosen by the user.

The response orbit at each station is, in general, of elliptical shape. The response ellipses can well be described by four spatial parameters: major and minor radii, inclination angle, and one time-related parameter (the phase reference angle). In the output a quantity, the ellipticity, which can be derived from the first two is also provided.

The major and minor radii set the size of the ellipse. The inclination angle is the angle between the major axis and the horizontal axis. The ellipticity is defined as the ratio between the difference of the two radii (major and minor) and the sum of them; if the ratio is greater than one, its reciprocal is used. Hence, a zero ellipticity implies a circular orbit while a unit ellipticity defines a straight line. The time-related phase reference angle at the station is measured with respect to the time reference point chosen by the user. This angle is that between the axial plane passing through both the z axis and reference point and the axial plane passing through both the z axis and displaced center of the rotor. The angle is positive when the displaced center of the rotor leads the reference point.

19.2

Iteration record of the complex eigenvalue calculation is printed out if the parameter IGEN of card 14 is a negative integer. The absolute value of IGEN represents the number of damped natural modes to be sought in each speed group, and its negative sign requests that the record of iteration be included in the output.

The iteration record consists of nine columns. The first column, under the heading "I" counts the number of iteration cycles which have been executed. The next two columns are respectively current estimates of the natural frequency (Hz) and the system damping (lb.-sec./in.). For the unbalance response problem, the synchronous constraint is imposed; therefore, the natural frequency is simply (1/60) of the damped critical speed. Modal mass and critical damping ratio occupy the next two

columns, they are defined in Section 6.2. Estimated corrections of the natural frequency and the system damping are indicated in columns 6 and 7. The computer program restricts the iterative process to a confined frequency range; therefore, actual change of the natural frequency for the next cycle of iteration may be only a fraction of the estimated correction; when this happens, the change of damping is proportionally reduced. Under the heading "Residue" the sum of squares of real and imaginary parts of the residue is listed. "D Check" heads up the "consistency parameter" given by Equation (6.18), it is used to achieve the correct factorization operation if another eigenvalue in the same frequency range is to be sought.

While the internal logic of the computer program to verify convergence is quite reliable, premature termination of the iterative process can occur. Display of the iteration record allows the user to decide whether or not an additional "search" effort is warranted. Typically, if a trend of monotonic decrease of "Residue" is shown while the iterative process is interrupted by internally set limits, another trial should be made with either or both of the following options:

- Shift the frequency range if one of the frequency bounds is repeatedly reached.
- Use user furnished starting values which are extrapolated from the current iteration record.

The iteration record also allows the user to verify if all the necessary convergence conditions are satisfied; i.e.,

- The value of "Residue" should be reducing by larger amounts upon approaching convergence.
- Estimated corrections of frequency and damping are small in comparison with their current values.
- The modal mass is approaching a positive limiting value.
- "D Check" is becoming stabilized.

The computer program presently tests the first two conditions.

19.3

The damped natural mode shape, accompanied by the listing of speed, frequency, and critical damping ratio, is tabulated in the same manner as the response orbit and is printed at each frequency point for every mode found in the designated speed range. This calculation may be suppressed if the parameter IGEN in input card 14 is set to zero.

The resonant response including excitation data is also tabulated in the same way as the response orbit. For a non-conservative system (e.g., a system with significant damping) the resonant response of the rotor can be quite different from its corresponding natural mode shape.

Item 20.0 Asynchronous Resonance

The set of output is similar to that of critical speed calculations, except, in addition to co-rotational modes, counter-rotational modes are printed out. Also, the rotor speed is different from the excitation frequency.

Item 21.0 Asynchronous Response

The output of this case is similar to that of unbalance response except for the non-synchronism; namely, the rotor speed and the excitation frequency are different from each other and the excitation may be either forward and backward whirl components or vertical and horizontal components depending on the value assigned to the parameter ITYPE in input card 14.

21.1

Excitation data and response orbits at each combination of rotor speed and excitation frequency are printed out.

21.2

Iteration record of the complex eigenvalue calculation is printed out if the parameter IGEN of card 14 is a negative integer. Remarks under 19.2 are applicable except that the computation is performed at a fixed shaft speed while the sought natural frequency has no direct relationship with the shaft speed.

21.3

Results for the damped, asynchronous natural modes in each frequency group and resonant response are printed out. Again this calculation would be suppressed if IGEN = 0 on input card 14.

Item 22.0 Stability Analysis

The output of stability analysis is essentially that portion of Item 21.3 pertaining to the damped, asynchronous, natural modes. The iteration record described under 21.2 is also available in a stability analysis.

3.4 Output Samples

The output samples have been taken from computer runs made for the rotor defined by the data in Section 3.2. Each output sample corresponds to an input sample previously given.

3.4.1 Critical Speed (IRUN = 1)

The results of the critical speed analysis are listed in Table 13. Items 1 - 5 relate to various aspects of rotor data under self-explanatory headings. Items 6 - 13 summarize Level I results. The column of co-rotational end determinants shows a change of sign near 5920 rpm which should be the location of a critical speed. Although a change of sign is also indicated by the column of counter-rotational determinants, the counter-rotational motion is not excitable by rotor unbalance unless there is significant anisotropy in the rotor system. Item 14.0 is mainly an identification table. Item 16.0 lists the prevailing bearing data used in Level II analysis in the eight-coefficient format. In this case, the data of Level I bearings as described by Item 13.0 are retained. Item 18.0 summarizes the results associated with the established critical speeds. First, the determinants of the system represented by the selected action stations are listed alongside the speed. Then the mode shape, co-rotational in this case, is given in the normalized representation. The amplitude distribution is typical of a "flexible translational" mode. Peak amplitude is near mid-span. Motion at each of the two bearing stations is less than 1/5 of the peak amplitude. Symmetry about the mid-span is nearly preserved.

3.4.2 Unbalance Response (IRUN = 2)

A preliminary run limited to Level I (KRUN = 0 in input card 1) yielded results highlighted in Table 14. The important information is contained in the list of co-rotational determinants (Item 11.0). Reversal of sign is seen between 4243 rpm and 4931 rpm and also between 6662 rpm and 7743 rpm. Simple interplation yields estimated critical speeds of 4350 rpm and 7000 rpm. They are expected to be respectively the first translational and angular modes.

TABLE 13

COMPUTER OUTPUT FOR CRITICAL SPEED ANALYSIS

(From Input Data of Table 4)

5 Sheets

	Item Number
LUND ROTOR WITH OVERHUNG DISK AND RIGID SUPPORT ON BALL BRGS SHAFT SEGMENTS = 32 SHAFT MATERIALS = 1 LUMPED INERTIA STATIONS = 18 BEARINGS = 2	1.0
SHAFT DIMENSIONS	2.0

SEGMENT NO	LENGTH (IN)	I D (IN)	O.D. (IN)	M.I.D. (IN)	M.O D (IN)
1	3.1496	0.0000	2.4724	0.0000	2.4724
2	0.7874	0.0000	2.4724	0.0000	2.4724
3	2.4016	0.0000	3.1496	0.0000	3.1496
4	0.4291	0.0000	3.1496	0.0000	3.4803
5	1.1417	0.0000	4.9016	0.0000	3.8189
6	1.1417	0.0000	4.9016	0.0000	3.8386
7	0.3976	0.0000	3.1496	0.0000	3.5000
8	2.2244	0.0000	3.1496	0.0000	3.1496
9	0.5276	0.0000	3.1496	0.0000	3.7087
10	1.3387	0.0000	5.5118	0.0000	3.8976
11	1.3387	0.0000	5.5118	0.0000	3.9764
12	0.4961	0.0000	3.0866	0.0000	3.7795
13	1.7914	0.0000	3.0866	0.0000	3.2283
14	1.0040	0.0000	3.9764	0.0000	4.0551
15	1.3741	0.0000	3.8583	0.0000	3.9370
16	0.5787	0.0000	3.0866	0.0000	3.3465
17	1.4449	0.0000	5.1969	0.0000	4.0157
18	1.4449	0.0000	5.1969	0.0000	3.9094
19	2.6850	0.0000	3.1496	0.0000	3.1496
20	0.5983	0.0000	3.1496	0.0000	3.5039
21	0.7402	0.0000	4.8819	0.0000	3.8386
22	0.7402	0.0000	4.8819	0.0000	3.8189
23	0.5669	0.0000	3.1496	0.0000	3.4646
24	2.7677	0.0000	3.1496	0.0000	3.1496
25	1.4449	0.0000	4.8819	0.0000	3.7008
26	1.4449	0.0000	4.8819	0.0000	3.5433
27	3.0669	0.0000	3.1496	0.0000	3.1496
28	0.7874	0.0000	2.4724	0.0000	2.4724
29	6.9482	0.0000	2.4724	0.0000	2.4724
30	0.7233	0.0000	3.3071	0.0000	2.4409
31	0.6890	0.0000	3.3071	0.0000	2.4016
32	0.6890	0.0000	1.5748	0.0000	1.7323

TABLE 13 (CONT.)

Item Number

SHAFT MATERIALS — 3.0

STARTING NODE	DENSITY (LBS/CU-IN)	YOUNG'S MOD (PSI)	SHEAR MOD (PSI)
1	0.2830	2.8508E 07	8.9520E 06

LUMPED INERTIAS — 4.0

NODAL STATION	WEIGHT (LB)	POLAR INERTIA (LB-SQ IN)	TRANS INERTIA (LB-SQ IN)	CG OFF-SET (IN)
1	0.6600	1.3640	0 6820	0.0000
5	10.2080	99.2310	48.4220	0.0000
6	10.7800	0.9855	48.0810	0.0000
7	10.0100	98.5490	48 0810	0 0000
10	34.9580	698.7100	467.1700	0 0000
11	34.8040	654.0400	448.4200	0.0000
12	34.6500	697.3500	467.1700	0.0000
17	12.6060	126.8500	66.1540	0.0000
18	13 4640	127.1900	66.1540	0.0000
19	12.9800	128 5600	66.8360	0.0000
21	29.2380	473.3800	244.1600	0.0000
23	29.4140	477.0600	244.5000	0.0000
25	13.3760	130.2600	67.5180	0.0000
26	14.2340	130.2600	67.8590	0.0000
27	13.7280	131.2900	68.2000	0.0000
30	13.8820	46 1350	23.8700	0.0000
32	13.9480	46.0350	23.8700	0.0000
33	0.6600	1.3640	0 6820	-0 1575

5.0

BRG NO.	ST NO.	LOCATION (IN)	**********VERTICAL PLANE********** MISALGN(IN)	LOAD(LB)	MOM(IN-LB)	**********HORIZONTAL PLANE********** MISALGN(IN)	LOAD(LB)	MOM(IN-LB)
1	2	3.1496	0.000000	187.9521	0.0003	0.000000	0.0000	0 0000
2	29	37.8547	0.000000	225.3712	0.0000	0 000000	0 0000	0.0000

TOTAL WEIGHT = 4.1332E 02 (LB)
LOCATION OF C.G. = 2.2073E 01 (IN)

SHEET 2 OF 5

TABLE 13 (CONT.)

Content	Item Number
NUMBER OF SPEED GROUPS = 1	6.0
DYNAMIC RESPONSE OF THE BENDING MODE	7.0
SPEED GROUP NUMBER 1	8.0

SPEED (RPM)	FREQUENCY (HZ)	END DETERMINANTS CO-ROTATIONAL	CTR-ROTATIONAL
4 5000E 03	7 5000E 01	2 486406E 14	2 010496E 14
4 9333E 03	8 2222E 01	1.677736E 14	1.204599E 14
5 4083E 03	9.0139E 01	7 932025E 13	3 563824E 13
5 9291E 03	9.8818E 01	-1 468409E 13	-4.998437E 13
6 5000E 03	1.0833E 02	-1.104602E 14	-1 311220E 14

Item Number 11.0

BRG NO	1		2		3		4	
STN NO	2		29					
LOC (IN)	3.1496E 00		3.7855E 01					
SPEED (RPM)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)
4 5000E 03	7.7987E 05	0 0000E-01	8 2618E 05	0 0000E-01				
4.9333E 03	7 8246E 05	0 0000E-01	8 2852E 05	0 0000E-01				
5 4083E 03	7 8549E 05	0.0000E-01	8 3128E 05	0 0000E-01				
5 9291E 03	7 8902E 05	0 0000E-01	8.3450E 05	0 0000E-01				
6 5000E 03	7.9310E 05	0.0000E-01	8 3826E 05	0 0000E-01				

Item Number 13.0

SHEET 3 OF 5

TABLE 13 (CONT.)

Item Number: 14.0

```
*******************************CALCULATION SUMMARY******************************

TEST IRUN=1     LUND ROTOR WITH OVERHUNG DISK AND RIGID SUPPORT ON BALL BRGS

CRITICAL SPEED CALCULATION

BENDING EXCITATION STATIONS
   2    11    18    29
```

Item Number: 16.0

```
LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 4 5000E 03 RPM      FREQUENCY = 7 5000E 01 HZ

BEARING  **** TYPE ****  ******** XX ********   ******** XY ********   ******** YX ********   ******** YY ********
NO. STN  RADIAL ANGULAR      K           C           K           C           K           C           K           C
 1   2     *             7 7987E 05 0 0000E-01 0 0000E-01 0 0000E-01 0.0000E-01 0 0000E-01 7 7987E 05 0 0000E-01
 2  29     *             8.2618E 05 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 8 2618E 05 0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 4 9333E 03 RPM      FREQUENCY = 8 2222E 01 HZ

BEARING  **** TYPE ****  ******** XX ********   ******** XY ********   ******** YX ********   ******** YY ********
NO. STN  RADIAL ANGULAR      K           C           K           C           K           C           K           C
 1   2     *             7.8246E 05 0.0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 7 8246E 05 0 0000E-01
 2  29     *             8.2852E 05 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 8 2852E 05 0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 5.4083E 03 RPM      FREQUENCY = 9.0139E 01 HZ

BEARING  **** TYPE ****  ******** XX ********   ******** XY ********   ******** YX ********   ******** YY ********
NO  STN  RADIAL ANGULAR      K           C           K           C           K           C           K           C
 1   2     *             7 8549E 05 0.0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 7 8549E 05 0 0000E-01
 2  29     *             8 3128E 05 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 8 3128E 05 0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 5 9291E 03 RPM      FREQUENCY = 9 8818E 01 HZ

BEARING  **** TYPE ****  ******** XX ********   ******** XY ********   ******** YX ********   ******** YY ********
NO  STN  RADIAL ANGULAR      K           C           K           C           K           C           K           C
 1   2     *             7 8902E 05 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 7 8902E 05 0 0000E-01
 2  29     *             8.3450E 05 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 8 3450E 05 0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 6 5000E 03 RPM      FREQUENCY = 1 0833E 02 HZ

BEARING  **** TYPE ****  ******** XX ********   ******** XY ********   ******** YX ********   ******** YY ********
NO  STN  RADIAL ANGULAR      K           C           K           C           K           C           K           C
 1   2     *             7 9310E 05 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 7 9310E 05 0 0000E-01
 2  29     *             8 3826E 05 0 0000E-01 0.0000E-01 0.0000E-01 0 0000E-01 0 0000E-01 8 3826E 05 0 0000E-01
```

SHEET 4 OF 5

TABLE 13 (CONT.)

CRITICAL S?EED DETERMINANTS

Item Number: 18.0

SPEED (RPM)	DETERMINANT
4.5000E 03	7.4142E 01
4.9333E 03	2.4505E 01
5.4083E 03	5.7031E 00
5.9291E 03	-5.2299E-01
6.5000E 03	-1.9644E 00

CO-ROTATIONAL MODE

RPM	LOCATION (IN)	DEFLECTION	SLOPE
5.8657E 03	3.1496E 00	1.7640E-01	
	1.3539E 01	8.4602E-01	
	2.1567E 01	1.0000E 00	
	3.7855E 01	1.4037E-01	

NO OF CO-ROTATIONAL ROOTS = 1

SHEET 5 OF 5

TABLE 14

COMPUTER OUTPUT FOR PRELIMINARY UNBALANCE RESPONSE ANALYSIS

(From Input Data of Table 5)

Output	Item Number
LUND ROTOR UNBALANCE RESPONSE WITH SOFT-MOUNTED DAMPER SHAFT SEGMENTS = 32 SHAFT MATERIALS = 1 LUMPED INERTIA STATIONS = 18 BEARINGS = 2	1.0
NUMBER OF SPEED GROUPS = 1	6.0
DYNAMIC RESPONSE OF THE BENDING MODE	7.0
SPEED GROUP NUMBER 1	8.0
End determinants table (below)	11.0
Bearing table (below)	13.0

Item 11.0:

SPEED (RPM)	FREQUENCY (HZ)	END DETERMINANTS CO-ROTATIONAL	END DETERMINANTS CTR-ROTATIONAL
2.0000E 03	3.3333E 01	2.154221E 13	2 078425E 13
2.3246E 03	3.8744E 01	1.864353E 13	1 773822E 13
2 7019E 03	4.5032E 01	1.501517E 13	1 399574E 13
3 1405E 03	5.2341E 01	1.062206E 13	9 585269E 12
3 6502E 03	6 0836E 01	5 575487E 12	4 722123E 12
4 2426E 03	7.0711E 01	2 816836E 11	-4 558320E 10
4 9313E 03	8.2188E 01	-4 325013E 12	-3.657621E 12
5.7316E 03	9.5527E 01	-6.473211E 12	-4.429604E 12
6 6619E 03	1.1103E 02	-3 299339E 12	-3.242122E 11
7.7432E 03	1.2905E 02	8 861613E 12	9.621702E 12
9.0000E 03	1.5000E 02	3 219663E 13	2.120175E 13

Item 13.0:

BRG NO	1		2		3		4	
STN NO	2		29					
LOC (IN)	3.1496E 00		3.7855E 01					
SPEED (RPM)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)
2 0000E 03	1 6000E 05	0.0000E-01	1 6000E 05	0 0000E-01				
2 3246E 03	1.6000E 05	0.0000E-01	1.6000E 05	0 0000E-01				
2.7019E 03	1 6000E 05	0 0000E-01	1 6000E 05	0 0000E-01				
3.1405E 03	1.6000E 05	0.0000E-01	1.6000E 05	0 0000E-01				
3 6502E 03	1.6000E 05	0 0000E-01	1 6000E 05	0 0000E-01				
4 2426E 03	1.6000E 05	0.0000E-01	1 6000E 05	0 0000E-01				
4.9313E 03	1 6000E 05	0.0000E-01	1 6000E 05	0.0000E-01				
5.7316E 03	1.6000E 05	0 0000E-01	1 6000E 05	0.0000E-01				
6.6619E 03	1 6000E 05	0 0000E-01	1 6000E 05	0.0000E-01				
7 7432E 03	1 6000E 05	0 0000E-01	1 6000E 05	0 0000E-01				
9.0000E 03	1.6000E 05	0.0000E-01	1 6000E 05	0.0000E-01				

The complete computer output for the unbalance response analysis is given in Table 15. Two speed groups are included, which respectively cover the range 4000 - 5000 rpm and the range 6000 - 8000 rpm. Level II bearing data (Item 16.0) in the eight coefficient format, together with the excitation description and response (Item 19.1) are tabulated together on the same page at each speed point. Note that unbalance is specified in units of (in.-oz.) in input card 16A but is represented by the corresponding centrifugal force at each speed in the output. Circularity in the response orbit is indicated by equal magnitudes of major and minor radii at all stations. The amplitude peaks mildly between 4230 rpm and 4349 rpm with a nearly symmetrical mode shape, corresponding to a unit unbalance (1 in.-oz.) near mid-span (station 18). Symmetry of the mode shape is reflected not only by the amplitude distribution but is also indicated by the nearly uniform phase reference. In the second speed group, the unit unbalance is shifted to station 29 which is a bearing support. This is done because an angular mode shape is anticipated. Peaking in the second speed group is observed between 6928 rpm and 7182 rpm, again very mildly. The angular mode shape is indicated by the difference in the phase reference of nearly 180 degrees between stations 2 and 29.

3.4.3 Asynchronous Resonance (IRUN = 3)

Two runs of asynchronous resonance analysis were made. The first run was limited to Level I, but dealt with both torsional and bending motions. The descriptive data of the rotor system have already been discussed in previous examples, the essential output of the present run are the lists of Item 11.0, the bending end determinants, and Item 12.0, the torsional end stiffness, which are tabulated one after another and followed by Level I bearing data, Item 13.0, within each speed-frequency group in Table 16. Change of sign indicates the location of a resonance condition. It is seen that bending asynchronous resonances of both co-rotational and counter-rotation types are located within each of the three frequency groups. Torsional resonance is found near 324 Hz.

TABLE 15

COMPUTER OUTPUT FOR UNBALANCE RESPONSE ANALYSIS

(From Input Data of Table 6)

23 Sheets

	Item Number
LUND ROTOR UNBALANCE RESPONSE WITH SOFT-MOUNTED DAMPER SHAFT SEGMENTS = 32 SHAFT MATERIALS = 1 LUMPED INERTIA STATIONS = 18 BEARINGS = 2	1.0

TABLE 15 (CONT.)

	Item Number
NUMBER OF SPEED GROUPS = 2	6.0
DYNAMIC RESPONSE OF THE BENDING MODE	7.0
SPEED GROUP NUMBER 1	8.0

SPEED (RPM)	FREQUENCY (HZ)	END DETERMINANTS CO-ROTATIONAL	END DETERMINANTS CTR-ROTATIONAL	Item Number
4.0000E 03	6.6667E 01	2.339302E 12	1.755535E 12	
4.1131E 03	6.8552E 01	1.357425E 12	8.863425E 11	
4.2295E 03	7.0491E 01	3.884523E 11	4 586776E 10	
4 3491E 03	7.2485E 01	-5.601040E 11	-7.576682E 11	
4 4721E 03	7.4536E 01	-1 479750E 12	-1.515250E 12	11.0
4.5986E 03	7.6644E 01	-2.360931E 12	-2.217054E 12	
4.7287E 03	7.8812E 01	-3 192960E 12	-2.852458E 12	
4.8625E 03	8.1041E 01	-3.963935E 12	-3.410054E 12	
5.0000E 03	8.3333E 01	-4.660675E 12	-3.877703E 12	

BRG NO	1		2		3		4		Item Number
STN NO	2		29						
LOC (IN)	3.1496E 00		3.7855E 01						
SPEED (RPM)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	
4.0000E 03	1.6000E 05	0.0000E-01	1.6000E 05	0.0000E-01					
4.1131E 03	1.6000E 05	0.0000E-01	1.6000E 05	0 0000E-01					
4.2295E 03	1.6000E 05	0.0000E-01	1 6000E 05	0.0000E-01					
4.3491E 03	1.6000E 05	0.0000E-01	1 6000E 05	0 0000E-01					13.0
4.4721E 03	1.6000E 05	0.0000E-01	1 6000E 05	0 0000E-01					
4.5986E 03	1.6000E 05	0.0000E-01	1 6000E 05	0 0000E-01					
4.7267E 03	1.6000E 05	0.0000E-01	1.6000E 05	0 0000E-01					
4.8625E 03	1.6000E 05	0.0000E-01	1.6000E 05	0.0000E-01					
5.0000E 03	1.6000E 05	0.0000E-01	1.6000E 05	0 0000E-01					

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TABLE 15 (CONT.)

SPEED GROUP NUMBER 2				Item Number
				8.0

SPEED (RPM)	FREQUENCY (HZ)	END DETERMINANTS CO-ROTATIONAL	CTR-ROTATIONAL	Item Number
6.0000E 03	1.0000E 02	-6.232070E 12	-3 784382E 12	11.0
6.2197E 03	1.0366E 02	-5.636002E 12	-2 921234E 12	
6 4474E 03	1.0746E 02	-4 625200E 12	-1 721174E 12	
6.6835E 03	1.1139E 02	-3.145846E 12	-1 703442E 11	
6.9282E 03	1.1547E 02	-1.144314E 12	1 734439E 12	
7.1819E 03	1.1970E 02	1.430474E 12	3.981556E 12	
7.4448E 03	1.2403E 02	4.623807E 12	6 539538E 12	
7.7174E 03	1.2862E 02	8.470818E 12	9 350980E 12	
8.0000E 03	1.3333E 02	1.299069E 13	1 232531E 13	

BRG NO	1		2		3		4		Item Number
STN NO	2		29						
LOC (IN)	3.1496E 00		3.7855E 01						
SPEED (RPM)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	
6.0000E 03	1.6000E 05	0.0000E-01	1.6000E 05	0.0000E-01					13.0
6.2197E 03	1.6000E 05	0.0000E-01	1.6000E 05	0 0000E-01					
6.4474E 03	1.6000E 05	0.0000E-01	1.6000E 05	0 0000E-01					
6.6835E 03	1.6000E 05	0.0000E-01	1.6000E 05	0.0000E-01					
6.9282E 03	1.6000E 05	0.0000E-01	1.6000E 05	0.0000E-01					
7.1819E 03	1.6000E 05	0.0000E-01	1 6000E 05	0.0000E-01					
7.4448E 03	1.6000E 05	0.0000E-01	1.6000E 05	0 0000E-01					
7.7174E 03	1.6000E 05	0.0000E-01	1 6000E 05	0.0000E-01					
8.0000E 03	1.6000E 05	0.0000E-01	1 6000E 05	0 0000E-01					

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TABLE [illegible]5 (CONT.)

	Item Number	Remarks
******************************CALCULATION SUMMARY****************************** TEST IRUN=2B LUND ROTOR UNBALANCE RESPONSE WITH SOFT-MOUNTED DAMPER UNBALANCE RESPONSE BENDING EXCITATION STATIONS 2 11 18 29	14.0	Speed Group No. 1

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TABLE 15 (CONT.)

Computer Output	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 4.0000E 03 RPM FREQUENCY = 6 6667E 01 HZ	16.0	Speed Point No. 1 of 9
EXCITATION DATA	19.1	

```
LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 4.0000E 03 RPM      FREQUENCY = 6 6667E 01 HZ

BEARING  **** TYPE ****  ******** XX ********  ******** XY ********  ******** YX ********  ******** YY ********
NO. STN  RADIAL ANGULAR      K          B          K          B          K          B          K          B
 1   2      *             1 5282E 05 1 0056E 02 0 0000E 01 0 0000E-01 0 0000E-01 0 0000E-01 1 5282E 05 1 0056E 02
 2  29      *             1 5282E 05 1.0056E 02 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01 1 5282E 05 1 0056E 02

EXCITATION DATA

         **** TYPE ****  ***** FORWARD *****   ***** BACKWARD *****
NO  STN  FORCE  MOMENT   IN-PHASE     LAG       IN-PHASE     LAG
 1   2    *              0.0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 2  11    *              0 0000E-01 0.0000E-01 0 0000E-01 0 0000E-01
 3  18    *              2 8380E 01 0.0000E-01 0 0000E-01 0 0000E-01
 4  29    *              0.0000E-01 0 0000E-01 0 0000E-01 0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 4 0000E 03 RPM      FREQUENCY = 6 6667E 01 HZ

                                 ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO  STN  LOCATION  DISPL SLOPE      MAJOR      MINOR       (DEG)     REFERENCE     ELLIPTICITY
 1   2     7 1496    *            4 8863E-04 4 8863E-04  2 7893E 01-6 1270E 01   7 1004E-12
 2  11    13 5395    *            8 2599E-04 8 2599E-04  2 8793E 01-5 3658E 01   7 0230E-12
 3  18    21 5673    *            9.0178E-04 9.0178E-04  2 9191E 01-5 2653E 01   6 9406E-12
 4  29    37 8547    *            4 9422E-04 4 9422E-04  2 9897E 01 6.3138E 01   6 6409E-12
```

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TABLE 15 (CONT.)

Item Number	Remarks
16.0	Speed Point No. 2 of 9
19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 4.1131E 03 RPM   FREQUENCY = 6.8552E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1.5282E 05	1.0056E 02	0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01	1.5282E 05	1.0056E 02
2	29	*		1.5282E 05	1.0056E 02	0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01	1.5282E 05	1.0056E 02

EXCITATION DATA

NO.	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	11	*		0.0000E-01	0 0000E-01	0.0000E-01	0.0000E-01
3	18	*		3.0009E 01	0.0000E-01	0.0000E-01	0.0000E-01
4	29	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 4.1131E 03 RPM   FREQUENCY = 6.8552E 01 HZ

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		5.7318E-04	5.7318E-04	3.3203E 01	-7 4677E 01	5.1572E-12
2	11	13.5395	*		9.6049E-04	9.6049E-04	3.6097E 01	-6.6853E 01	4.9342E-12
3	18	21.5673	*		1.0455E-03	1.0455E-03	3.7384E 01	-6.5816E 01	4.8447E-12
4	29	37.8547	*		5.7643E-04	5.7643E-04	3.9219E 01	-7.6783E 01	4.5142E-12

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TABLE 15 (CONT.)

Item Number	Remarks
16.0	Speed Point No. 3 of 9

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 4 2295E 03 RPM   FREQUENCY = 7 0491E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1 5282E 05	1 0056E 02	0.0000E-01	0.0000E-01	0.0000E-01	0 0000E-01	1.5282E 05	1.0056E 02
2	29	*		1.5282E 05	1.0056E 02	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5282E 05	1.0056E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	11	*		0 0000E-01	0.0000E-01	0.0000E-01	0 0000E-01
3	18	*		3 1730E 01	0.0000E-01	0.0000E-01	0.0000E-01
4	29	*		0 0000E-01	0.0000E-01	0.0000E-01	0.0000E-01

Item Number	Remarks
19.1	

RESPONSE ORBIT
ROTATIONAL SPEED = 4 2295E 03 RPM   FREQUENCY = 7.0491E 01 HZ

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL PADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		6.3538E-04	6.3538E-04	4 2557E 01	-9.0850E 01	9.9380E-12
2	11	13.5395	*		1.0550E-03	1.0550E-03	4 4414E 01	-8.2811E 01	9.9056E-12
3	18	21.5673	*		1.1448E-03	1.1448E-03	4 5410E 01	-8 1741E 01	1.0486E-11
4	29	37 8547	*		6.3514E-04	6.3514E-04	4.6732E 01	-9.3230E 01	9.9637E-12

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TABLE 15 (CONT.)

LEVEL 2 BEARING DATA	Item Number	Remarks
ROTATIONAL SPEED = 4.3491E 03 RPM FREQUENCY = 7 2485E 01 HZ	16.0	Speed Point No. 4 of 9

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1 5282E 05	1.0056E 02	0 0000E-01	0.0000E-01	0 0000E-01	0 0000E-01	1.5282E 05	1.0056E 02
2	29	*		1.5292E 05	1.0056E 02	0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5282E 05	1.0056E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	11	*		0.0000E-01	0.0000E-01	0.0000E-01	0 0000E-01
3	18	*		3.3551E 01	0.0000E-01	0 0000E-01	0 0000E-01
4	29	*		0.0000E-01	0 0000E-01	0.0000E-01	0.0000E-01

RESPONSE ORBIT

ROTATIONAL SPEED = 4 3491E 03 RPM   FREQUENCY = 7.2485E 01 HZ   19.1

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		6 4866E-04	6 4866E-04	5 2396E 01	-1 0779E 02	9 2745E-12
2	11	13 5395	*		1.0666E-03	1.0666E-03	5.3245E 01	-9 9535E 01	9.4983E-12
3	18	21.5673	*		1.1535E-03	1 1535E-03	5.3554E 01	-9 8433E 01	9.5645E-12
4	29	37 8547	*		6.4431E-04	6 4431E-04	5.4303E 01	-1.1049E 02	9.3049E-12

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TABLE 15 (CONT.)

Item Number	Remarks
16.0	Speed Point No. 5 of 9

```
LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 4.4721E 03 RPM      FREQUENCY = 7.4536E 01 HZ

BEARING  **** TYPE ****  ******** XX ********  ******** XY ********  ******** YX ********  ******** YY ********
NO. STN  RADIAL ANGULAR       K          B          K          B          K          B          K          B
 1   2      *            1.5282E 05 1 0056E 02 0.0000E-01 0 0000E-01 0 0000E-01 0.0000E-01 1.5282E 05 1.0056E 02
 2  29      *            1.5282E 05 1.0056E 02 0 0000E-01 0.0000E-01 0 0000E-01 0.0000E-01 1.5282E 05 1.0056E 02
```

Item Number
19.1

```
EXCITATION DATA

        **** TYPE ****  ***** FORWARD  *****  ***** BACKWARD *****
NO. STN  FORCE  MOMENT   IN-PHASE      LAG     IN-PHASE      LAG
 1   2     *            0.0000E-01 0.0000E-01 0.0000E-01 0.0000E-01
 2  11     *            0.0000E-01 0.0000E-01 0.0000E-01 0.0000E-01
 3  18     *            3.5476E 01 0.0000E-01 0.0000E-01 0.0000E-01
 4  29     *            0.0000E-01 0.0000E-01 0.0000E-01 0.0000E-01


RESPONSE ORBIT
ROTATIONAL SPEED = 4.4721E 03 RPM      FREQUENCY = 7 4536E 01 HZ

                                   ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO   STN  LOCATION  DISPL SLOPE      MAJOR      MINOR       (DEG)     REFERENCE     ELLIPTICITY
 1    2     3.1496    *             6.1486E-04 6.1486E-04  5.7196E 01-1.2291E 02    6.8720E-12
 2   11    13.5395    *             1.0007E-03 1.0007E-03  5 9512E 01-1 1443E 02    7.2303E-12
 3   18    21.5673    *             1.0783E-03 1.0783E-03  6.0759E 01-1.1330E 02    7.2972E-12
 4   29    37.8547    *             6.0663E-04 6.0663E-04  6.3242E 01-1.2598E 02    7.2291E-12
```

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TABLE 15 (CONT.)

Content	Item Number	Remarks
LEVEL 2 BEARING DATA	16.0	Speed Point No. 6 of 9
EXCITATION DATA		
RESPONSE ORBIT	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 4.5986E 03 RPM   FREQUENCY = 7.6644E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1.5282E 05	1.0056E 02	0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01	1.5282E 05	1.0056E 02
2	29	*		1.5282E 05	1.0056E 02	0 0000E-01	0.0000E-01	0.0000E-01	0.0000E-01	1 5282E 05	1.0056E 02

EXCITATION DATA

NO.	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	11	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	18	*		3.7511E 01	0.0000E-01	0 0000E-01	0 0000E-01
4	29	*		0 0000E-01	0.0000E-01	0.0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 4.5986E 03 RPM   FREQUENCY = 7 6644E 01 HZ

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		5.5923E-04	5.5923E-04	6.3481E 01	-1.3492E 02	5.2734E-12
2	11	13.5395	*		9.0031E-04	9 0031E-04	6 5568E 01	-1 2621E 02	5.6108E-12
3	18	21.5673	*		9.6639E-04	9.6639E-04	6 6813E 01	-1 2505E 02	5.7585E-12
4	29	37.8547	*		5.4780E-04	5.4780E-04	6.8778E 01	-1 3841E 02	5.6747E-12

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TABLE 15 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 4.7287E 03 RPM FREQUENCY = 7 8812E 01 HZ	16.0	Speed Point No. 7 of 9

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1.5282E 05	1.0056E 02	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5282E 05	1 0056E 02
2	29	*		1.5282E 05	1.0056E 02	0.0000E-01	0 0000E-01	0.0000E-01	0.0000E-01	1.5282E 05	1.0056E 02

EXCITATION DATA

NO.	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0 0000E-01	0 0000E-01
2	11	*		0.0000E-01	0.0000E-01	0 0000E-01	0 0000E-01
3	18	*		3.9663E 01	0.0000E-01	0 0000E-01	0 0000E-01
4	29	*		0.0000E-01	0.0000E-01	0 0000E-01	0 0000E-01

	Item Number	Remarks
RESPONSE ORBIT ROTATIONAL SPEED = 4.7287E 03 RPM FREQUENCY = 7.8812E 01 HZ	19.1	

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	** MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		5.0253E-04	5.0253E-04	6 7150E 01	-1.4396E 02	4 0043E-12
2	11	13.5395	*		7.9980E-04	7 [illegible]380E-04	6 9673E 01	-1.3503E 02	4.3552E-12
3	18	21.5673	*		8.5488E-04	8.5488E-04	7 0838E 01	-1 3384E 02	4.4886E-12
4	29	37.8547	*		4.8850E-04	4 8850E-04	7 3477E 01	-1 4795E 02	4 5170E-12

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TABLE 15 (CONT.)

Printout	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 4.8625E 03 RPM FREQUENCY = 8 1041E 01 HZ	16.0	Speed Point No. 8 of 9
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 4.8625E 03 RPM FREQUENCY = 8 1041E 01 HZ	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 4.8625E 03 RPM   FREQUENCY = 8 1041E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1.5282E 05	1.0056E 02	0.0000E-01	0.0000E-01	0.0000E-01	0 0000E-01	1 5282E 05	1 0056E 02
2	29	*		1.5282E 05	1 0056E 02	0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1.5282E 05	1 0056E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	11	*		0.0000E-01	0.0000E-01	0 0000E-01	0 0000E-31
3	18	*		4.1938E 01	0.0000E-01	0 0000E-01	0 0000E-01
4	29	*		0.0000E-01	0 0000E-01	0.0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 4.8625E 03 RPM   FREQUENCY = 8 1041E 01 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		4.5329E-04	4.5329E-04	6 9522E 01	-1.5072E 02	4 6230E-12
2	11	13.5395	*		7.1270E-04	7.1270E-04	7.2394E 01	-1.4156E 02	5 1893E-12
3	18	21.5673	*		7.5829E-04	7.5829E-04	7 3826E 01	-1.4034E 02	5 3898E-12
4	29	37.8547	*		4.3697E-04	4 3697E-04	7.6523E 01	-1.5529E 02	5.4387E-12

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TABLE 15 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 5.0000E 03 RPM FREQUENCY = 8 3333E 01 HZ	16.0	Speed Point No. 9 of 9

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1.5282E 05	1.0056E 02	0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01	1 5282E 05	1.0056E 02
2	29	*		1 5282E 05	1.0056E 02	0 0000E-01	0 0000E-01	0 0000E-01	9 0000E-01	1 5282E 05	1 0056E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	11	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0030E-01
3	18	*		4.4344E 01	0.0000E-01	0.0000E-01	0.0000E-01
4	29	*		0.0000E-01	0.0000E-01	0.0000E-01	0.0000E-01

RESPONSE ORBIT — Item Number 19.1

ROTATIONAL SPEED = 5.0000E 03 RPM FREQUENCY = 8 3333E 01 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		4.1302E-04	4.1302E-04	7.0886E 01	-1.5582E 02	2 1672E-12
2	11	13.5395	*		6.4104E-04	6.4104E-04	7 3948E 01	-1 4644E 02	2 4571E-12
3	18	21.5673	*		6.7863E-04	6 7863E-04	7 5508E 01	-1 4519E 02	2 5767E-12
4	29	37.8547	*		3.9453E-04	3.9453E-04	7.8962E 01	-1.6108E 02	2.6645E-12

TABLE 15 (CONT.)

	Item Number	Remarks
********************************CALCULATION SUMMARY******************************** TEST IRUN=2B LUND ROTOR UNBALANCE RESPONSE WITH SOFT-MOUNTED DAMPER UNBALANCE RESPONSE BENDING EXCITATION STATIONS 2 11 18 29	14.0	Speed Group No. 2

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TABLE 15 (CONT.)

Item Number	Remarks
16.0	Speed Point No. 1 of 9

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 6 0000E 03 RPM   FREQUENCY = 1 0000E 02 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	YY K	YY B	YX K	YX B	YY K	YY B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	TYPE FORCE	TYPE MOMENT	FORWARD IN-PHASE	FORWARD LAG	BACKWARD IN-PHASE	BACKWARD LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		6 3856E 01	0 0000E-01	0 0000E-01	0 0000E-01

Item Number
19.1

RESPONSE ORBIT
ROTATIONAL SPEED = 6 0000E 03 RPM   FREQUENCY = 1 0000E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	PRINCIPAL RADII MAJOR	PRINCIPAL RADII MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		4.4769E-04	4 4769E-04	3 0993E 01	1 2832E 02	5 8122E-13
2	11	13 5395	*		3 9782E-04	3 9782E-04	6 8696E 01	1 4868E 02	1 0029E-12
3	18	21 5673	*		2 5284E-04	2 5284E-04	8 7245E 01	1 7340E 02	1 6740E-12
4	29	37 8547	*		5 8982E-04	5 8982E-04	3 1002E 00	-5 6864E 01	5 9999E-13

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TABLE 15 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 6 2197E 03 RPM FREQUENCY = 1 0366E 02 HZ	16.0	Speed Point No. 2 of 9
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 6 2197E 03 RPM FREQUENCY = 1 0366E 02 HZ	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 6 2197E 03 RPM   FREQUENCY = 1 0366E 02 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0.0000E-01	0 0000E-01	0 0000E-01
4	29	*		6 8618E 01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 6 2197E 03 RPM   FREQUENCY = 1 0366E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		4 8256E-04	4.8256E-04	6 6694E 01	1 1831E 02	5 7517E-13
2	11	13.3395	*		4.0357E-04	4 0357E-04	7 6671E 01	1 3932E 02	9 4565E-13
3	18	21 5673	*		2 3638E-04	2 3638E-04	-8 5046E 01	1.6862E 02	1 6145E-12
4	29	37.9547	*		6.9998E-04	6 9998E-04	8 9765E 00	-6.3807E 01	3 2713E-13

TABLE 15 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 6 4474E 03 RPM FREQUENCY = 1 0746E 02 HZ	16.0	Speed Point No. 3 of 9
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 6 4474E 03 RPM FREQUENCY = 1 0746E 02 HZ	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 6 4474E 03 RPM FREQUENCY = 1 0746E 02 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		7 3735E 01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 6 4474E 03 RPM FREQUENCY = 1 0746E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		5 1540E-04	5 1540E-04	5 8093E 01	1 0679E 02	7 1355E-13
2	11	13 5395	*		4 0563E-04	4 0563E-04	7 2725E 01	1 2646E 02	1 2317E-12
3	18	21 5677	*		2 1857E-04	2 1857E-04	-8 3414E 01	1 6398E 02	2 1667E-12
4	29	37 8547	*		8 1214E-04	8 1214E-04	1 9122E 01	-7 2737E 01	4 6992E-13

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TABLE 15 (CONT.)

Item Number	Remarks
16.0	Speed Point No. 4 of 9
19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 6 6835E 03 RPM   FREQUENCY = 1 1139E 02 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1.4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	TYPE FORCE	TYPE MOMENT	FORWARD IN-PHASE	FORWARD LAG	BACKWARD IN-PHASE	BACKWARD LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0300E-01
4	29	*		7 9232E 01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 6 6835E 03 RPM   FREQUENCY = 1 1139E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	PRINCIPAL RADII MAJOR	PRINCIPAL RADII MINOR	INCLINATION DEG	PHASE REFERENCE	ELLIPTICITY
1	2	2 1496	*		5 3785E-04	5 3785E-04	5 3058E 01	9 3834E 01	5 0314E-13
2	11	13 5395	*		3 9823E-04	3 9823E-04	6 757E 01	1 1619E 02	3 4962E-13
3	18	21 5673	*		1 9809E-04	1 9809E-04	-8 256E 01	1 5914E 02	1 7690E-12
4	29	37.8547	*		9 1092E-04	9 1092E-04	2454E 01	-6 3419E 01	3 1993E-13

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TABLE 15 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 6 9282E 03 RPM FREQUENCY = 1 1547E 02 HZ	16.0	Speed Point No. 5 of 9
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 6 9282E 03 RPM FREQUENCY = 1 1547E 02 HZ	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 6 9282E 03 RPM FREQUENCY = 1 1547E 02 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		8 5141E 01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 6 9282E 03 RPM FREQUENCY = 1 1547E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		5 4116E-04	5 4116E-04	4 5043E 01	7 9982E 01	1 6669E-13
2	11	13 5395	*		3 7676E-04	3 7676E-04	5 5732E 01	1 0305E 02	2 5784E-13
3	18	21.5673	*		1 7519E-04	1 7519E-04	8 5196E 01	1 5541E 02	4 6540E-13
4	29	37 8547	*		9.7684E-04	9 7684E-04	-3 7937E 00	-9 5204E 01	1 4207E-14

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TABLE 15 (CONT.)

Content	Item Number	Remarks
LEVEL 2 BEARING DATA	16.0	Speed Point No. 6 of 9
EXCITATION DATA; RESPONSE ORBIT	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 7 1819E 03 RPM FREQUENCY = 1 1970E 02 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	16	*		0 0000E-01	0 0000E-01	0 0000E 01	0 0000E-01
4	29	*		9 1490E 01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 7 1819E 03 RPM FREQUENCY = 1 1970E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	** MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		5 2174E-04	5 2174E-04	6 4804E 01	6 6173E 01	3 9899E-13
2	11	13 5395	*		3 4128E-04	3 4128E-04	7 5597E 01	8.9973E 01	8 4379E-13
3	18	21 5673	*		1 5230E-04	1 5230E-04	-6 9453E 01	1.5347E 02	1 6972E-12
4	29	37 8547	*		9 9734E-04	9 9734E-04	3 3116E 01	-1 0708E 02	1 3915E-13

TABLE 15 (CONT.)

Item Number	Remarks
16.0	Speed Point No. 7 of 9
19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 7 4448E 03 RPM   FREQUENCY = 1 2408E 02 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	TYPE FORCE	TYPE MOMENT	FORWARD IN-PHASE	FORWARD LAG	BACKWARD IN-PHASE	BACKWARD LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		9 8313E 01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 7 4448E 03 RPM   FREQUENCY = 1 2408E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	PRINCIPAL RADII MAJOR	PRINCIPAL RADII MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		4 8411E-04	4 8411E-04	4 0119E 01	5 3331E 01	4 3000E-14
2	11	13 5395	*		2 9715E-04	2 9715E-04	5 8865E 01	7 7885E 01	9 3407E-14
3	18	21 5673	*		1 3270E-04	1 3270E-04	-6 9605E 01	1 5390E 02	1 8301E-13
4	29	37 8547	*		9 7544E-04	9 7544E-04	2 0011E 01	-1 1805E 02	2 1341E 14

TABLE 15 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA	16.0	Speed Point No. 8 of 9
EXCITATION DATA		
RESPONSE ORBIT	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 7 7174E 03 RPM   FREQUENCY = 1 2862E 02 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD ***** IN-PHASE	LAG	***** BACKWARD ***** IN-PHASE	LAG
1	2	*		0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		1 0564E 02	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 7 7174E 03 RPM   FREQUENCY = 1 2862E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG.)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		4 3708E-04	4 3708E-04	6 8586E 01	4 1978E 01	1 3494E-13
2	11	13.5395	*		2 5129E-04	2 5129E-04	8 2886E 01	6 7308E 01	4 1420E-13
3	18	21.5673	*		1 1828E-04	1 1828E-04	-4 8237E 01	1 5624E 02	9 7527E-13
4	29	37.8547	*		9 2559E-04	9 2559E-04	5 0942E 01	-1 2756E 02	5 2477E-14

TABLE 15 (CONT.)

LEVEL 2 BEARING DATA ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 3333E 02 HZ	Item Number	Remarks
	16.0	Speed Point No. 9 of 9

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01
2	29	*		1 4545E 05	9 1544E 01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 4545E 05	9 1544E 01

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** FORWARD IN-PHASE	***** LAG	***** BACKWARD IN-PHASE	***** LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		1 1352E 02	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT — Item Number 19.1

ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 3333E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION DEG	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		3 8818E-04	3 8818E-04	2 1818E 01	3 2176E 01	2 6913E-14
2	11	13 5395	*		2 0849E-04	2 0849E-04	3 4830E 01	5 6063E 01	5 8244E-14
3	18	21 5673	*		1 0877E-04	1 0877E-04	-3 3422E 01	1 5945E 02	2 3922E-14
4	29	37 8547	*		8 6306E-04	8 6306E-04	7 2847E 01	-1 3543E 02	3 2159E-14

TABLE 16

PARTIAL OUTPUT OF ASYNCHRONOUS RESONANCE ANALYSIS FOR TORSIONAL AND BENDING MOTIONS

LEVEL I ONLY

(From Input Data of Table 7)

3 Sheets

	Item Number
LUND ROTOR WITH SIMULATED FLUID FILM BRGS AND TORS ANALYSIS	
SHAFT SEGMENTS = 32 SHAFT MATERIALS = 1 LUMPED INERTIA STATIONS = 18 BEARINGS = 2	1.0

TABLE 16 (CONT.)

Output	Item Number
NUMBER OF SPEED GROUPS = 2	6.0
DYNAMIC RESPONSES OF BOTH TORSIONAL AND BENDING MODES	7.0
SPEED GROUP NUMBER 1	8.0
EXCITATION FREQUENCY GROUP 1 HZ 63.0000 64.6396 66.3218 68.0478 69.8187 71.6357 73.5000	9.0

Item Number 11.0

SPEED (RPM)	FREQUENCY (HZ)	END DETERMINANTS CO-ROTATIONAL	END DETERMINANTS CTR-ROTATIONAL
8.0000E 03	6.3000E 01	2.515243E 12	1.548479E 12
8.0000E 03	6.4640E 01	1.747326E 12	9.578401E 11
8.0000E 03	6.6322E 01	9.827314E 11	3.824287E 11
8.0000E 03	6.8048E 01	2.258965E 11	-1.735230E 11
8.0000E 03	6.9819E 01	-5.181722E 11	-7.053880E 11
8.0000E 03	7.1636E 01	-1.243858E 12	-1.208133E 12
8.0000E 03	7.3500E 01	-1.944890E 12	-1.676312E 12

Item Number 12.0

FREQUENCY (HZ)	TORSIONAL END STIFF
6.3000D 01	-1.611507D 06
6.4640D 01	-1.690822D 06
6.6322D 01	-1.773718D 06
6.8048D 01	-1.860323D 06
6.9819D 01	-1.950763D 06
7.1636D 01	-2.045162D 06
7.3500D 01	-2.143647D 06

Item Number 13.0

BRG NO	1		2		3		4	
STN NO	2		29					
LOC (IN)	3.1496E 00		3.7855E 01					
SPEED (RPM)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)	RAD (LB/IN)	ANG (IN-LB/RAD)
8.0000E 03	1.2950E 05	0.0000E-01	1.5350E 05	0.0000E-01				

SHEET 2 OF 3

TABLE 16 (CONT.)

	Item Number
SPEED GROUP NUMBER 2	8.0
EXCITATION FREQUENCY GROUP 1 HZ 92 8000 95 1234 97.5049 99.9461 102.4484 105.0134 107 6425 110.3375 113.1000	9.0
SPEED (RPM) | FREQUENCY (HZ) | END DETERMINANTS CO-ROTATIONAL | END DETERMINANTS CTR-ROTATIONAL 8.0000E 03 | 9.2800E 01 | 1.679913E 14 | 3 423157E 13 8.0000E 03 | 9 5123E 01 | 1.155819E 14 | -1 025850E 13 8.0000E 03 | 9 7505E 01 | 6.246683E 13 | -5.473874E 13 8.0000E 03 | 9.9946E 01 | 8.797742E 12 | -9 904060E 13 8.0000E 03 | 1.0245E 02 | -4.525121E 13 | -1.429772E 14 8.0000E 03 | 1.0501E 02 | -9.948163E 13 | -1 863424E 14 9.0000E 03 | 1.0764E 02 | -1.536690E 14 | -2.289098E 14 8.0000E 03 | 1.1034E 02 | -2.075608E 14 | -2.704320E 14 8.0000E 03 | 1.1310E 02 | -2.608747E 14 | -3.106400E 14	11.0
FREQUENCY (HZ) | TORSIONAL END STIFF 9.2800D 01 | -3.243267D 06 9.5123D 01 | -3.383256D 06 9.7505D 01 | -3 527914D 06 9.9946D 01 | -3.677243D 06 1 0245D 02 | -3.831224D 06 1.0501D 02 | -3.989812D 06 1.0764D 02 | -4.152936D 06 1.1034D 02 | -4.320494D 06 1.1310D 02 | -4 492347D 06	12.0
EXCITATION FREQUENCY GROUP 2 HZ 300.0000 311.7870 324.0370 336.7684 350.0000	9.0
SPEED (RPM) | FREQUENCY (HZ) | END DETERMINANTS CO-ROTATIONAL | END DETERMINANTS CTR-ROTATIONAL 8.0000E 03 | 3.0000E 02 | 1.004226E 15 | 4.205831E 13 8.0000E 03 | 3.1179E 02 | 4.230628E 14 | -5.598046E 14 8.0000E 03 | 3.2404E 02 | -5.051822E 14 | -1.395417E 15 8.0000E 03 | 3.3677E 02 | -1.865942E 15 | -2.495496E 15 8.0000E 03 | 3.5000E 02 | -3.749671E 15 | -3.881267E 15	11.0
FREQUENCY (HZ) | TORSIONAL END STIFF 3.0000D 02 | -3.214928D 06 3.1179D 02 | -1.803141D 06 3.2404D 02 | -1.982565D 05 3.3677D 02 | 1.591056D 06 3.5000D 02 | 3.546048D 06	12.0
BRG NO | 1 | | 2 | | 3 | | 4 | STN NO | 2 | | 29 | | | | | LOC (IN) | 3.1496E 00 | | 3.7855E 01 | | | | | SPEED (RPM) | RAD (LB/IN) | ANG (IN-LB/RAD) | RAD (LB/IN) | ANG (IN-LB/RAD) | RAD (LB/IN) | ANG (IN-LB/RAD) | RAD (LB/IN) | ANG (IN-LB/RAD) 8.0000E 03 | 9.8440E 05 | 0.0000E-01 | 1.2720E 06 | 0.0000E-01	13.0

SHEET 3 OF 3

In order to gain some additional insight of the bending modes in preparation for subsequent analyses, a second asynchronous resonance analysis was dedicated to the bending modes of the two lower frequency ranges, this time invoking Level II analysis to obtain the mode shapes. Level II output of the second asynchronous resonance analysis is given in Table 17. In the first frequency range, resonances are found at 68.6 Hz and 67.5 Hz, respectively, for the co-rotational and counter-rotational modes. The relative frequency relationship is due to the effect of gyroscopic inertia. The amplitude distributions of these two modes are practically the same. It should be recalled that the relatively low bearing stiffness values assigned to this frequency range is an attempt to emulate the "half-speed" behavior of a fluid film bearing. Since the reduced effective bearing stiffness at "half-speed" is associated only with the co-rotational motion, the counter-rotational mode found in this frequency range should be disregarded for the present study. In the second frequency range, again a co-rotational mode and a counter-rotational mode are found, respectively, at 100.3 Hz and 94.6 Hz. Amplitude distributions of these modes are again very similar to each other. In comparison with the 68.6 Hz mode, the amplitudes at the bearing supports, stations 2 and 29, are much reduced at the higher frequency modes.

3.4.4 Asynchronous Response (IRUN = 4)

The example for damped asynchronous response was calculated with the same speed-frequency combinations used in the previous example; consequently, the input was set up to access the pre-stored intermediate data file in lieu of repeating Level I analysis. The complete output of the example is shown in Table 18.

Excitation is specified to be the space-fixed type. For the first frequency range a unit excitation force, aimed in the direction of the static load, is assigned at station 18, which is near the mid-span of the rotor and is the peak deflection point of the undamped co-rotational mode in Table 17. For the second frequency range, the excitation force is shifted to station 29, which is a bearing support where deflection is minimal for both undamped modes in this frequency range.

TABLE 17

COMPUTER OUTPUT OF ASYNCHRONOUS BENDING RESONANCE ANALYSIS

LEVEL II RESULTS

(From Input Data of Table 8)

7 Sheets

	Item Number
LUND ROTOR ASYNCHRONOUS BENDING ANALYSIS SHAFT SEGMENTS = 32 SHAFT MATERIALS = 1 LUMPED INERTIA STATIONS = 18 BEARINGS = 2	1.0

TABLE 17 (CONT.)

	Item Number
******************************CALCULATION SUMMARY****************************** TEST IRUN=3,4,5 LUND ROTOR ASYNCHRONOUS BENDING ANALYSIS ASYNCHRONOUS RESONANCE CALCULATION BENDING EXCITATION STATIONS 2 11 18 29	14.0

SHEET 2 OF 7

TABLE 17 (CONT.)

Item Number
16.0

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 3000E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1 2950E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2950E 05	0 0000E-01
2	29	*		1 5350E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5350E 05	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 4640E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1 2950E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2950E 05	0 0000E-01
2	29	*		1 5350E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5350E 05	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 6322E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1 2950E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2950E 05	0 0000E-01
2	29	*		1 5350E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5350E 05	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 8048E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1.2950E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2950E 05	0 0000E-01
2	29	*		1 5350E 05	0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5350E 05	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 6 9819E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1.2950E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2950E 05	0 0000E-01
2	29	*		1 5350E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5350E 05	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 7 1636E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1 2950E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2950E 05	0 0000E-01
2	29	*		1 5350E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5350E 05	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 7 3500E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		1 2950E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2950E 05	0 0000E-01
2	29	*		1 5350E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 5350E 05	0 0000E-01

TABLE 17 (CONT.)

Item Number: 20.0

```
UNDAMPED ASYNCHRONOUS STIFFNESS DETERMINANTS
ROTATIONAL SPEED = 8.0000E 03 RPM

FREQUENCY       ****** DETERMINANTS *******
  (HZ)             CO -ROT          CTR-ROT
6.3000E 01      2.8755E 00       1.9106E 00
6.4640E 01      1.6326E 00       9.6797E-01
6.6322E 01      7.5066E-01       3.1663E-01
6.8048E 01      1.4110E-01      -1.1774E-01
6.9819E 01     -2.6473E-01      -3.9241E-01
7.1636E 01     -5.1994E-01      -5.5120E-01
7.3500E 01     -6.6537E-01      -6.2748E-01

CO-ROTATIONAL MODE

                   FREQ (HZ)      LOCATION (IN)  DEFLECTION      SLOPE
                   6.8621E 01     3.1496E 00     6.2909E-01
                                  1.3539E 01     9.4850E-01
                                  2.1567E 01     1.0000E 00
                                  3.7855E 01     5.2182E-01

NO OF  CO-ROTATIONAL ROOTS =    1

CTR-ROTATIONAL MODE

                   FREQ (HZ)      LOCATION (IN)  DEFLECTION      SLOPE
                   6.7536E 01     3.1496E 00     6.2607E-01
                                  1.3539E 01     9.5105E-01
                                  2.1567E 01     1.0000E 00
                                  3.7855E 01     5.0063E-01

NO OF CTR-ROTATIONAL ROOTS =    1
```

SHEET 4 OF 7

TABLE 17 (CONT.)

	Item Number
******************************CALCULATION SUMMARY****************************** TEST IRUN=3,4,5 LONG ROTOR ASYNCHRONOUS BENDING ANALYSIS ASYNCHRONOUS RESONANCE CALCULATION BENDING EXCITATION STATIONS 2 11 18 29	14.0

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TABLE 17 (CONT)

Item Number: 16.0

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 2600E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 5123E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 7505E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 9946E 01 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 0245E 02 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 0501E 02 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 0764E 02 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 1034E 02 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 1310E 02 HZ

BEARING NO.	STN	TYPE RADIAL	TYPE ANGULAR	XX K	XX B	XY K	XY B	YX K	YX B	YY K	YY B
1	2	*		9 8440E 05	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	9 8440E 05	0 0000E-01
2	29	*		1 2720E 06	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01	1 2720E 06	0 0000E-01

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TABLE 17 (CONT.)

Item Number: 20.0

UNDAMPED ASYNCHRONOUS STIFFNESS DETERMINANTS
ROTATIONAL SPEED = 8.0000E 03 RPM

FREQUENCY (HZ)	****** DETERMINANTS ******* CO -ROT	CTR-ROT
9.2800E 01	9.4961E 00	2 1743E 00
9 5123E 01	5.4109E 00	-5 4145E-01
9.7505E 01	2.4233E 00	-2.4025E 00
9.9946E 01	2.8299E-01	-3 6174E 00
1 0245E 02	-1 2077E 00	-4 3494E 00
1.0501E 02	-2.2046E 00	-4.7252E 00
1.0764E 02	-2.8298E 00	-4 8432E 00
1.1034E 02	-3.1788E 00	-4.7788E 00
1.1310E 02	--3.3257E 00	-4 5897E 00

CO-ROTATIONAL MODE

FREQ (HZ)	LOCATION (IN)	DEFLECTION	SLOPE
1.0034E 02	3 1496E 00	1 5738E-01	
	1.3539E 01	8 5190E-01	
	2.1567E 01	1 0000E 00	
	3.7855E 01	8 9878E-02	

NO OF CO-ROTATIONAL ROOTS = 1

CTR-ROTATIONAL MODE

FREQ (HZ)	LOCATION (IN)	DEFLECTION	SLOPE
9.4609E 01	3.1496E 00	1 3981E-01	
	1.3539E 01	8 4282E-01	
	2.1567E 01	1 0000E 00	
	3.7855E 01	7 7683E-02	

NO OF CTR-ROTATIONAL ROOTS = 1

SHEET 7 OF 7

TABLE 18

COMPUTER OUTPUT OF DAMPED ASYNCHRONOUS RESPONSE ANALYSIS

(From Input Data of Table 9)

25 Sheets

	Item Number	Remarks
******************************CALCULATION SUMMARY****************************** TEST IRUN=3,4,5 LUND ROTOR ASYNCHRONOUS BENDING ANALYSIS ASYNCHRONOUS RESPONSE EXCITATION IS STATIONARY BENDING EXCITATION STATIONS 2 11 16 29	14.0	Freq. Group No. 1

TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 6 3000E 01 HZ	16.0	Freq. Point No. 1 of 7

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1 5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2 1097E 05	1 7124E 02

EXCITATION DATA

NO.	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01

	Item Number	Remarks
RESFONSE ORBIT ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 6 3000E 01 HZ	19.1	

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE PEFERENCE	ELLIPTICITY
1	2	3 1496	*		2 6178E-06	-1 5643E-06	4 4426E 00	7 8439E 01	2 5192E-01
2	11	13 5395	*		6 1706E-06	-2 7853E-06	3 9304E 01	4 4923E 01	3 7800E-01
3	18	21 5673	*		7 0586E-06	-2 9171E-06	4 1539E 01	4 5691E 01	4 1516E-01
4	29	37.8547	*		2 0453E-06	-1 3042E-06	-9 9503E-01	4 0772E 01	2 2126E-01

TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA	16.0	Freq. Point No. 2 of 7
EXCITATION DATA		
RESPONSE ORBIT	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 4640E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1 5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2.7264E 02	4 9946E 04	2 7264E 02	2 1097E 05	1 7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 4640E 01 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		3 3899E-06	2 1898E-06	-7 9435E-01	5 0094E 01	2 1509E-01
2	11	13 5395	*		7 0643E-06	4 0561E-06	4 6659E 01	5 0743E 01	2.7051E-01
3	18	21 5673	*		8 0044E-06	4 2533E-06	4 3481E 01	5 0988E 01	3 0596E-01
4	29	37 8547	*		2 6737E-06	1 7885E-06	-7 5031E 00	5 3163E 01	1 9838E-01

TABLE 18 (CONT.)

Item Number	Remarks
16.0	Freq. Point No. 3 of 7
19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEEC = 8 0000E 03 RPM FREQUENCY = 6 6322E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1.7531E 05	1 5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9546E 04	2.7264E 02	2.1097E 05	1.7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		0 0000E-01	0 0000E-01	0.0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 6 6322E 01 HZ

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		4 7180E-06	-3 2735E-06	-8 9652E 00	7 9685E 01	1 8075E-01
2	11	13 5395	*		8.3055E-06	-6 3203E-06	5 0100E 01	6 7998E 01	1 3573E-01
3	18	21.5673	*		9 3374E-06	-6 5829E-06	5 3881E 01	6 6896E 01	1 7302E-01
4	29	37 8547	*		3.7795E-06	-2 6087E-06	-1 6350E 01	8.3285E 01	1.8328E-01

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TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA	16.0	Freq. Point No. 4 of 7
EXCITATION DATA		
RESPONSE ORBIT	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 8048E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	7 0312E 04	2 2607E 02	1 7531E 05	1 5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2 1097E 05	1 7124E 02

EXCITATION DATA

NO.	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		0.0000E-01	0 0000E-01	0 0000F-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 8048E 01 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		5 7556E-06	-4 1588E-06	-2 1772E 01	1 2775F 02	1 6106E-01
2	11	13 5395	*		9.4813E-06	-6 9299E-06	-8 3064E 01	1 0431E 02	1 5546E-01
3	18	21.5673	*		1 0533E-05	-7 0766E-06	-8 7689E 01	1 0083E 02	1 9629E-01
4	29	37 8547	*		4 7002E-06	-3 2327E-06	-2 7389E 01	1 2818E 02	1 8498E-01

TABLE 18 (CONT.)

Item Number	Remarks
16.0	Freq. Point No. 5 of 7
19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8.0000E 03 RPM   FREQUENCY = 6 9819E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9.2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1.7531E 05	1.5177E 02
2	29	*		8 9933E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2.1097E 05	1.7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0 0000E-01	0 0000E-01
2	11	*		0.0000E-01	0.0000E-01	0 0000E-01	0 0000E-01
3	18	*		1.0000E 00	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8.0000E 03 RPM   FREQUENCY = 6 9819E 01 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	7.1496	*		4 2657E-06	2 9735E-06	-3 7733E 01	1 5312E 02	1 7850E-01
2	11	13.5395	*		7 8418E-06	2 8614E-06	-7 5037E 01	1 1812E 02	4 6532E-01
3	18	21 5673	*		8.7060E-06	2 6423E-06	-7 3469E 01	1.1232E 02	5 3433E-01
4	29	37 8547	*		3.5232E-06	2 2849E-06	-3 8796E 01	1.5208E 02	2.1319E-01

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TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 7 1636E 01 HZ	16.0	Freq. Point No. 6 of 7
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 7 1636E 01 HZ	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM   FREQUENCY = 7 1636E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6.6956E 05	9.2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1.7531E 05	1 5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2 1097E 05	1.7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0.0000E-01	0.0000E-01	0 0000E-01	0 0000E-01
3	18	*		1 0000E 00	0.0000E-01	0 0000E-01	0 0000E-01
4	29	*		0.0000E-01	0 0000E-01	0 0000E-01	0.0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8.0000E 03 RPM   FREQUENCY = 7 1636E 01 HZ

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		2.8595E-06	1 7262E-06	-5 0936E 01	1.6147E 02	2 4715E-01
2	11	13.5395	*		5.9412E-06	1 6432E-07	-7 9193E 01	1 0223E 02	9 4617E-01
3	18	21.5673	*		6.7280E-06	2 1545E-07	-8 3147E 01	9 5370E 01	9 3794E-01
4	29	37 8547	*		2.3532E-06	1 3443E-06	-4 8476E 01	1 6051E 02	2 7287E-01

TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 7 3500E 01 HZ	16.0	Freq. Point No. 7 of 7
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 7 3500E 01 HZ	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 7 3500E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9.2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1 5177E 02
2	29	*		8.9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2 1097E 05	1 7124E 02

EXCITATION DATA

NO.	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0.0000E-01	0 0000E-01	0 0000E-01
3	18	*		1.0000E 00	0 0000E-01	0.0000E-01	0 0000E-01
4	29	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8.0000E 03 RPM FREQUENCY = 7 3500E 01 HZ

NO.	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*		2.1193E-06	9 8326E-07	-5 9842E 01	1 6218E 02	3 6617E-01
2	11	13.5395	*		5.1027E-06	1.0871E-06	-8 8506E 01	8 1504E 01	6.4874E-01
3	18	21.5673	*		5.9637E-06	1 4509E-06	8 6572E 01	7 6998E 01	6 0863E-01
4	29	37.8547	*		1.7219E-06	7 9500E-07	-5 5973E 01	1 6207E 02	3 6610E-01

TABLE 18 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	PESIDUE	@ CHECK	Item Number	Remarks
1	6 811D 01	1 357D 00	3 323D-01	2.998D-02	4 675D-04	2 631D-04	1 913D-03	-3 667D-02	21.2	Mode No. 1
2	6.811D 01	1 357D 00	3 322D-01	2 999D-02	4 486D-08	3 235D-08	2 596D-11	-3 672D-02		

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TABLE 18 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	6.811D 01	1 357D 00	2.562D-02	3 888D-01	1 769D-02	5 276D-04	1 833D 03	-4 380D-01	21.2	Mode No. 2
2	6.813D 01	1 357D 00	2.569D-02	3 877D-01	-3.050D-04	4 612D-04	4 788D 02	-4 267D-01		
3	6 813D 01	1 358D 00	2.569D-02	3 879D-01	-2 977D-04	4 086D-04	3 768D 02	-4 271D-01		
4	6 813D 01	1 358D 00	2.568D-02	3 879D-01	-2 639D-04	3 619D-04	2 955D 02	-4 271D-01		
5	6.813D 01	1.359D 00	2.568D-02	3 883D-01	-2.339D-04	3 205D-04	2 318D 02	-4.279D-01		
6	6 813D 01	1 359D 00	2 567D-02	3 885D-01	-2 072D-04	2 839D-04	1 818D 02	-4 282D-01		
7	6 813D 01	1.359D 00	2 567D-02	3 886D-01	-1 836D-04	2 515D-04	1 426D 02	-4 285D-01		
8	6.813D 01	1 360D 00	2 567D-02	3 887D-01	-1 627D-04	2 227D-04	1 118D 02	-4 287D-01		
9	6 813D 01	1 360D 00	2 566D-02	3 889D-01	-1 441D-04	1 973D-04	8 773D 01	-4 289D-01		
10	6 813D 01	1.360D 00	2.566D-02	3 890D-01	-1 277D-04	1 747D-04	6 881D 01	-4 291D-01		
11	6.813D 01	1 360D 00	2.566D-02	3.890D-01	-1 131D-04	1 548D-04	5 397D 01	-4 293D-01		
12	6.813D 01	1.360D 00	2.566D-02	3.891D-01	-1 002D-04	1 371D-04	4 233D 01	-4 294D-01		
13	6.813D 01	1.360D 00	2.565D-02	3.891D-01	-8 879D-05	1 214D-04	3 321D 01	-4.294D-01		
14	6.813D 01	1.361D 00	2.565D-02	3.892D-01	-7 865D-05	1 075D-04	2 605D 01	-4 297D-01		
15	6.813D 01	1.361D 00	2.565D-02	3 892D-01	-6.967D-05	9 524D-05	2 043D 01	-4 297D-01		
16	6.813D 01	1 361D 00	2 565D-02	3 893D-01	-6 172D-05	8 435D-05	1 602D 01	-4 299D-01		
17	6 813D 01	1 361D 00	2.565D-02	3 894D-01	-5 467D-05	7 471D-05	1 257D 01	-4 300D-01		
18	6.813D 01	1.361D 00	2.565D-02	3 894D-01	-4.842D-05	6 617D-05	9 859D 00	-4 301D-01		
19	6 813D 01	1.361D 00	2.565D-02	3.895D-01	-4.289D-05	5 860D-05	7 734D 00	-4 301D-01		
20	6.813D 01	1 361D 00	2.565D-02	3.895D-01	-3.799D-05	5 190D-05	6 066D 00	-4 302D-01		

TABLE 18 (CONT.)

Item Number	Remarks
21.3	Freq. Group No. 1

```
ROTATIONAL SPEED = 6 0000E 03 RPM
FREQUENCY        = 6 8114E 01  HZ
CRIT DAMP RATIO  = 2 9986E-02
PRECISION INDEX  =-3 6718E-02
NATURAL MODE SHAPE
                                    ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO  STN  LOCATION  DISPL SLOPE        MAJOR      MINOR        (DEG)     REFERENCE    ELLIPTICITY
 1   2     3 1496    *              1 1335E 00 2 0729E-01 -5 5521E 01-4 [illegible] 01    6 9080E-01
 2  11    13 5395    *              1 6911E 00 2 3654E-01 -5 6117E 01-4 [illegible]356E 01    7 5458E-01
 3  18    21 5673    *              1.7703E 00 2 2968E-01 -5 6353E 01-4 5323E 01    7 7032E-01
 4  29    37 8547    *              9 0723E-01 1.5079E-01 -5 2265E 01-4 4390E 01    7 1496E-01


RESONANT RESPONSE



EXCITATION DATA

       **** TYPE ****  ***** VERTICAL *****  **** HORIZONTAL ****
NO  STN  FORCE  MOMENT   IN-PHASE      LAG      IN-PHASE      LAG
 1   2     *             0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 2  11     *             0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 3  18     *             1 0000E 00 0 0000E-01 0 0000E-01 0 0000E-01
 4  29     *             0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01

RESPONSE ORBIT
                                    ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO. STN  LOCATION  DISPL SLOPE        MAJOR      MINOR        (DEG)     REFERENCE    ELLIPTICITY
 1   2     3 1496    *              2.1225E-04 3 8641E-05 -5 5217E 01-6 1226E 01    6 9197E-01
 2  11    13 5395    *              3.1892E-04 4 1284E-05 -5 5200E 01-5 9150E 01    7 7078E-01
 3  18    21 5673    *              3.3433E-04 3 9296E-05 -5 5310E 01-5 8857E 01    7 8965E-01
 4  29    37 8547    *              1.6985E-04 2 3095E-05 -5 6997E 01-6 2514E 01    7 1614E-01
```

TABLE 18 (CONT.)

	Item Number	Remarks
******************************CALCULATION SUMMARY****************************** TEST IRUN=3,4,5 LUND ROTOR ASYNCHRONOUS BENDING ANALYSIS ASYNCHRONOUS RESPONSE EXCITATION IS STATIONARY BENDING EXCITATION STATIONS 2 11 18 29	14.0	Freq. Group No. 2

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TABLE 18 (CONT.)

Item Number	Remarks
16.0	Freq. Point No. 1 of 9
19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM   FREQUENCY = 9 2800E 01 HZ

BEARING		**** TYPE ****		******** XX ********		******** XY ********		******** YX ********		******** YY ********	
NO.	STN	RADIAL	ANGULAR	K	B	K	B	K	B	K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1.5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2.1097E 05	1 7124E 02

EXCITATION DATA

		**** TYPE ****		***** VERTICAL *****		**** HORIZONTAL ****	
NO	STN	FORCE	MOMENT	IN-PHASE	LAG	IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8 0000E 03 RPM   FREQUENCY = 9 2800E 01 HZ

					** PRINCIPAL RADII	**	INCLINATION	PHASE	
NO	STN	LOCATION	DISPL	SLOPE	MAJOR	MINOR	(DEG)	REFERENCE	ELLIPTICITY
1	[illegible]	3 1-96	*		9 1949E-07	-6 4262E-07	-4 9329E 01	-1 3008E 02	1 7724E-01
2	11	13 5795	*		1 7983E-06	-2 1088E-09	7 7209E 01	-1 0449E 02	9 9766E-01
3	18	21 5473	*		2 1406E-06	1 1553E-07	6 1558E 01	4 3153E 01	8 9758E-01
4	29	37 8547	*		1 6241E-06	-7 6957E-07	6 2728E 00	4 6829E 01	3 5699E-01

TABLE 18 (CONT.)

Printout	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 5123E 01 HZ	16.0	Freq. Point No. 2 of 9
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 5123E 01 HZ	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 5123E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1.5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2 1097E 05	1 7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 5123E 01 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		9 3254E-07	-7 5038E-07	-6 9942E 01	-1 2324E 02	1 0824E-01
2	11	13 5395	*		2 5096E-06	1 1653E-08	6 7558E 01	-1 1153E 02	9 9076E-01
3	18	21 5673	*		2 9970E-06	1 8191E-07	5 6160E 01	-9 6158E 01	8 8555E-01
4	29	37.3547	*		1 8222E-06	-8 7550E-07	8 3613E-01	5 5606E 01	3 5094E-01

TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA	16.0	Freq. Point No. 3 of 9
EXCITATION DATA		
RESPONSE ORBIT	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 7505E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1.5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 3944E 04	2 7264E 02	2 1097E 05	1.7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0.0000E-01	0 0000E-01	0 0000E-01
3	18	*		0.0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 9 0000E 03 RPM FREQUENCY = 9 7505E 01 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	** MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		1 0792E-06	-8 6912E-07	3 8740E 01	-1 2516E 02	1 0781E-01
2	11	13 5395	*		3 3334E-06	-1 6528E-07	5 3177E 01	-1 2422E 02	9 0552E-01
3	18	21 5673	*		3 8293E-06	2 1807E-07	5 1173E 01	-1 2519E 02	8 9224E-01
4	29	37 8547	*		1.9481E-06	-9 9021E-07	-8 3248E 00	7 7868E 01	3 2599E-01

TABLE 18 (CONT.)

Item Number	Remarks
16.0	Freq. Point No. 4 of 9
19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 9946E 01 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1 5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2 1097E 05	1 7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		1.0000E 00	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 9946E 01 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		1 2906E-06	-8 2420E-07	6 7394E 00	-1 1508E 0[illegible]	2 2054E-01
2	11	13 5395	*		2 8061E-06	-4 3374E-07	5 0806E 01	-1 2928E 0[illegible]	7 3225E-01
3	18	21 5673	*		2 9898E-06	1 3474E-07	4 6615E 01	-1 3299E 02	9 1376E-01
4	29	37 8547	*		1 9069E-06	-1 1629E-06	-1 6004E 01	9 2867E 01	2 4237E-01

TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 0245E 02 HZ	16.0	Freq. Point No. 5 of 9
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 0243E 02 HZ	19.1	

LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 0245E 02 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5345E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1 5177E 02
2	29	*		8 9902E 05	1 1572E 03	5 6963E 05	2 7264E 02	4 9946E 04	2 7264E 02	2 1097E 05	1 7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 0243E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		1 3431E-06	-7 9524E-07	-6 2136E 00	-9 2227E 01	2 5622E-01
2	11	17 5395	*		1 9939E-06	-5 6788E-07	4 6292E 01	-1 2707E 02	5 5665E-01
3	18	21 5673	*		1 9408E-06	5 2465E-08	4 2481E 01	-1 3783E 02	9 4736E-01
4	29	37 8547	*		1 9782E-06	-1 3348E-06	-1 8512E 01	1 0128E 02	1 9421E-01

TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA (see printout below)	16.0	Freq. Point No. 6 of 9
RESPONSE ORBIT (see printout below)	19.1	

```
LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM      FREQUENCY = 1 0501E 02 HZ

BEARING  **** TYPE ****  ******** XX ********  ******** XY ********  ******** YX ********  ******** YY ********
NO. STN  RADIAL ANGULAR      K          B          K          B          K          B          K          B
 1   2      *           6 6956E 05 9 2412E 02 4 5395E 05 2 2607E 02 3 0312E 04 2 2607E 02 1.7531E 05 1.5177E 02
 2  29      *           8 9993E 05 1 1572E 03 5 6968E 05 2 7264E 02 4 9946E 04 2 7264E 02 2.1097E 05 1.7124E 02
```

```
EXCITATION DATA

       **** T/PE ****  ***** VERTICAL *****  **** HORIZONTAL ****
NO  STN  FORCE  MOMENT   IN-PHASE      LAG      IN-PHASE      LAG
 1   2     *            0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 2  11     *            0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 3  18     *            0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 4  29     *            1 0000E 00 0 0000E-01 0 0000E-01 0 0000E-01
```

```
RESPONSE ORBIT
ROTATIONAL SPEED = 8 0000E 03 RPM      FREQUENCY = 1 0501E 02 HZ

                                ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO  STN  LOCATION  DISPL SLOPE    MAJOR       MINOR      (DEG)     REFERENCE    ELLIPTICITY
 1   2    3 1496     *         1 3434E-06-7 8087E-07 -1 2856E 01-6 '379E 01    2 6481E-01
 2  11   13.5395     *         1 4916E-06-6 2152E-07  4 4883E 01-1 1827E 02    4 1177E-01
 3  18   21.5673     *         1 3397E-09 6 3160E-09  3 8757E 01 1 4135E 02    9.9062E-01
 4  29   ?7.8547     *         2.0634E-06-1 4480E-06 -2 2064E 01 1 1216E 02    1 7525E-01
```

TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA	16.0	Freq. Point No. 7 of 9
EXCITATION DATA		
RESPONSE ORBIT	19.1	

```
LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM      FREQUENCY = 1 0764E 02 HZ

BEARING  **** TYPE ****  ******** XX ********  ******** XY ********  ******** YX ********  ******** YY ********
NO. STN  RADIAL ANGULAR      K          B          K          B          K          B          K          B
 1   2     *             6 6956E 05 9 2412E 02 4 5395E 05 2 2607E 02 3 0312E 04 2 2607E 02 1.7531E 05 1 5177E 02
 2  29     *             8 9993E 05 1 1572E 03 5 6968E 05 2 7264E 02 4.9946E 04 2.7264E 02 2 1097E 05 1 7124E 02
```

```
EXCITATION DATA

        **** TYPE ****  ***** VERTICAL *****  **** HORIZONTAL ****
NO  STN  FORCE  MOMENT   IN-PHASE     LAG      IN-PHASE     LAG
 1   2     *           0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 2  11     *           0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 3  18                 0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 4  29     *           1 0000E 00 0 0000E-01 0 0000E-01 0 0000E-01
```

```
RESPONSE ORBIT
ROTATIONAL SPEED = 8 0000E 03 RPM      FREQUENCY = 1 0764E 02 HZ

                                 ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO  STN  LOCATION  DISPL SLOPE     MAJOR      MINOR      (DEG)      REFERENCE    ELLIPTICITY
 1   2    3 1496     *           1 2915E-06-7 4782E-07 -1 6883E 01-5 0777E 01   2 5661E-01
 2  11   13 5395     *           1 1586E-06-6 1553E-07  4 6899E 01-1 0483E 02   3 0609E-01
 3  18   21 5673     *           9 8842E-07-1 9568E-08  3 5405E 01-1 4405E 02   9 6117E-01
 4  29   37 8547                 2 0700E-06-1 4802E-06 -2 7358E 01 1 2248E 02   1 6614E-01
```

TABLE 18 (CONT.)

```
LEVEL 2 BEARING DATA
ROTATIONAL SPEED = 8 0000E 03 RPM      FREQUENCY = 1 1034E 02 HZ

BEARING  **** TYPE ****  ******** XX ********  ******** XY ********  ******** YX ********  ******** YY ********
NO. STN  RADIAL ANGULAR      K          B          K          B          K          B          K          B
 1   2     *            6 6956E 05 9 2412E 02 4 5395E 05 2 2607E 02 3 0312E 04 2 2607E 02 1.7531E 05 1.5177E 02
 2  29     *            8 9993E 05 1.1572E 03 5 0968E 05 2 7264E 02 4 9946E 04 2 7264E 02 2 1097E 05 1.7124E 02
```

```
EXCITATION DATA

        **** TYPE ****  ***** VERTICAL *****  **** HORIZONTAL ****
NO. STN  FORCE  MOMENT   IN-PHASE     LAG       IN-PHASE     LAG
 1   2     *            0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 2  11     *            0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 3  18     *            0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 4  29     *            1 0000E 00 0 0000E-01 0 0000E-01 0 0000E-01
```

```
RESPONSE ORBIT
ROTATIONAL SPEED = 9 0000E 03 RPM      FREQUENCY = 1 1034E 02 HZ

                                  ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO  STN  LOCATION  DISPL SLOPE      MAJOR       MINOR        (DEG)    REFERENCE     ELLIPTICITY
 1   2    3.1496     *          1.1854E-06-6 8638E-07 -1 9650E 01-3 9871E 01    2 6659E-01
 2  11   13.5395     *          9.1316E-07-5 5726E-07  5 2537E 01-9 0971E 01    2 4203E-01
 3  18   21.5673     *          7 6545E-07-3 4113E-08  3 2330E 01-1 4626E 02    9 1467E-01
 4  29   37.8547     *          1 9784E-06-1 4219E-06 -3 3774E 01 1 3053E 02    1 6367E-01
```

Item Number	Remarks
16.0	Freq. Point No. 8 of 9
19.1	

TABLE 18 (CONT.)

	Item Number	Remarks
LEVEL 2 BEARING DATA ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 1310E 02 HZ	16.0	Freq. Point No. 9 of 9
EXCITATION DATA		
RESPONSE ORBIT ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 1310E 02 HZ	19.1	

LEVEL 2 BEARING DATA

ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 1310E 02 HZ

BEARING NO.	STN	**** TYPE **** RADIAL	ANGULAR	******** XX ******** K	B	******** XY ******** K	B	******** YX ******** K	B	******** YY ******** K	B
1	2	*		6 6956E 05	9 2412E 02	4 5395E 05	2 2607E 02	3 0312E 04	2 2607E 02	1 7531E 05	1 5177E 02
2	29	*		8 9993E 05	1 1572E 03	5 6968E 05	2 7264E 02	4 9946E 04	2 7264E 02	2.1097E 05	1.7124E 02

EXCITATION DATA

NO	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0.0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0.0000E-01	0.0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0.0000E-01
4	29	*		1 0000E 00	0 0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT

ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 1 1310E 02 HZ

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII ** MAJOR	MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		1.0459E-06	-6 0628E-07	-2 1725E 01	-3.2197E 01	2.6609E-01
2	11	13 5395	*		7 2882E-07	-4 6638E-07	6 0389E 01	-8 0184E 01	2 1958E-01
3	18	21.5673	*		6 1434E-07	-4 2263E-08	2 9393E 01	-1 4825E 02	8 7127E-01
4	29	37 8547	*		1 8182E-06	-1 2950E-06	-4 0660E 01	1 3621E 02	1 6804E-01

TABLE 18 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	B CHECK	Item Number	Remarks
1	9.757D 01	2 256D 00	3.602D-01	3 210D-02	2 491D-05	-1 985D-06	8 829D-12	2 543D-02	21.2	Mode No. 1

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TABLE 18 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	1 027D 02	4 001D 00	1 500D-01	1 299D-01	3 930D-04	-4 559D-06	1 285D-04	-2 286D-01	21.2	Mode No. 2
2	1 027D 02	4 001D 00	1 498D-01	1 301D-01	1 494D-05	-6 913D-07	2 014D-07	-2 287D-01		

TABLE 18 (CONT.)

Item Number	Remarks
21.3	Mode No. 1 of 2 in Freq. Group No. 1

ROTATIONAL SPEED = 8 0000E 03 RPM
FREQUENCY = 9 7573E 01 HZ
CRIT DAMP RATIO = 3 2096E-02
PRECISION INDEX = 2 5428E-02

NATURAL MODE SHAPE

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		3 0138E-01	4 9344E-02	4 7018E 01	4 3296E 01	7 1861E-01
2	11	13 5395	*		1 5780E 00	-6 1414E-02	1 4430E 01	1 4009E 01	9 2508E-01
3	18	21.5673	*		1 8817E 00	-1.1828E-01	1 3263E 01	1 2414E 01	8 8172E-01
4	29	37 8547	*		1 8765E-01	8 5406E-02	5 7390E 01	4 2713E 01	3 7444E-01

RESONANT RESPONSE

EXCITATION DATA

NO.	STN	**** TYPE **** FORCE	MOMENT	***** VERTICAL ***** IN-PHASE	LAG	**** HORIZONTAL **** IN-PHASE	LAG
1	2	*		0.0000E-01	0.0000E-01	0.0000E-01	0 0000E-01
2	11	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
3	18	*		0 0000E-01	0 0000E-01	0 0000E-01	0 0000E-01
4	29	*		1 0000E 00	0.0000E-01	0 0000E-01	0 0000E-01

RESPONSE ORBIT

NO	STN	LOCATION	DISPL	SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*		1 9591E-06	4 5282E-08	-3 8989E 01	1 4069E 02	9 5482E-01
2	11	13.5395	*		4 5969E-06	-1 0704E-06	-3 1652E 00	1.6623E 02	6.2227E-01
3	18	21.5673	*		5 3932E-06	-5.7947E-07	3 3357E 00	1 7633E 02	8 0596E-01
4	29	37 8547	*		2.9524E-06	9 1331E-07	-5 0903E 01	-6 2222E 01	5 2749E-01

TABLE 18 (CONT.)

```
ROTATIONAL SPEED = 8.0000E 03 RPM
FREQUENCY        = 1 0266E 02  HZ
CRIT DAMP RATIO  = 1.3008E-01
PRECISION INDEX  =-2 2869E-01
NATURAL MODE SHAPE
                               ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO  STN  LOCATION  DISPL SLOPE     MAJOR       MINOR       (DEG)     REFERENCE     ELLIPTICITY
 1   2    3.1496     *          2 4206E-01-1 1432E-01  4 5042E 01 5 1253E 01    3.5845E-01
 2  11   13.5395     *          1.4276E 00-2 4464E-01  1 5216E 01 1 3154E 01    7.0741E-01
 3  18   21.5673     *          1.7127E 00-2.8725E-01  1 3353E 01 1 1073E 01    7 1275E-01
 4  29   37.8547     *          1.3626E-01 9 1211E-03  3 5805E 01 3 4396E 01    8 7452E-01


RESONANT RESPONSE



EXCITATION DATA

        **** TYPE ****  ***** VERTICAL *****  **** HORIZONTAL ****
NO  STN  FORCE  MOMENT   IN-PHASE      LAG      IN-PHASE      LAG
 1   2     *            0 0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 2  11     *            0.0000E-01 0 0000E-01 0 0000E-01 0 0000E-01
 3  18     *            0 0000E-01 0.0000E-01 0 0000E-01 0 0000E-01
 4  29     *            1 0000E 00 C.0000E-01 0 0000E-01 0 0000E-01

RESPONSE ORBIT
                               ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO  STN  LOCATION  DISPL SLOPE     MAJOR       MINOR       (DEG)     REFERENCE     ELLIPTICITY
 1   2    3 1496     *          2.1417E-06 5 3578E-07 -4 8344E 01 1 1967E 02    5.9279E-01
 2  11   13.5395     *          2 9368E-06-5 2480E-08 -1 5849E 01 1 6459E 02    9 6489E-01
 3  18   21.5673     *          2.7486E-06 1 2159E-07  1 5922E 00-1 7894E 02    9.1527E-01
 4  29   37.8547     *          3.3333E-06 5 0853E-07 -5.1830E 01-6 0985E 01    7.3527E-01
```

21.3

Mode No. 2 of 2 in Freq. Group No. 1

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Response orbits at frequencies which span each frequency range are listed in order. The input set-up required the response to be calculated at damped natural frequencies and allowed up to two damped natural modes to be found in each frequency range. The record of iteration progression in the process of finding the damped natural mode is to be printed out and starting values for the iterative processes are furnished according to the results of stability analysis, which will be discussed in the next section.

For the first frequency range, the iteration record shows convergence to be reached in two cycles for the first mode. The rapid convergence is expected because the starting values are actually the converged solution obtained from the stability analysis. Improvement achieved by iteration steps is beyond the resolution of the output format. There is no other mode expected in this frequency range, therefore, the same starting values are used for finding the "non-existing" second mode. The iteration record shows 20 cycles without achieving convergence. Additional discussions on the iteration record will be found in Section 3.4.5. The damped natural frequency of 68.11 Hz is seen to be quite close to the undamped approximation of 68.62 Hz. The critical damping ratio of 0.030 suggests a rather modest stability margin. Amplitude distribution of the major radii is not too different from the amplitude distribution of the undamped, isotropic mode shape. All orbit parameters are fairly uniform. The minor radius is less than (1/5) of the major radius at all stations, indicating substantial anisotropy of the system. Since the minor radii are all positive, the orbits are co-rotational. Response amplitude at station 18, where the unit excitation is located, shows a major radius of 3.34×10^{-4} in. This is over 30 times larger than that calculated at 68.05 Hz, which is the closest assigned frequency. Clearly, if the system is not highly damped, it is important that response be calculated precisely at the natural frequency.

In the second frequency range, convergence is similarly rapid for both modes. The natural mode shapes have very similar distributions of major radii. The minor radii of the mode at 102.66 Hz are algebraically smaller than those of the mode at 97.57 Hz at all stations, however the differences are very small. There are no clear-cut shape features to distinguish those two modes. The values of the critical damping ratio are quite different. The mode at 97.57 Hz is not well damped, critical damping ratio = 0.032, and the resonant response to the "misplaced" excitation is more or less similar to the natural mode shape and the amplitudes at all stations are significantly higher than those at the nearest assigned frequency of 97.51 Hz. The mode at 102.66 Hz is fairly well damped. The resonant response to the "misplaced" excitation no longer resembles the natural mode shape, and peaks only moderately above the response at the nearest assigned frequency of 102.45 Hz.

3.4.5 Stability Analysis (IRUN = 5)

From the second run of asynchronous resonance analysis, estimates can be made on the approximate frequencies of the damped natural modes. In fact, one might as well use the same speed-frequency combinations of the asynchronous resonance analysis. Pre-stored intermediate data can be accessed in lieu of repeating Level I calculations. One mode is expected in the first frequency range, while two modes are expected in the second frequency range. Thus, the single input set-up, which covers both frequency ranges, specifies two modes to be sought in each frequency range. The second frequency range is somewhat wider than if only one mode is to be sought (see Section 2.2), nine frequency points are used to divide up this range, two more points than the number used for the first frequency range to ensure computation accuracy.

Since there is as yet no clue on the approximate value of damping associated with various modes, a first trial was run allowing the program to use its internal logic to initiate the iteration process. The output of this run is given as Table 19.

TABLE 19

COMPUTER OUTPUT OF STABILITY ANALYSIS

FIRST TRIAL WITH INTERNAL STARTING LOGIC

(From Input Data of Table 10)

8 Sheets

Output	Item Number	Group
******************************CALCULATION SUMMARY****************************** TEST IRUN=3,4,5 LUND ROTOR ASYNCHRONOUS BENDING ANALYSIS WHIRL STABILITY ANALYSIS BENDING EXCITATION STATIONS 2 11 18 29	14.3	Freq. Group No. 1

TABLE 19 (CONT.)

i	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	δ CHECK	Item Number	Remarks
1	6 805D 01	0 000D-01	3 627D-01	0 000D-01	2 484D-01	1 246D 00	2 754D 04	1 724D-01	21.2	Mode No. 1
2	6 830D 01	1 246D 00	3 172D-01	2 876D-02	-1 892D-01	1 182D-01	3 558D 02	-1 509D-02		
3	6 811D 01	1 364D 00	3 328D-01	2 876D-02	7 413D-03	-7 264D-03	1 132D 00	-1 509D-02		
4	6 811D 01	1 357D 00	3 322D-01	2 999D-02	6 991D-06	-1 799D-05	4 941D-06	-3 672D-02		

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TABLE 19 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	6 585D 01	0 0000-01	-4 360D-02	0 000D-01	-2 996D 00	-9 775D-01	4 997D 08	0 000D-01	21.2	Mode No. 2
LOWER-BOUND RESET										
2	6 300D 01	-9 288D-01	-1 414D-02	5 211D-01	-4 447D 00	-1 572D 00	1 185D 09	2 440D 00		
3	6.805D 01	-2 501D 00	-2 584D-01	7 110D-02	-7 366D-01	7 336D 00	2 313D 08	-1 490D 00		
4	6.731D 01	4 836D 00	-2.664D-02	-1 348D 00	4 698D 00	1 642D-01	4 408D 07	1 808D 00		
5	7 201D 01	5 000D 00	-2 866D-02	-1 212D 00	4 953D 00	3 406D-02	4 792D 06	1 241D 00		
UPPER-BOUND RESET										
6	7 350D 01	5 010D 00	-5 086D-02	-6 702D-01	4 115D 00	6 766D-03	2 479D 06	9 101D-01		
7	6 805D 01	5 017D 00	-2 557D-02	-1 442D 00	4 832D 00	1 434D-01	3 056D 07	1 739D 00		
8	7 288D 01	5 160D 00	-3 465D-02	-1 442D 00	4 604D 00	1 415D-02	3 325D 06	1 739D 00		
UPPER-BOUND RESET										
9	7 350D 01	5 162D 00	-4 593D-02	-7 645D-01	4 101D 00	1 035D-02	2 491D 06	9 545D-01		
10	6 805D 01	5 172D 00	-2 759D-02	-1 378D 00	4 705D 00	1 297D-01	3 134D 07	1 647D 00		
11	7 275D 01	5 302D 00	-3 188D-02	-1 378D 00	4 654D 00	1 716D-02	3 544D 06	1 647D 00		
UPPER-BOUND RESET										
12	7 350D 01	5 305D 00	4 239D-02	-8 513D-01	4 086D 00	1 262D-02	2 505D 06	9 943D-01		
ITERATION OSCILLATES AFTER 12 TRIALS FOR ROOT# 2										

TABLE 19 (CONT.)

Computer output	Item Number	Remarks
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 6 8114E 01 HZ CRIT DAMP RATIO = 2 9986E-02 PRECISION INDEX =-3 6721E-02 NATURAL MODE SHAPE	21.3	Freq. Group No. 1

NO	STN	LOCATION	DISPL SLOPE	PRINCIPAL RADII MAJOR	PRINCIPAL RADII MINOR	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3 1496	*	1 0790E 00	2 1341E-01	-5 5313E 01	-4 1353E 01	6 6976E-01
2	11	13 5395	*	1 6280E 00	2 8776E-01	-5 4080E 01	-4 3643E 01	6 9959E-01
3	18	21 5673	*	1 7083E 0C	2 9165E-01	-5 3956E 01	-4 0753E 01	7 0835E-01
4	29	37 8547	*	8 6442E-01	1 5431E-01	-5 7041E 01	-4 4088E 01	6 9706E-01

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TABLE 19 (CONT.)

	Item Number	Remarks
******************************CALCULATION SUMMARY****************************** TEST IRUN=3,4,5 LUND ROTOR ASYNCHRONOUS. BENDING ANALYSIS WHIRL STABILITY ANALYSIS BENDING EXCITATION STATIONS 2 11 18 29	14.0	Frequency Group No. 2

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TABLE 19 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	1 024D 02	0 0000D-01	2 507D-01	0 0000D-01	-4 200D 00	4 156D 00	1 558D-01	3 808D-01	21.2	Mode No. 1
2	9.825D 01	4 156D 00	3.277D-01	6 453D-02	-4 084D-01	-2 402D 00	6 757D-02	-2 280D-01		
3	9.784D 01	1.754D 00	3 579D-01	2.505D-02	-2 411D-01	5 035D-01	7 373D-03	7 216D-02		
4	9.760D 01	2 263D 00	3 594D-01	2 505D-02	-2 614D-02	-6 581D-03	1 133D-05	7 216D-02		
5	9 757D 01	2.256D 00	3 602D-01	3 210D-02	1 266D-04	1 247D-05	2 389D-10	2 543D-02		
6	9 757D 01	2.256D 00	3 602D-01	3 210D-02	2 678D-09	-2 572D-10	9 925D-20	2 543D-02		

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TABLE 19 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	D-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	1 024D 02	0 0000-01	2 1550-03	0 0000-01	2 3200 01	2 0620-01	6 1500 01	-1 1590 00		
UPPER-BOUND RESET										
2	1 131D 02	9 4690-02	5 3770-03	7 7850-02	3 5110 00	1 0750-01	1 3980 01	-2 3630-01		
3	1 024D 02	2 0220-01	2 1360-03	7 7650-02	2 2230 01	1 8180-01	5 9420 01	-2 3630-01		
UPPER-BOUND RESET										
4	1 131D 02	2 8290-01	5 3600-03	7 7850-02	3 4710 00	1 1820-01	1 3480 01	-2 3630-01	21.2	Mode No. 2
5	1 024D 02	3 9630-01	2 2510-03	8 6570-01	2 1220 01	1 5710-01	5 7630 01	-1 3670 00		
UPPER-BOUND RESET										
6	1 131D 02	4 7580-01	5 3460-03	8 6070-01	8 3490 00	1 0860-01	1 3010 01	-1 3670 00		
7	1 024D 02	5 8440-01	2 3520-03	1 2130 00	1 9930 01	1 3290-01	5 6100 01	-1 4240 00		
UPPER-BOUND RESET										
8	1 131D 02	6 5540-01	5 3330-03	5 4340-01	8 2690 00	1 0920-01	1 2590 01	-4 7030-01		

ITERATION OSCILLATES AFTER 8 TRIALS FOR ROOT# 2

TABLE 19 (CONT.)

	Item Number	Remarks
ROTATIONAL SPEED = 8 0000E 03 RPM FREQUENCY = 9 7573E 01 HZ CRIT DAMP RATIO = 3 2096E-02 PRECISION INDEX = 2 5428E-02 NATURAL MODE SHAPE	21.3	Mode No. 1

NO	STN	LOCATION	DISPL SLOPE	** PRINCIPAL RADII MAJOR	MINOR	** INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*	3 8172E-01	4 7744E-02	[illegible]	4 2296E 01	7 1961E-01
2	11	13 5295	*	1 5726E 00	[illegible]	4 4892E 01	1 4002E 01	9 2508E-01
3	18	21 5672	*	1 8817E 00	1 9282E-01	1 3267E 01	1 2414E 01	8 8172E-01
4	29	37 8547	*	1 8765E-01	[illegible] 5406E-02	5 7390E 01	4 2713E 01	3 7444E-01

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In the frequency range, convergence of iteration for the first mode was reached after four cycles. There being no second mode, the search effort resulted in repeated attempts to exceed the upper bound of the frequency range. The residue shows no definite trend of reducing. The modal mass consistently shows an anomalous negative value. The natural frequency and the critical damping ratio are exactly equal to those given in Table 18 as far as the available digits reveal. The mode shape parameters are slightly different owing to a somewhat larger residue at the last cycle of iteration. In the second frequency range, convergence of iteration for the first mode was reached after six cycles, with an extremely small residue at the end. Search for the second mode, however, did not succeed. The iteration process developed an oscillating pattern. A closer look revealed that after each oscillating cycle, the residue does reduce monotonically. Therefore, the attempt to find the second mode in this frequency range was continued afterwards. Modal parameters of the first mode at 97.57 Hz are identical to those given in Table 18.

A second trial was made using converged solutions of the first modes in each frequency range as the respective starting values. For the second set of starting values, the first set was repeated in the first frequency range since another mode is really not expected. The designated values for continuation as shown by the previous iteration record was used as the second set in the second frequency range. The second trial still failed to establish the second mode in the second frequency range. Previously noted trend of a monotonically reducing residue persisted at a moderate pace. Therefore, a third trial was made with the starting damping value "accelerated" beyond the trend indicated by the iteration record of the second trial. This time convergence was reached in four cycles. Iteration records pertaining to the second mode in the second frequency range are shown in Table 20.

TABLE 20

COMPUTER OUTPUT FOR STABILITY ANALYSIS

ITERATION RECORDS OF SECOND AND THIRD TRIALS WITH USER FURNISHED STARTING VALUES

(From Input Data of Tables 11 and 12)

11 Sheets

	Item Number	Remarks
******************************CALCULATION SUMMARY****************************** TEST IRUN=3,4, LUND ROTOR ASYNCHRONOUS BENDING ANALYSIS WHIRL STABILITY ANALYSIS BENDING EXCITATION STATIONS 2 11 18 29	14.0	Freq. Group No. 2, 2nd Trial

TABLE 23 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	6.811D 01	1.357D 00	3.323D-01	2.998D-02	4 075D-04	2 631D-04	1.913D-03	-3 667D-02	21.2	Mode No. 1, 2nd Trial
2	6.811D 01	1.357D 00	3.322D-01	2.999D-02	4.486D-08	3.235D-08	2 596D-11	-3 672D-02		

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TABLE 20 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK
1	6.811D 01	1.357D 00	2.562D-02	3.888D-01	1.769D-02	5.276D-04	1.833D 03	-4.380D-01
2	6.813D 01	1.357D 00	2.569D-02	3.877D-01	-3.050D-04	4.612D-04	4.788D 02	-4.267D-01
3	6.813D 01	1.358D 00	2.569D-02	3.879D-01	-2.977D-04	4.086D-04	3.768D 02	-4.271D-01
4	6.813D 01	1.358D 00	2.568D-02	3.879D-01	-2.639D-04	3.619D-04	2.955D 02	-4.271D-01
5	6.813D 01	1.359D 00	2.568D-02	3.883D-01	-2.339D-04	3.205D-04	2.318D 02	-4.279D-01
6	6.813D 01	1.359D 00	2.567D-02	3.885D-01	-2.072D-04	2.839D-04	1.818D 02	-4.282D-01
7	6.813D 01	1.359D 00	2.567D-02	3.886D-01	-1.836D-04	2.515D-04	1.426D 02	-4.285D-01
8	6.813D 01	1.360D 00	2.567D-02	3.887D-01	-1.627D-04	2.227D-04	1.118D 02	-4.287D-01
9	6.813D 01	1.360D 00	2.566D-02	3.889D-01	-1.441D-04	1.973D-04	8.773D 01	-4.289D-01
10	6.813D 01	1.360D 00	2.566D-02	3.890D-01	-1.277D-04	1.747D-04	6.881D 01	-4.291D-01
11	6.813D 01	1.360D 00	2.566D-02	3.890D-01	-1 131D 4	1.548D-04	5.397D 01	-4.293D-01
12	6.813D 01	1.360D 00	2.566D-02	3.891D-01	-1.002D-04	1.371D-04	4.233D 01	-4.294D-01
13	6.813D 01	1.360D 00	2.565D-02	3.891D-01	-8.879D-05	1.214D-04	3.321D 01	-4.294D-01
14	6.813D 01	1.361D 00	2.565D-02	3.892D-01	-7.865D-05	1.075D-04	2.605D 01	-4.297D-01
15	6.813D 01	1.361D 00	2.565D-02	3.892D-01	-6.967D-05	9.524D-05	2.043D 01	-4.297D-01
16	6.813D 01	1.361D 00	2.565D-02	3.893D-01	-6.172D-05	8.435D-05	1.602D 01	-4.299D-01
17	6.813D 01	1.361D 00	2.565D-02	3.894D-01	-5.467D-05	7.471D-05	1.257D 01	-4.300D-01
18	6.813D 01	1.361D 00	2.565D-02	3.894D-01	-4.842D-05	6.617D-05	9.859D 00	-4.301D-01
19	6.813D 01	1.361D 00	2.565D-02	3.895D-01	-4.289D-05	5.860D-05	7.734D 00	-4.301D-01
20	6.813D 01	1.361D 00	2.565D-02	3.895D-01	-3.799D-05	5.190D-05	6.066D 00	-4.302D-01

AFTER 20 ITERATIONS, NO EIGENVALUE IS FOUND.

Item Number	Remarks
21.2	Mode No. 2, 2nd Trial

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TABLE 20 (CONT.)

Item Number	Remarks
21.3	Mode No. 1, 2nd Trial

```
ROTATIONAL SPEED = 8.0000E 03 RPM
FREQUENCY        = 6.8114E 01  HZ
CRIT DAMP RATIO  = 2.9986E-02
PRECISION INDEX  =-3.6718E-02
NATURAL MODE SHAPE

NO. STN  LOCATION  DISPL SLOPE  ** PRINCIPAL RADII  ** INCLINATION   PHASE
                                  MAJOR      MINOR       (DEG)      REFERENCE    ELLIPTICITY
 1   2    3.1496       *        1.1335E 00 2.0729E-01 -5.5521E 01-4.1272E 01   6.9089E-01
 2  11   13.5395       *        1.6911E 00 2.3654E-01 -5.6117E 01-4.4358E 01   7.5458E-01
 3  18   21.5673       *        1.7703E 00 2.2968E-01 -5.6353E 01-4.5323E 01   7.7032E-01
 4  29   37.8547       *        9.0723E-01 1.5079E-01 -5.7265E 01-4.4390E 01   7.1496E-01
```

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TABLE 20 (CONT.)

	Item Number	Remarks
********************************CALCULATION SUMMARY******************************* TEST IRUN=3,4,5 LUND ROTOR ASYNCHRONOUS BENDING ANALYSIS WHIRL STABILITY ANALYSIS BENDING EXCITATION STATIONS 2 11 18 29	14.0	Freq. Group No. 2, 2nd Trial

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TABLE 20 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	9.757D 01	2.256D 00	3.602D-01	3.209D-02	-3.751D-04	2.380D-04	3.274D-09	2.545D-02	21.2	Mode No. 1, 2nd Trial
2	9.757D 01	2.256D 00	3.602D-01	3.210D-02	3.588D-09	-1 878D-08	9.627D-18	2.543D-02		

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TABLE 20 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	1.024D 02	7.646D-01	2.446D-03	3.895D-01	1 879D 01	1.108D-01	5 516D 01	-4 302D-01	21.2	Mode No. 2, 2nd Trial
UPPER-BOUND FESET										
2	1.131D 02	8.277D-01	5.321D-03	3.895D-01	8.189D 00	1.099D-01	1.217D 01	-4.302D-01		
3	1.024D 02	9 376D-01	2 675D-03	1.711D 00	1 699D 01	8 918D-02	5 374D 01	-1.443D 00		
UPPER-BOUND RESET										
4	1.131D 02	9.935D-01	5.311D-03	8 269D-01	8.109D 00	1.106D-01	1.178D 01	-6.140D-01		
5	1.024D 02	1.104D 00	2.905D-03	1.855D 00	1.546D 01	7.065D-02	5.286D 01	-1.413D 00		
UPPER-BOUND RESET										
6	1.131D 02	1 153D 00	5.303D-03	1.855D 00	8 029D 00	1.113D-01	1.142D 01	-1 413D 00		
7	1.024D 02	1.264D 00	3.185D-03	1.855D 00	1.397D 01	5.475D-02	5 216D 01	-1.413D 00		
UPPER-BOUND RESET										
8	1.131D 02	1.306D 00	5.295D-03	1.090D 00	7.949D 00	1.120D-01	1.108D 01	-7.481D-01		
ITERATION OSCILLATES AFTER 8 TRIALS FOR ROOT# 2										

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TABLE 20 (CONT.)

```
ROTATIONAL SPEED = 8.0000E 03 RPM
FREQUENCY        = 9.7573E 01  HZ
CRIT DAMP RATIO  = 3.2096E-02
PRECISION INDEX  = 2 5428E-02
NATURAL MODE SHAPE
                              ** PRINCIPAL RADII  ** INCLINATION   PHASE
NO. STN  LOCATION  DISPL SLOPE    MAJOR       MINOR       (DEG)     REFERENCE    ELLIPTICITY
 1   2    3.1496      *        3.0138E-01 4 9344E-02  4.7018E 01 4.3296E 01   7.1861E-01
 2  11   13.5395      *        1 5780E 00-6.1414E-02  1.4480E 01 1.4009E 01   9.2508E-01
 3  18   21.5673      *        1.8817E 00-1 1828E-01  1.3263E 01 1.2414E 01   8 8172E-01
 4  29   37.8547      *        1.8765E-01 8 5406E-02  5 7390E 01 4.2713E 01   3.7444E-01
```

Item Number	Remarks
21.3	Mode No. 1, 2nd Trial

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TABLE 20 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	9.757D 01	2.256D 00	3.602D-01	3.209D-02	-3 751D-04	2 380D-04	3 274D-09	2.545D-02	21.2	Mode No. 1, Freq. Group No. 2, 3rd Trial
2	9.757D 01	2.256D 00	3.602D-01	3.210D-02	3 588D-09	-1 878D-08	9 627D-18	2.543D-02		

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TABLE 20 (CONT.)

I	FREQ	DAMP	MODAL MASS	DAMP RATIO	F-CHANGE	C-CHANGE	RESIDUE	D CHECK	Item Number	Remarks
1	1.024D 02	4.000D 00	3.428D-01	5.698D-02	2 237D-01	1 724D-03	8 079D 01	-1 980D-01	21.2	Mode No. 2, Freq. Group No. 2, 3rd Trial
2	1.026D 02	4.002D 00	1.660D-01	1.175D-01	3 504D-02	-2.249D-04	1 134D 00	-2.206D-01		
3	1.027D 02	4.001D 00	1.505D-01	1.295D-01	1.714D-03	-2.035D-05	2.456D-03	-2 282D-01		
4	1.027D 02	4.001D 00	1.498D-01	1.301D-01	6.845D-05	-3.052D-06	4 208D-06	-2.287D-01		

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TABLE 20 (CONT.)

Item Number	Remarks
21.3	Mode No. 1, Freq. Group No. 2, 3rd Trial

```
ROTATIONAL SPEED = 8.0000E 03 RPM
FREQUENCY        = 9.7573E 01  HZ
CRIT DAMP RATIO  = 3.2096E-02
PRECISION INDEX  = 2.5428E-02
NATURAL MODE SHAPE
```

NO.	STN	LOCATION	DISPL SLOPE	** PRINCIPAL RADII MAJOR	MINOR **	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*	3.0138E-01	4.9344E-02	4.7018E 01	4.3296E 01	7.1861E-01
2	11	13.5395	*	1.5780E 00	-6.1414E-02	1.4480E 01	1.4009E 01	9.2508E-01
3	18	21.5673	*	1.8817E 00	-1.1828E-01	1 3263E 01	1.2414E 01	8.8172E-01
4	29	37.8547	*	1.8765E-01	8 5406E-02	5 7390E 01	4.2713E 01	3.7444E-01

Item Number	Remarks
21.3	Mode No. 2, Freq. Group No. 2, 3rd Trial

```
ROTATIONAL SPEED = 8.0000E 03 RPM
FREQUENCY        = 1 0266E 02  HZ
CRIT DAMP RATIO  = 1.30[illegible]E-01
PRECISION INDEX  =-2.2868E-01
NATURAL MODE SHAPE
```

NO.	STN	LOCATION	DISPL SLOPE	** PRINCIPAL RADII MAJOR	MINOR **	INCLINATION (DEG)	PHASE REFERENCE	ELLIPTICITY
1	2	3.1496	*	2.4206E-01	-1.1432E-01	4.5042[illegible] 01	5.1253E 01	3.5845E-01
2	11	13.5395	*	1.4276E 00	-2.4464E-01	1.5216E 01	1 3154E 01	7 0742E-01
3	18	21.5673	*	1 7127E 00	-2.8725E-01	1.3353E 01	1.1073E 01	7.1275E-01
4	29	37.8547	*	1.3626E-01	9.1212E-03	3 5805E 01	3.4396E 01	8.7452E-01

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SECTION IV

THEORETICAL OVERVIEW

Rotordynamic analysis, in the manner treated here, deals with a linear system. As such, much of the analytical methods which were developed for linear control systems would be applicable. One can indeed go quite far to enumerate analogous concepts between the fields of rotordynamics and linear controls. It is quite appropriate to regard a rotor system as a multi-terminal "black box." Much of the mathematical operations involved in rotordynamic analysis concerns the "transfer functions" of the "black box." The practical aspects of rotor engineering, however, require an understanding of the restrictions of the simplifying assumptions which underlie the method of analysis and an understanding of the general characteristics of rotor systems which may take on special significance as dictated by particular design features and operating environments. To address these issues, the subject matter has to be identified as a high speed rotor system, not merely a linear black box. The present section will examine the "anatomy" of rotordynamic analysis. While the accompanying software specifically utilizes the "frequency-response" approach as a starting point, the physical ideas to be discussed here are less restrictive. Details of mathematical derivations are contained in Sections V and VI. Immediate attention here is directed toward basic concepts.

4.1 Idealized Rotor Structure

While the historical evolution of rotordynamic studies began by emulating the analysis of beam-like structures, the contemporary point of view recognizes two important distinctive features in the dynamic features of a high speed rotor; they are

- gyroscopic inertia, and
- support compliance

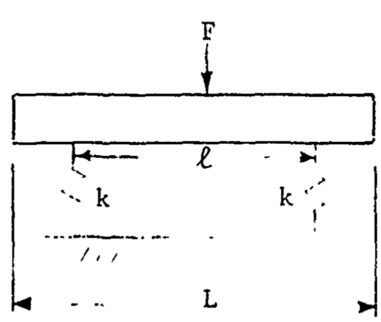
4.1.1 Gyroscopic Effect

In normal operations of a rotor, even during startup and shutdown transients, the rotational speed can be treated as a constant for

lateral motions of the rotor. Since functional requirements of the machinery generally include a well defined rotational axis relative to its platform, any inclination-deflection of the shaft-axis is presumably small. Furthermore, in comparison with the rotational speed, the angular rate of any platform motion is probably quite small. Under these circumstances, Euler's Law for angular motions can be reduced to a cross-axis coupling of the gyroscopic inertia. A thorough derivation of this idea is presented in Appendix A. The consequence of cross-axis gyroscopic coupling is two fold. First of all, lateral motions of the rotor generally would not be planar -- the shaft center motion would typically describe an elliptic orbit. In fact, if rotational symmetry prevails in the rotor system, the orbit would be circular. Secondly, each free vibration mode of the equivalent non-rotating shaft is split into two modes which are distinguishable not only in the natural frequency, but also in the sense of rotation of the whirl motion (relative to the shaft rotation).

4.1.2 Rotor Flexibility

Rotor flexibility is increasingly of importance as mechanical equipment keeps up its pace with technological sophistication. The word "flexibility", however, is a relative quantity in the context of rotordynamic phenomena. Since the main external constraints on the rotor axis are the bearing supports, bearing compliance is in fact the natural reference for rotor flexibility. This idea can be illustrated by a simple example as depicted in the following sketch. Here, a concentrated load is applied to the midpoint of a uniform beam which is supported from the ground by two springs of equal stiffness.



The total deflection at the applied force is the sum of beam deflection $\frac{F\ell^3}{48EI}$ and spring deflection $\frac{F}{2k}$. The ratio of these two quantities, $\frac{k\ell^3}{24EI}$, can be regarded as the "static flexibility index." It is noted that in such an example the overhung portions of the beam have no contribution to static "flexibility". Also, the applied force F is not an explicit parameter. However, it is evident that its location, as well as the span between the springs, defines the physical setup. In a realistic rotor, the rigidity parameter EI would have to represent an average, and the "static flexibility index" reflects the relative insensitivity of the influence of the bearing stiffness on the two lowest natural modes of the rotor. For higher natural modes, the ratio of the bearing span to the characteristic wave length of bending vibration may be considered to be the "dynamic flexibility index" $\ell\sqrt{\omega}/(EI/\rho A)^{\frac{1}{4}}$. This index represents the relative ineffectiveness of support bearings in controlling lateral vibrations of the rotor.

4.1.3 Modeling of Shafting Structure

Shafting can be structurally modeled as a sucession of beam segments which are rigidly connected to each other.

Division of the rotor into interconnected segments is necessary to accomodate variations in shaft cross-sections and/or materials, the attachment of wheel-like appendages as rigid bodies, the presence of bearing constraints, and sources of dynamic excitation. A common first approximation is to assume the inter-segmental junction as a boundary across which deflection, slope, and twist are continuous, whereas jumps in various other physical quantities are allowed to take place as required. Theories of torsion and flexure for slender prismatic bars are commonly used to deal with the structural aspects of the shaft segments. In the original treatments of Holzer and Myklestad, lineal inertia effects are

assigned to the ends of the segment while the segment itself is regarded as a massless torsion or flexure spring. In so doing, each inter-segmental junction becomes automatically a triplet of degrees of freedom, one for torsion and one each for flexure in the two lateral planes. Since closed-form solutions of a uniform shaft with distributed inertia are available for both the torsion and the flexure problems, they can be incorporated into a rotor-dynamic analysis. The main advantage of the latter approach is to allow rotor modeling of slender shaft segments without subdividing. Otherwise, employing the lumped-parameter method in the manner of Holzer and Myklestad, it is necessary to subdivide slender segments such that the length of each subdivided segment is about equal to its diameter. The second order flexure theory, which includes the effects of rotary inertias and shear deformation can also be incorporated to improve accuracy of analysis for higher order flexure modes. In actual practice, however, uncertainties in the treatments of junctions and attached appendages probably would obscure the improvements gained through the use of second order flexure theory. Accuracy of the slender bar theory of flexure also becomes degraded when a rotor of rather stubby proportions is analyzed. In such a circumstance, the important rotor motions are probably rigid body-like; and the slender bar theory is amiss in neglecting the rotary inertias. Shear deformation may be of increased importance relative to flexure, but since the motion is predominently due to bearing support compliance, it is not essential to refine the elastic aspect of the flexure theory. The software prepared here makes use of the distributed analyses of slender bars, while lumped correction is made for rotary inertias. Omission of possible contribution of shear deformation on high order flexure modes is recognized as an inherent shortcoming; this should appear mainly in some uncertainty in the estimated natural frequencies of the higher modes but should not contribute significantly to the estimated mode shapes.

4.1.4 Dynamic Degrees of Freedom

The rotor model described above allows estimates to be made on responses to excitations and the conditions of resonances. Each inter-segmental junction may be considered to be a location of potential excitation. For the present discussion simple harmonic temporal dependence is assumed for all excitations. An excitation may be any combination of torsion, lateral force, and lateral moment. Since the lateral force and the lateral moment may be in either of the two mutually perpendicular planes, there can be up to four independent degrees of freedom for flexure excitation. The torsional motion is basically uncoupled from the flexural motions in the sense that a torsional moment would not induce a flexural motion and a lateral force or moment would not result in a torsional twist. There may of course be simultaneous torsional and flexural excitions such as through a gear mesh. Flexural motions in the two lateral planes are coupled through gyroscopic inertia in the conventional Cartesian coordinate system. Prevailing symmetry, however, allows the flexural motion to be described in a pair of whirl components, which have opposite senses of rotation and are uncoupled from each other, provided the bearing support characteristics are isotropic .

4.1.5 Mobility and Stiffness Matrices

Corresponding to the excitation components of torsional moment, lateral force, and lateral moment, the response consists of displacement components of torsional twist, lateral deflection, and lateral slope at each junction. A complete description of the response of the system per unit of each excitation is given by the influence coefficients which form the mobility or response matrix. This matrix is the analog of a transfer function of a control system. The inverse of the mobility matrix is the impedance or stiffness matrix. For a given shafting model, these matrices are dependent on both shaft rotation speed and the frequency of excitation.

Because the model is conservative and isotropic, these matrices are symmetrical and are also quasi-diagonal; i.e., the influence coefficients among torsional, lateral forward whirl, and lateral backward whirl degrees of freedom are all zero. At resonant conditions, the mobility matrix is both unbounded and degenerate, all columns are proportional to each other, and because of symmetry all rows are proportional to each other. The stiffness matrix also becomes singular at resonance, but its elements remain finite. These properties are confined to the applicable partitioned submatrices because these submatrices are uncoupled from one another.

4.2 Bearing Support Characteristics

4.2.1 Matrix Representation

The function of a bearing is to restrict the rotor axis to a nominal axis under realistic static and dynamic load environments. Deviation of any particular point of the rotor axis from the nominal line can be characterized by three lineal and two angular displacements. These may be designated as (δ_x, δ_y, δ_z, s_x, s_y) in accordance with a right-handed Cartesian reference system. The z-coordinate is coincident with the reference axis and is directed toward the spin vector. (s_x, s_y) are rotor axis inclinations respectively in the z-x and z-y planes. The x-coordinate is directed toward the predominant static load; e.g., earth gravity. The bearing would resist the occurrence of any displacement so that the reaction force system imparted by the rotor to the bearing is generally expressed in matrix notation as

$$\underline{R} = \underline{\underline{Z}} \cdot \underline{X}$$

$\underline{R}$ is a column vector comprising the five reaction components (F_x, F_y, F_z, M_x, M_y) while $\underline{X}$ is the displacement vector (δ_x, δ_y, δ_z, s_x, s_y). $\underline{\underline{Z}}$ is a (5 x 5) matrix containing the elements Z_{ij} with both indices (i, j) ranging from 1 to 5. The values of Z_{ij} characterize how rotor displacements are being resisted by the bearing.

Ideally it would be desirable to design the bearing such that Z_{ij} is diagonal (e.g., $Z_{ij} = 0$ if $i \neq j$). In reality, some cross-coupling always exists; although it is commonly reasonable to assume independence between the linear (δ_x, δ_y) and the angular (s_x, s_y) as well as between the axial (δ_z) and the transverse (δ_x, δ_y) lineal degrees of freedom. That is:

$$Z_{1j} = Z_{2j} = Z_{j1} = Z_{j2} = 0 \text{ for } j > 2$$

$$Z_{3j} = Z_{j3} = 0 \text{ for } j < 3$$

Such null conditions are presumed in the present work. That is, cross-coupling exists between δ_x and δ_y and also between s_x and s_y only. The axial characteristics of the bearing support affect only the axial dynamics of the rotor, which are commonly quite benign in comparison with the much more compliant torsional and lateral degrees of freedom. Henceforth, no further attention will be given to the axial motion. Thus, bearing characteristics concern mainly the four lateral degrees of freedom.

Matrix elements Z_{ij} are often dependent on the displacement amplitudes and sometimes also on their time histories. Thus, there are circumstances when rotodynamics is studied by step-by-step numerical integration with respect to time. Although such methods are theoretically more rigorous, they have proven to have only limited usefulness because

- Computational cost consideration precludes execution of such methods except for the simplest mechanical models.
- Stability difficulty associated with numerical integration often masks credibility of such results.

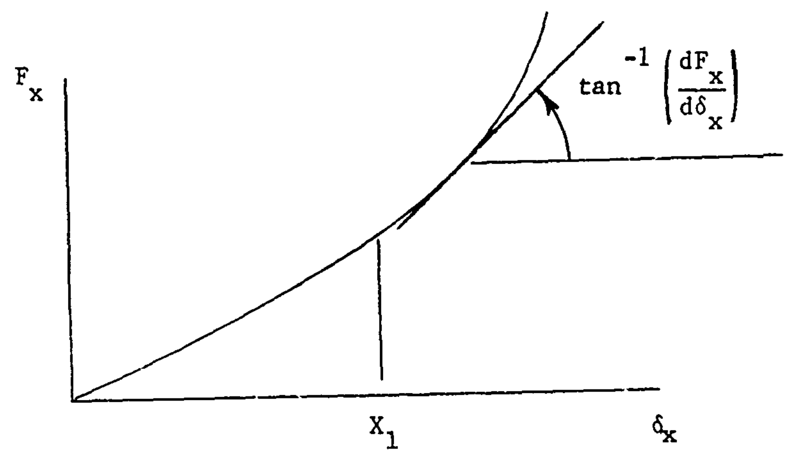
* The method of dynamic perturbation analysis, particularly if one includes the question of stability, has proven to be quite adequate in most realistic engineering problems. Questions of large amplitudes and anomalous histories can be relegated to the qualitative judgment of the experienced rotordynamic expert; in only extremely rare circumstances do such questions require serious consideration.

4.2.2 Linearization

From the standpoint of dynamic perturbation, distinction is made between a static equilibrium component and a dynamic perturbation component for both the displacements and the reactions. Thus,

$$\underline{X} = \underline{X}_0 + \underline{X}'; \quad \underline{F} = \underline{F}_0 + \underline{F}'$$

($\underline{X}'$, F') are respectively presumed to be infinitesimal in comparison with ($\underline{X}_0$, $\underline{F}_0$). Accordingly, Z_{ij} are regarded as dependent on $\underline{X}_0$ but not on $\underline{X}'$. To illustrate the idea of perturbation linearization one may examine the one-dimensional load-displacement curve.



According to the illustrated curve, a straight-line relationship appears to be quite adequate for $0 < \delta_x > X_1$. In this range Z_{11} may be regarded as a constant. However, for $\delta_x > X_1$, incremental change of F_x is

$$F_x' = \frac{dF_x}{d\delta_x} \delta_x'$$

where δ_x' is the incremental displacement. $\frac{dF_x}{d\delta_x}$ will depend on

the amplitude of δ_x for $\delta_x > X_1$.

The question of history dependence is circumvented by regarding $\underline{X}'$ as a periodic motion at any frequency ν of interest. The matrix elements Z_{ij} accordingly would have both real and imaginary parts and may also be dependent on both the rotor speed ω and the vibration frequency ν. In effect, the dynamic problems are treated with the frequency domain analysis.

To avoid notational clumsiness, the primes will be dropped from $\underline{F}'$ and $\underline{X}'$ so that $\underline{F}$ and $\underline{X}$ are understood to be dynamic perturbation quantities unless the subscript "0" is used to designate the static equilibrium condition.

4.2.3 Speed Dependence

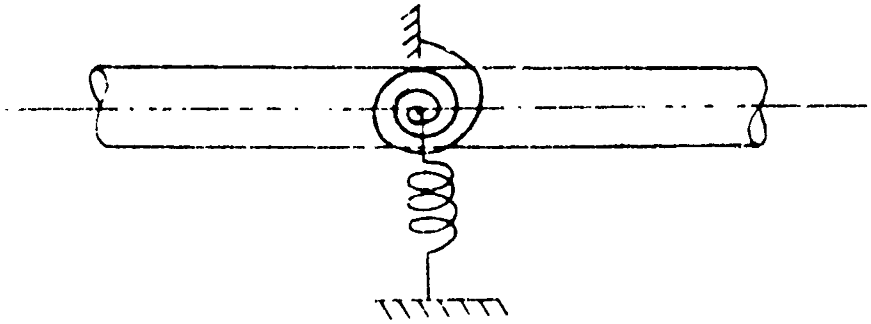
For the two most common bearing types, namely rolling-element and fluid-film bearings, speed is an important parameter.

Fluid-film bearing forces are explicitly proportional to the operating speed. A secondary speed effect is caused by the temperature sensitivity of lubricant viscosity. The effective lubricant viscosity usually decreases since a larger amount of heat is generated at higher speeds.

The deflection of a rolling-element bearing under load is due to the contact compliances at both inner and outer races. At a high operating speed, centrifugal acceleration of the rolling-element increases the contact load at the outer race while the contact with the inner race is accordingly relieved. Since the load-deflection characteristics of race contacts are highly non-linear, the stiffness of a rolling-element bearing becomes speed dependent.

4.2.4 Dynamic Characterization of Bearing Support

Accurate calculation of the lateral dynamic response of a high speed rotor depends on realistic characterization of the support bearings. In the most general case, both lineal and angular motions are restrained by the support bearings at the attachment location. In the analytical model, the reaction force and the reaction moment of each bearing are felt by the rotor through a single station of the rotor axis. As schematically illustrated below, a coil spring restraining the lateral displacement and a



Bearing Stiffness Model

torsion spring tending to oppose an inclination are attached to the same point of the rotor axis. A complete description of the characteristics of the support bearings, however, involves much more than the specification of the two spring constants. This is because:

- The lateral motion of the rotor axis is associated with two displacement components and two inclination components.
- The restraining characteristics may include cross coupling among various displacement/inclination coordinates.
- The restraining force/moment may not be temporally in phase with the displacement/inclination.
- The restraining characteristics of the bearing may be dependent on either the rotor speed or the frequency of

vibration or both.

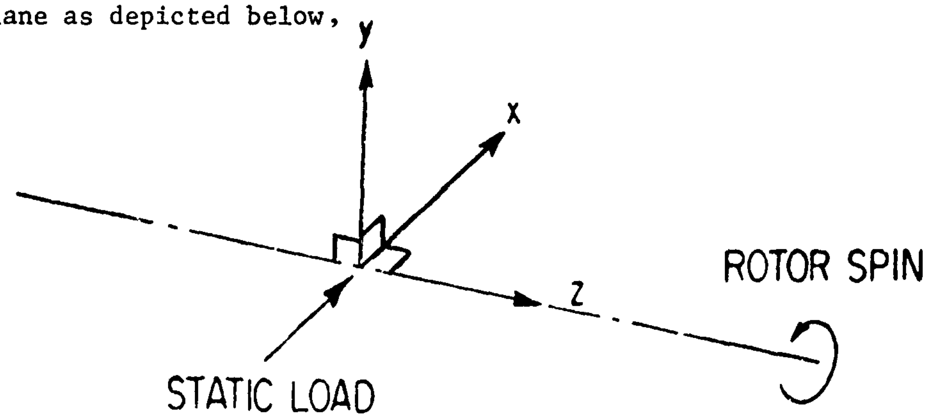
- Bearing pedestal compliance may not be negligible.

To accommodate the above considerations, the support bearing characteristics are described by a four-degrees-of-freedom impedance matrix as defined in the equation

$$\underline{R}_N = \underline{\underline{Z}}_N \cdot \underline{W}_N$$

where $\underline{W}_N$ is a column vector containing elements which are the two lateral displacements δ_x, δ_y, and the two lateral inclinations s_x, s_y of the rotor axis at the bearing station N.

Employing a right handed Cartesian representation in a lateral plane as depicted below,



Coordinate System

the z-axis is coincident with the spin vector of the rotor. The x-axis is oriented in the direction of the external static load, and the y-axis is perpendicular to both z and x axes forming the the right handed triad (x, y, z). δ_x, δ_y are respectively lateral lineal displacement components of the rotor axis al· ·g the x, y directions. s_x, s_y are lateral inclination components respectively

in the z-x, z-y planes. Note that s_x is a rotation about the y-axis, while s_y is a rotation about the negative x-axis.

$\underline{\underline{Z}}_N$ is a complex 4 X 4 matrix, and, in accordance with the common notation for stiffness and damping coefficients, may be expressed as

$$\underline{\underline{Z}}_N = \underline{\underline{K}}_N + i\nu\underline{\underline{B}}_N$$

where $\underline{\underline{K}}_N$ is the stiffness matrix, $\underline{\underline{B}}_N$ is the damping matrix, and ν is the frequency of vibration. Most commonly, lateral linear and angular displacements do not interact with each other, so that the non-vanishing portions of $\underline{\underline{K}}_N$ and $\underline{\underline{B}}_N$ are separate 2 X 2 matrices. That is

$$\underline{\underline{K}}_N = \begin{bmatrix} \multicolumn{2}{c}{(\underline{\underline{K}}_N)_{\text{lineal}}} & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{(\underline{\underline{K}}_N)_{\text{angular}}} \\ 0 & 0 & & \end{bmatrix}$$

$$\underline{\underline{B}}_N = \begin{bmatrix} \multicolumn{2}{c}{(\underline{\underline{B}}_N)_{\text{lineal}}} & 0 & 0 \\ & & 0 & 0 \\ 0 & 0 & \multicolumn{2}{c}{(\underline{\underline{B}}_N)_{\text{angular}}} \\ 0 & 0 & & \end{bmatrix}$$

Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 X 2) matrices:

$$(\underline{\underline{K}})_{lineal} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{lineal}$$

$$(\underline{\underline{B}})_{lineal} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{lineal}$$

$$(\underline{\underline{K}})_{angular} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{angular}$$

$$(\underline{\underline{B}})_{angular} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{angular}$$

In the event that the pedestal compliance is significant, then the effective support impedance can be calculated from

$$\underline{\underline{Z}}_N = \left[\underline{\underline{Z}}_p \cdot (\underline{\underline{Z}}_b + \underline{\underline{Z}}_p)^{-1} \cdot \underline{\underline{Z}}_b \right]_N$$

where subscripts "p" and "b" refer to the pedestal and bearing respectively. Note that both pedestal inertia and damping may be included in $\underline{\underline{Z}}_p$.

4.2.5 Impedance Alteration

From above discussions, it is clear that an isotropic stiffness representation for a bearing support is a gross simplification. A realistic description of bearing support characteristics should include:

- speed dependence,
- anisotropy or directional sensitivity with respect to the equilibrium load,
- distinction of stiffness and damping elements, and
- cross-coupling effects.

It is, however, not necessary to redo the analysis of a rotor because the representation of the bearing support needs refinement. As shown in Section 5.3.2, for the appropriate degrees-of-freedom associated with the bearing support in question, a direct adjustment of the already-computed impedance matrix suffices. This principle can be used to add any proportional type constraint to the rotor. Therefore, one can conveniently evaluate the characteristics of the rotor system due to

- varied bearing characteristics,
- relocation of bearing support,
- electromagnetic and aerodynamic excitations.

4.3 Natural Modes

4.3.1 Conservative Rotor System

If the bearing damping elements and cross coupling effects are negligible, despite whether or not isotropy is preserved, the rotor system would be conservative. At discrete frequency-speed combinations, self-sustained oscillations are possible. Such conditions are determined as the roots of the impedance determinant. Critical speeds are special cases of the natural modes of a conservative rotor system; the special conditions include restricting the excitation frequency to be in synchronism with shaft rotation and the motion to be in the forward sense of whirl.

4.3.2 Non-Conservative Rotor System

With realistic representation of fluid-film bearing characteristics, the rotor system becomes non-conservative. The natural mode of a non-conservative system can be established in a frequency domain analysis through the concept of system damping. The mathematical analysis required to determine the non-conservative natural modes involves the diagonalization of the impedance matrix, which leads to the formulation of a two-parameter complex eigenvalue problem. Details of the analysis are given in Section 6.2.

The non-conservative natural mode analysis is required to assess potential rotor instabilities which may stem from the improper use of fluid film bearings or the presence of aerodynamic excitations. It is also useful to predict the maximum forced response conditions of both synchronous and asynchronous type and to give an accurate estimate of the amplitudes involved.

4.4 Computational Strategy

In a comprehensive dynamic analysis of a rotor system, the required computation procedure can be logically divided into two steps. The first step concerns the computation of the mobility and impedance

matrices of the idealized structure. The second step deals with the determination of particular dynamic characteristics of the rotor system with the option to include realistic characterization of bearing supports as well as various identifiable excitation sources. Recognition of the logical breakdown of these two steps made it possible to devise a strategy for the computational procedure which is cost effective not only in the tangible sense, but also by providing flexibility to suit the particular needs of the prevailing circumstances. Specifically, efficiency is realized by

- system truncation to omit irrelevant details,
- use of "real" frequency domain analysis for all problem situations,
- judicial use of whirl coordinates, and
- employment of high accuracy interpolation of smooth functions in the frequency domain.

These ideas are amplified separately below.

4.4.1 System Truncation and Active Degrees of Freedom

In the analysis of the rotor structure, if the distributed inertia approach is used for shaft segments, the mathematical representation is inherently truncated. While deflection components can be identified at each intersegmental junction, their distributions along the lengths of the segments are left obscure. For instance, if idealized bearing support characteristics are assumed so that the rotor system is conservative, an infinite number of natural lateral modes can be computed. Since only a finite set of deflection representations is available to describe the various mode shapes, the dimensionality of representation is insufficient and the mode shape description is not unique. This deficiency is not serious, because only a finite number of the lower modes are of practical interest. By heuristic reasoning, one may be convinced that ambiguity in truncated modal representation is

legitimized if the frequency domain is also truncated to limit the total number of natural modes.

This argument can be extended one more step. Since the range of frequency domain of interest is quite readily definable, only a relatively small number of natural modes is of interest. This usually means that there are more than enough inter-segmental junctions to satisfy completeness in modal representation.

System truncation leads to considerable savings of computational cost if the analysis effort goes beyond that for the idealized rotor structure. While reduction of the dimensionality of mode shape representation may enhance computation economy, there are explicit reasons to identify particular locations of the rotor and the particular degree of freedom where dynamic interaction of various type may be of interest. Examples of active degrees of freedom include the following:

Bearing Stations — Isotropic springs represent bearings in the idealized rotor model. In order to allow more realistic characterization of the particular bearing, identification of the corresponding station is necessary so that impedance alteration may be performed. This involves the lateral lineal deflections for a radial bearing and the lateral angular deflection for a thrust bearing.

Alternate Bearing Stations — If the need to consider relocation of bearing support is anticipated, the alternate bearing stations should be identified even though no bearing constraint is assigned there in the idealized rotor model.

Mass Unbalance Station — This is where a mass unbalance type of forcing function is assigned for study of unbalance response.

Gears — For rotors with gears, gear mesh and its harmonic excitation of both lateral and torsional vibrations can

be important. For a spur gear the applicable degrees of freedom are lateral lineal deflections and the torsional twist; the lateral angular deflections are also involved for a bevel gear.

Impellers — Impellers are always potential locations for mass unbalance, there are also possibilities of aerodynamic cross-coupling excitation of shaft whirl. The latter problem can be emulated by assigning equivalent bearing coefficients at the impeller stations.

4.4.2 Frequency Domain Analysis

Use of frequency domain analysis is equivalent to the solution of time-dependent problem by the method of Laplace transformation. The efficacy of this approach for conservative systems is widely accepted, no further elaboration is needed here. By introducing the concept of system damping, non-conservative systems can also be treated. Full details of this method are given in Section 6.2. Both damped response and stability analysis can be accordingly calculated. In the present work, the frequency domain is confined to the real frequency axis. Non-conservative aspects are described in terms of phase shift instead of an exponential coefficient. The well accepted concept of critical damping ratio is suitably defined. Avoidance of the exponential temporal behavior has obvious advantages in computational economy. An additional benefit is its direct correspondence to practical measurements. By and large, experimental data of rotor dynamics pertain to steady-state oscillatory phenomena. Interpretive ease is an important advantage of the frequency response analysis.

4.4.3 Whirl Coordinates

In the analysis of an idealized rotor structure, use of whirl coordinates simplifies the description of gyroscopic inertia and associated computation effort. Even when non-conservative elements are incorporated, it is still advantageous to retain whirl

coordinates in the computation process, so long as isotropy prevails. Verification of isotropy is performed automatically in the computer program so that the simplified computation procedure can be utilized whenever it is permissible.

4.4.4 Interpolation on Impedance Matrices

The computer program allows the user to specify frequencies at which response calculations are desired. Within the specified frequency range, a search is always made to identify natural modes. Since truncated representation reduces the rank of the applicable matrices, interpolation from the available matrices with respect to frequency is likely to be less costly than the execution of a complete calculation cycle at a new frequency. The impedance matrices are used here because its elements always have smoothly varying values. A cubic spline procedure was found to be very effective for this purpose. The cubic spline is described in Appendix B.

SECTION V

FLEXIBLE ROTOR VIBRATION ANALYSIS

This section contains a description of the analytical background in the "structural" treatment of a flexible rotor. The basic approach adopted is patterned after Holzer [17] and Myklestad [18]. The rotor is described in terms of a number of uniform shaft segments which are rigidly attached to each other and share a common geometrical and rotational axis. Rigid body inertia elements can be attached to stations between the shaft segments or at rotor ends. Bearing supports, which affect lateral vibration of the rotor, can also be assigned to intersegmental stations or rotor ends.

It may also be necessary to identify a rotor station for the purpose of recognizing the presence of external forcing or excitation. Most commonly, however, such a station would coincide with the location of a rigid body inertia element. For instance, an aerodynamic excitation would be located at an impeller which calls for a rigid body inertia in the rotor model. Thus, the rotor would be described as a succession of M shaft segments. Accordingly, there are M + 1 rotor stations at which rigid body inertia elements, bearing supports, and/or external excitations may be assigned. Shaft segment no. K is enclosed between Stations no. K and K+1.* The last shaft segment, no. M, is terminated by Station no. M+1.

To allow flexibility in rotor modeling, distinction is made between mass and stiffness diameters. Thus, a complete description of a shaft segment includes:

ℓ	length of shaft segment (in)
D_i	inner stiffness diameter (in)
D_o	outer stiffness diameter (in)
d_i	inner mass diameter (in)
d_o	outer mass diameter (in)

*Station K+1 is regarded as being to the right of station K.

ρ density of shaft material (lb/in^3)

E Young's modulus (psi)

G shear modulus (psi)

A major departure from the Holzer-Myklestad treatment is the use of distributed analysis of shaft segments. Thus it is not necessary to subdivide a slender and long shaft segment in order to realize accuracy of higher modes. The use of distributed analysis may not always achieve computational economy since it involves transcendential functions. It is preferred because rotor model input setup is simplified and the results are least sensitive to input setup. The distributed analysis makes it easier for the user to achieve good results without extensive modelling experience.

The analysis deals with small displacement periodic motions. The rotor materials are assumed to obey linear elastic constitutive laws. Non-conservative effects, however, can be introduced at bearing support stations. Periodic time dependence is represented by the exponential factor, $e^{i\nu t}$. All variables are allowed to assume complex values, with the absolute value and argument of the complex number giving respectively the amplitude and the temporal phase. Although one cannot directly address the problems of transient motions and limit cycle vibration, most commonly encountered rotor dynamic problems can be treated. The following are state-of-the-art features included to permit an exhaustive analysis of an advanced high speed turbo rotor:

- The vibration frequency may be different from the rotational speed

- The bearing support characteristics can include up to 16 coefficients, representing both lineal and angular constraints.

- The bearing coefficients may be dependent on both rotor speed and vibration frequency. Thus, not only can one study a gas bearing supported rotor, it is also possible to treat the speed dependence of rolling element bearings, speed-frequency dependence of tilting-pad bearings, and to emulate pedestal compliance and inertia.

- Forced response is automatically calculated at damped resonant frequencies.
- Rotor system critical damping ratios can be calculated within user specified frequency ranges at all natural modes. When a negative critical damping ratio is encountered, a self-excited instability is indicated.

The analytical formulation is structured to minimize computation cost when calculations are to be repeated many times for the same speed-frequency combinations but with variations of external excitations and bearing support characteristics. Two crucial steps make this possible.

- A truncated computation system is reduced from the stations of the rotor model. Only such stations, where external excitations are applied and where variations of bearing support characteristics are desired, need be included in the truncated system.
- A nominal impedance matrix is computed at each speed-frequency combination by inverting the mobility matrix of the truncated system. Variation of bearing support characteristics is achieved by altering specific elements of the nominal impedance matrix without repeating the time-consuming computation of the rotor response from the rotor model.

Due to the small displacement assumption, torsional and lateral vibrations are uncoupled (except that a lateral excitation may simultaneously exert a torsional moment).

In Sections 5.1 and 5.2, computation of torsional and lateral mobility matrices will be separately treated. Also in Section 5.2, the concept of mass unbalance as a rotating excitation is examined. In Section 5.3, various aspects of the system impedance matrix as they are used in rotor dynamic studies are reviewed.

5.1 Torsional Vibration

In a torsional vibration problem, the primitive variables are (ϕ, M_z), the angle of twist and the torsional moment. ϕ is a continuous function and would be sectionally differentiable with respect to the axial coordinate in each shaft segment, but the derivative, which is proportional to the torsional moment, may be discontinous across a station at which a lumped polar moment of inertia is attached.

According to the theory of torsion of slender rods [19].

$$M_z = G\, I_p \frac{d\phi}{dz} \tag{5.1}$$

where, $I_p = \frac{\pi}{32}(D_o^{\,4} - D_i^{\,4})$. Equation (5.1) is applicable in each shaft segment. Using matrix notation, a torsional primitive vector, $\underset{\sim}{Q}_t$ may be defined to have (ϕ, M_z) as its elements. For shaft segment no. K, one can write $\underset{\sim}{Q}_t(z_K)$, indicating the dependence of the primitive variables on the local axial coordinate z_K. At either end of the shaft segment, one can define

$$(\underset{\sim}{Q}_t)_K = \underset{\sim}{Q}_t(z_K=0); \quad (\underset{\sim}{Q}'_t)_K = \underset{\sim}{Q}_t(z_K=\ell_K) \tag{5.2}$$

ℓ_K is the length of the shaft segment no. K.

5.1.1. Calculation at a Station

In the analysis of torsional vibration, the calculation at a station is mainly concerned with the d'Alembert effect of a lumped polar mass moment of inertia. This is described by the matrix equation

$$(\underset{\sim}{Q}_t)_K = (\underset{\approx}{T}_{t1})_K \cdot (\underset{\sim}{Q}'_t)_{K-1} \tag{5.3}$$

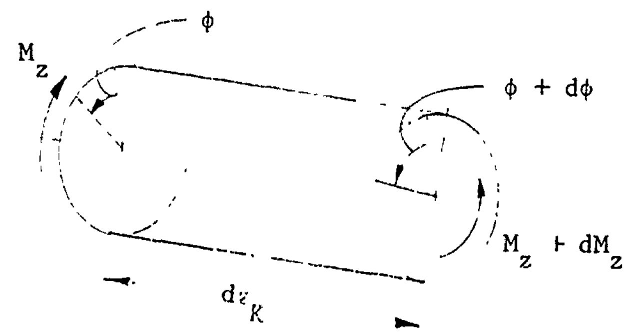
where

$$(\underline{\underline{T}}_{t1})_K = \begin{bmatrix} 1 & 0 \\ & \\ -\nu^2 I_{pK} & 1 \end{bmatrix} \tag{5.4}$$

I_{pK} is the polar mass moment of inertia at station K.

5.1.2 Calculation Along a Shaft Segment

In a shaft segment, $0 < z_K < \ell_K$, torsional dynamic equilibrium is given by



$$\frac{dM_z}{dz_K} = -\nu^2(\rho J_p/g)_K \cdot \phi \tag{5.5}$$

where $J_p = \frac{\pi}{32}(d_o^{\;4} - d_i^{\;4})$ is the cross-sectional polar moment of inertia and g is the gravitational constant (386.4 in/sec^2). Combining with Equation (5.1), one obtains

$$\frac{d^2\phi}{dz_K^2} = -(\gamma_K \nu)^2 \phi \tag{5.6}$$

where

$$\gamma_K = \sqrt{\left|\frac{\rho J_p}{g G I_p}\right|_K} \tag{5.7}$$

The general solution of Equation (5.6) is

$$\phi = a_1 \cos \gamma_K \nu z_K + a_2 \sin \gamma_K \nu z_K$$

Differentiating,

$$\frac{1}{\gamma_K \nu} \frac{d\phi}{dz_K} = -a_1 \sin \gamma_K \nu z_K + a_2 \cos \gamma_K \nu z_K$$

or, in matrix notation, one can write

$$\underline{\phi} = \underline{\underline{S}}_t(z_K) \cdot \underline{a} \tag{5.8}$$

where,

$$\underline{\phi} = \begin{bmatrix} \phi \\ \\ \frac{1}{\gamma_K \nu} \frac{d\phi}{dz_K} \end{bmatrix} ; \quad \underline{\underline{S}}_t = \begin{bmatrix} \cos\gamma_K \nu z_K & \sin\gamma_K \nu z_K \\ \\ -\sin\gamma_K \nu z_K & \cos\gamma_K \nu z_K \end{bmatrix} ; \quad \underline{a} = \begin{bmatrix} a_1 \\ \\ a_2 \end{bmatrix} \tag{5.9}$$

Also,

$$\underline{Q}_t = \underline{\underline{g}}_t \cdot \underline{\phi} \tag{5.10}$$

where

$$\underline{\underline{g}}_t = \begin{bmatrix} 1 & 0 \\ \\ 0 & (GI_p \gamma)_K \nu \end{bmatrix} \tag{5.11}$$

Equation (5.8) can be inverted at $z_K = 0$:

$$\underline{a} = \underline{\underline{S}}_t (0)^{-1} \cdot \underline{\phi}(0) \tag{5.12}$$

Substituting back into Equation (5.8) and setting $z_K = \ell_K$, one obtains

$$\underline{\phi} (\ell_K) = \underline{\underline{S}}_t (\ell_K) \cdot \underline{\underline{S}}_t (0)^{-1} \cdot \underline{\phi} (0) \tag{5.13}$$

Now, since

$$(\underline{Q}_t')_K = \underline{\underline{G}}_t \cdot \underline{\phi}(\ell_K)$$

and

$$\underline{\phi}(0) = \underline{\underline{G}}_t^{-1} \cdot (\underline{Q}_t)_K$$

one can thus rewrite Equation (5.13) as

$$(\underline{Q}')_K = (\underline{\underline{T}}_{t2})_K \cdot (\underline{Q}_t)_K \tag{5.14}$$

where

$$(\underline{\underline{T}}_{t2})_K = \left\{\left[\underline{\underline{G}}_t \cdot \underline{\underline{S}}_t (\ell_K) \cdot \underline{\underline{S}}_t (0)^{-1}\right] \cdot \underline{\underline{G}}_t^{-1}\right\}_K \tag{5.15}$$

5.1.3 Torsional Mobility Matrix

Combining Equations (5.3) and (5.14), one obtains

$$(\underline{Q}_t')_K = (\underline{\underline{C}}_t)_K \cdot (\underline{Q}_t')_{K-1} \tag{5.16}$$

where

$$(\underline{\underline{C}}_t)_K = (\underline{\underline{T}}_{t2} \cdot \underline{\underline{T}}_{t1})_K \tag{5.17}$$

$(\underline{\underline{C}}_t)_K$ is the torsional connection matrix of the primitive vector from the right end of station K-1 to the right end of segment K.*

---

*Station K+1 is regarded as being to the right of station K.

The left free end condition is given by

$$(\underline{Q}'_t)_0 = \phi_1 \begin{bmatrix} 1 \\ \\ 0 \end{bmatrix} \tag{5.18}$$

Thus the homogeneous solution for the torsional primitive vector at the left of station N + 1 is

$$(\underline{Q}'_t)_{N,\ \text{homogeneous}} = \phi_1 \cdot \prod_{K=1}^{N} (\underline{\underline{C}}_t)_{N+1-K} \begin{bmatrix} 1 \\ \\ 0 \end{bmatrix} \tag{5.19}$$

If a unit torsional moment is applied at station J, the corresponding particular solution for $N \geq J$ is

$$(\underline{Q}'_t)_{N,\ \text{particular}} = - \prod_{K=1}^{N+1-J} (\underline{\underline{C}}_t)_{N+1-K} \begin{bmatrix} 0 \\ \\ 1 \end{bmatrix} \tag{5.20}$$

For the right end of the shaft, on the right of station M + 1, the torsional primitive vector is

$$(\underline{Q}_t)_{M+1} = (\underline{\underline{T}}_{t1})_{M+1} \cdot \left\{ \phi_1 \prod_{K=1}^{M} (\underline{\underline{C}}_t)_{M+1-K} \begin{bmatrix} 1 \\ \\ 0 \end{bmatrix} - \prod_{K=1}^{M+1-J} (\underline{\underline{C}}_t)_{M+1-K} \begin{bmatrix} 0 \\ \\ 1 \end{bmatrix} \right\} \tag{5.21}$$

or, writing

$$(\underline{\underline{T}}_{t1})_{M+1} \cdot \prod_{K=1}^{M} (\underline{\underline{C}}_t)_{M+1-K} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

$$(\underline{\underline{T}}_{t1})_{M+1} \cdot \prod_{K=1}^{M+1-J} (\underline{\underline{C}}_t)_{M+1-K} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}_J \qquad (5.22)$$

then

$$(\underline{Q}_t)_{M+1} = \begin{bmatrix} \phi \\ M_z \end{bmatrix}_{M+1}$$

$$= \phi_1 \begin{bmatrix} A_{11} \\ A_{21} \end{bmatrix} - \begin{bmatrix} B_{12} \\ B_{22} \end{bmatrix}_J$$

Since the right end is free,

$$(M_z)_{M+1} = 0 = \phi_1 A_{21} - (B_{22})_J$$

Therefore,

$$\phi_1 = \frac{(B_{22})_J}{A_{21}} \qquad (5.23)$$

Substituting back into Equations (5.19) and (5.20), for $N < J$

$$(\underline{Q}_t')_N = \frac{(B_{22})_J}{A_{21}} \prod_{K=1}^{N} (\underline{\underline{C}}_t)_{N+1-K} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad (5.24a)$$

and for $N \geq J$

$$(Q_t')_N = \frac{(P_{22})_J}{A_{21}} \prod_{K=1}^{N} (\underline{\underline{C}}_t)_{N+1-K} \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix} - \prod_{K=1}^{N+1-J} (\underline{\underline{C}}_t)_{N+1-K} \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix} \tag{5.24b}$$

To obtain the torsional mobility matrix, one must first identify the set of active torsional stations S_t to which J belongs. Equations (5.24) are then used to calculate $(Q_t)_N$ which belong to S_t. The vector of $(\phi)_N$ thus obtained is one column of the torsional mobility matrix corresponding to J.

5.2 Lateral Vibration

The lateral vibration problem is more complicated in many ways:

- The lateral motion has two degrees of freedom, which are coupled together by gyroscopic inertia, fluid-film bearing effects, and/or forcing functions.

- It is a fourth order system. Lineal and angular displacements are distinct aspects of the motion. Together with the "internal forces", bending moment and shear force, they form a fourth rank primitive vector. "External force" may be either a lateral moment or a lateral force.

- Journal bearing characteristics influence lateral vibrations directly.

In the Cartesian coordinate system, the lateral lineal motion is described by (δ_x, δ_y). (x,y,z) form a right-handed triad with the rotational vector of the shaft directed along the z-axis. With the small displacement approximation, the lateral angular displacements are

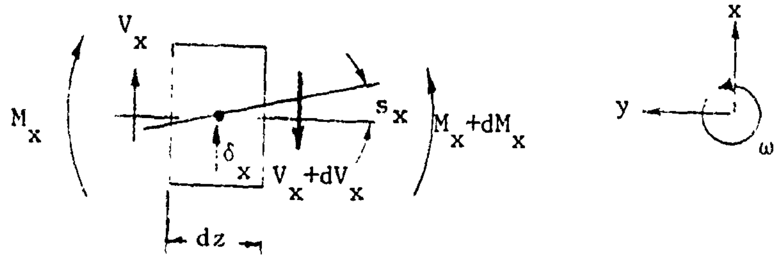
$$(s_x, s_y) = \frac{d}{dz} (\delta_x, \delta_y) \tag{5.25}$$

According to the flexure theory of slender rods [20], the bending moments and the shear forces in the shaft segment are given by

$$(M_x, M_y) = E\, I_t \frac{d^2}{dz^2} (\delta_x, \delta_y)$$

$$(V_x, V_y) = E\, I_t \frac{d^3}{dz^3} (\delta_x, \delta_y) \qquad (5.26)$$

where the symbols are defined as follows



and where I_t is the cross-section transverse moment of inertia

$$I_t = \frac{\pi}{64} (D_o^{\;4} - D_i^{\;4}) \qquad (5.27)$$

For years, it has been known that the flexure theory of slender rods can be improved if shear displacement and rotary inertias are also considered [21]. Such refinements are often proposed to improve calculation accuracy for higher flexural modes. In the study of rotor dynamics, however, such theoretical refinements are overshadowed by our inability to cope with the true conditions at the intersegmental regions. The questions here are concerned with three-dimensional stress fields which are further complicated by assembly and fabrication details such as shrink-fit, flange-bolting, welding, brazing, and/or key-ways. Even with the best available computer codes for three-dimensional elasticity, it is unlikely that these

effects can be treated with sufficient precision to justify improvement of the flexure analysis with shear displacement and rotary inertias.

The potential importance of rotary inertias, however, may also present itself in rigid body-like conical modes. The conventional flexure analysis neglects the cross-sectional rotary inertias and would thus incur inaccuracy in the computation of such modes. This situation can be remedied by adding one half of the cross-sectional rotary inertias of each shaft segment to the intersegmental inertias at either end of the shaft segment.

As indicated previously, gyroscopic inertia would cause coupling between (s_x, s_y). Thus if the Cartesian coordinate system were to be used to analyze the flexural vibration problem, the two sets of fourth order primitive vectors would have to be carried along simultaneously. Fortunately, gyroscope cross-coupling is of the isotropic type, and by employing forward and backward whirl coordinates, diagonalization of the system can be restored. A brief description of the forward and backward whirl coordinate system is given in Section 5.2.2, subsequent to discussion of lumped lateral inertias in Section 5.2.1, where the gyroscopic phenomenon is treated.

The flexural primitive vectors applicable to shaft segment K in the Cartesian coordinates are

$$\underset{\sim}{Q}_{fx}(z_K) = \begin{bmatrix} \delta_x \\ s_x \\ M_x \\ V_x \end{bmatrix} \quad ; \quad \underset{\sim}{Q}_{fy}(z_K) = \begin{bmatrix} \delta_y \\ s_y \\ M_y \\ V_y \end{bmatrix} \tag{5.28}$$

Following the same notation previously used in torsional vibration analysis, the flexural primitive vectors at either end of the Kth segment may be defined as

$$(Q_{fx})_K = Q_{fx}\,(z_K = 0);\ (Q'_{fx})_K = Q_{fx}\,(z_K = \ell_K)$$

$$(Q_{fy})_K = Q_{fy}\,(z_K = 0);\ (Q'_{fy})_K = Q_{fy}\,(z_K = \ell_K) \qquad (5.29)$$

Similarly, for individual elements of the flexural primitive vectors, one may write

$$(\delta_x)_K = \delta_x\,(z_K = 0);\ (\delta_x')_K = \delta_x\,(z_K = \ell_K) \qquad (5.30)$$

etc.

Lateral vibrations are affected by bearing supports, the lineal and angular stiffnesses of which respectively cause discontinuities in shear forces and in bending moments. A complete characterization of a bearing support would include damping, cross-coupling, and an-isotropy. These aspects are deferred to Section 5.3.2. In the following nominal bearing support effects are treated in terms of isotropic lineal and angular stiffnesses which are assigned at appropriate stations.

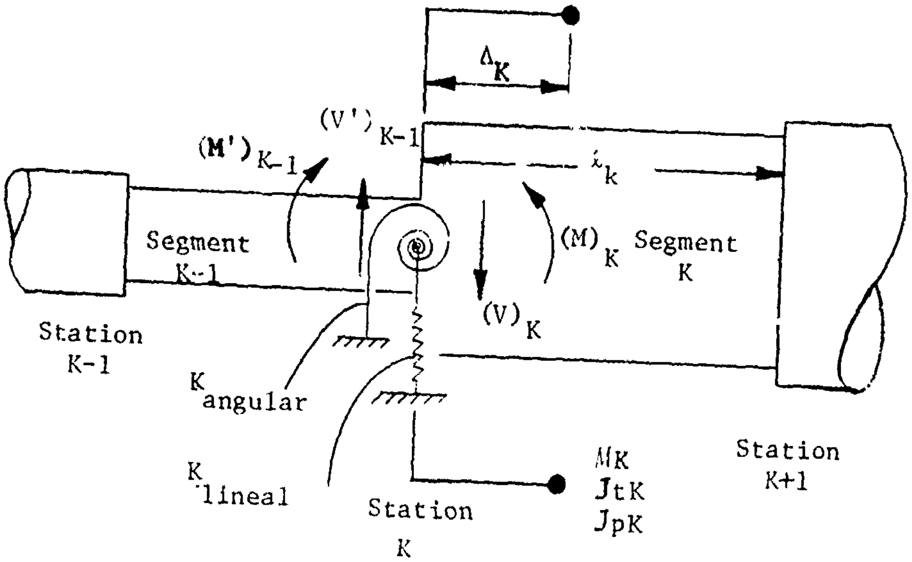
5.2.1 Calculations at a Station

d'Alembert Effects

At a rotor station, a total representation of rigid body inertia would include

- mass, M_K;
- axial offset of c.g., Δ_K, to the right of the station.
- transverse mass moment of inertia, J_{tK}; and
- polar mass moment of inertia, J_{pK}

The location of these inertias is shown in the following sketch.



The polar mass moment of inertia is the same as that used in torsional vibration analysis.

Lineal dynamic equilibrium which accounts for d'Alembert effects, at station K is satisfied by

$$\Delta \begin{bmatrix} (V_x)_K \\ \\ (V_y)_K \end{bmatrix}_{\text{d'Alembert}} = \nu^2 M_K \left\{ \begin{bmatrix} (\delta_x)_K \\ \\ (\delta_y)_K \end{bmatrix} + \Delta_K \begin{bmatrix} (s_x)_K \\ \\ (s_y)_K \end{bmatrix} \right\} \quad (5.31)$$

Angular dynamic equilibrium is satisfied by

$$\Delta \begin{bmatrix} (M_x)_K \\ \\ (M_y)_K \end{bmatrix}_{\text{d'Alembert}} = \nu^2 M_K \Delta_K \begin{bmatrix} (\delta_x)_K \\ \\ (\delta_y)_K \end{bmatrix} - \begin{bmatrix} -\nu^2 (J_{tK} + M_K \Delta^2_K) & -i\nu\omega J_{pK} \\ \\ i\nu\omega J_{pK} & \nu^2 (J_{tK} + M_K \Delta^2_K) \end{bmatrix} \begin{bmatrix} (s_x)_K \\ \\ (s_y)_K \end{bmatrix}$$

(5.32)

The off-diagonal elements of the rotary inertia matrix are gyroscopic effects. Detailed derivations are referred to Appendix A. They are responsible for an inherent inertial coupling between the lateral planes for a rotating shaft and cause complications in computation if the Cartesian coordinate system is retained for the lateral motion.

Whirl Coordinates

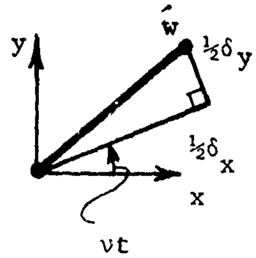
The rotary inertia matrix has identical main diagonal elements, while its off-diagonal elements have the same magnitude but opposite signs. Such a matrix is isotropic and can be diagonalized by using whirl coordinates. The transformation matrix from Cartesian to whirl coordinates is

$$\underline{\underline{W}} = \frac{1}{2} \begin{bmatrix} 1 & i \\ \\ 1 & -i \end{bmatrix} \tag{5.33}$$

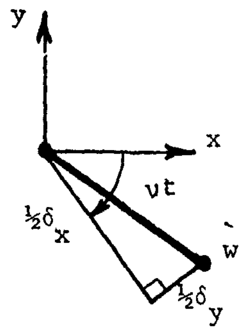
such that the lateral whirl displacements are

$$\begin{vmatrix} \overset{\nearrow}{w} \\ \\ \overset{\nwarrow}{w} \end{vmatrix} = \underline{\underline{W}} \begin{bmatrix} \delta_x \\ \\ \delta_y \end{bmatrix} \tag{5.34}$$

$\acute{w}$ represents a forward circular whirl motion while $\grave{w}$ is a backward circular whirl motion as illustrated below:



Forward Whirl



Backward Whirl

$|\hat{w}|$ ("^" designates either a forward or a backward whirl vector) is the amplitude of the circular whirl orbit, while Arg $\{\hat{w}\}$ is the phase advance in the same sense as the respective whirl motion relative to the chosen time reference.

The same transformation applies also to all other vectors in the lateral plane. The inverse whirl transformation is

$$\underline{\underline{W}}^{-1} = \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix} \tag{5.35}$$

If $\underline{\underline{A}}$ is an isotropic matrix in the Cartesian coordinate system,

$$\underline{\underline{A}} = \begin{bmatrix} a_1 & a_2 \\ -a_2 & a_1 \end{bmatrix}$$

then its representation in whirl coordinates is

$$\hat{A} = \underline{\underline{W}} \cdot \underline{\underline{A}} \cdot \underline{\underline{W}}^{-1} = \begin{bmatrix} a_1 - ia_2 & 0 \\ 0 & a_1 + ia_2 \end{bmatrix} \tag{5.36}$$

Applying this formula to the rotary inertia matrix in Eq. (5.32), one obtains

$$- \begin{bmatrix} \nu^2(J_{tK} + M_K\Delta^2_K) - \nu\omega J_{pK} & 0 \\ 0 & \nu^2(J_{tK} + M_K\Delta^2_K) + \nu\omega J_{pK} \end{bmatrix} \qquad (5.37)$$

One may observe that, upon transformation into whirl coordinates, the rotary inertia matrix becomes simplified in two ways:

- o off-diagonal terms are eliminated
- o all elements of the matrix are real.

Representation of the lateral motion in whirl coordinates also facilitates its kinematic description. Any simple harmonic motion in the lateral plane describes an elliptical orbit. Use of Equation (5.34) allows one to characterize the elliptical orbit by $(\acute{w}, \grave{w})$. Referring to the illustration on the next page, one readily observes:

$$R_1 = \text{Major Orbit Radius} = |\acute{w}| + |\grave{w}| \qquad (5.38)$$

$$R_2 = \text{Minor Orbit Radius} = |\acute{w}| - |\grave{w}| \qquad (5.39)$$

$$\text{Rotational Sense} = \text{Sg}\left\{|\acute{w}| - |\grave{w}|\right\} \qquad (5.40)$$

When the orbit reaches its major axis, the radii of the two counter-rotating motions are coincident; therefore

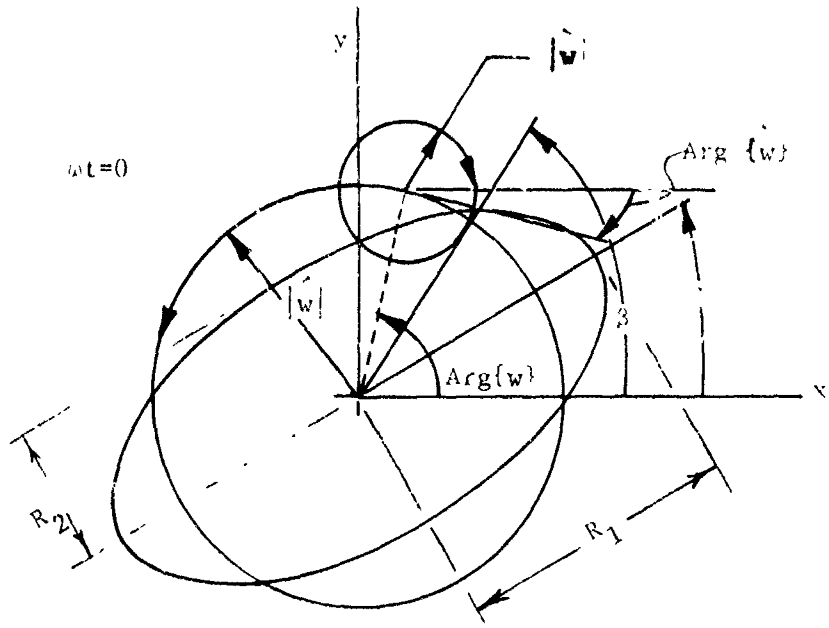
$$\alpha = \text{Inclination of the major axis}$$

$$= \nu t + \text{Arg}\ \acute{w} = -\left[\nu t + \text{Arg}\ \grave{w}\right] \qquad (5.41)$$

$$- \frac{1}{2}\left[\text{Arg}\ \acute{w} - \text{Arg}\ \grave{w}\right]$$

and the polar angular coordinate for the temporal reference, $t = 0$, is

$$\beta = \tan^{-1}\left[\frac{\mathrm{Im}(\hat{w} + \check{w})}{\mathrm{Re}(\hat{w} + \check{w})}\right] \tag{5.42}$$



Combined Effects of Inertias and Support Stiffnesses

Shear forces and bending moments at a station are also affected by bearing support characteristics, which are represented by isotropic lineal and angular springs here. Combining the effects of inertia elements and springs, the jump condition at station number K can be written in the whirl representation as

$$(\underline{Q}_f)_K = (\underline{\underline{T}}_{f1})_K \cdot (\underline{Q}_f')_{K-1} \tag{5.43}$$

where

$$(\underline{\underline{T}}_{f1})_K = \begin{bmatrix} \underline{\underline{1}} & \underline{\underline{0}} \\ \underline{\underline{M}} & \underline{\underline{1}} \end{bmatrix} \tag{5.44}$$

and

$$\underline{\underline{M}} = \begin{bmatrix} -\nu^2 M_K \Delta_K & -\nu^2(J_{tK} + M_K \Delta_K{}^2) \pm \nu\omega J_{pK} + (K_{\text{angular}})_K \\ \nu^2 M_K - (K_{\text{lineal}})_K & \nu^2 M_K \Delta_K \end{bmatrix}$$

5.2.2 Calculations Along a Shaft Segment – the Dynamical Flexure Solution

The dynamical flexure problem, written in whirl coordinates without rotary inertias and shear deflection is governed by

$$\frac{d\hat{V}}{dz} = \nu^2 \left(\frac{\rho A}{g}\right) \hat{w} \tag{5.45}$$

Combining with Equations (5.25 and 5.26), one obtains the ordinary differential equation for

$$(EI_t)_K \frac{d^4 \hat{w}}{dz_K^4} = \nu^2 \left(\frac{\rho A}{g}\right)_K \hat{w} \tag{5.46}$$

and its general solution is

$$\hat{w} = C_1 \cosh \zeta_K z_K + C_2 \sinh \zeta_K z_K + C_3 \cos \zeta_K z_K + C_4 \sin \zeta_K z_K \tag{5.47}$$

where

$$\zeta_K = \left(\nu^2 \frac{\rho A}{gEI_t}\right)^{\frac{1}{4}} \tag{5.48}$$

Elements in the flexural primitive vector are related to up-to-the-third-order derivatives of $\hat{w}$. Arranging them into a fourth rank column vector, one can write

$$\underline{w} = \underline{\underline{S}}_f(z_K) \cdot \underline{C} \tag{5.49}$$

where

$$\underline{\underline{S}}_f = \begin{bmatrix} \cosh \zeta_K z_K & \sinh \zeta_K z_K & \cos \zeta_K z_K & \sin \zeta_K z_K \\ \sinh \zeta_K z_K & \cosh \zeta_K z_K & -\sin \zeta_K z_K & \cos \zeta_K z_K \\ \cosh \zeta_K z_K & \sinh \zeta_K z_K & -\cos \zeta_K z_K & -\sin \zeta_K z_K \\ \sinh \zeta_K z_K & \cosh \zeta_K z_K & \sin \zeta_K z_K & -\cos \zeta_K z_K \end{bmatrix}$$

$$\underline{w} = \begin{bmatrix} \hat{w} \\ \frac{1}{\zeta_K} \frac{d\hat{w}}{dz_K} \\ \frac{1}{\zeta_K^{\,2}} \frac{d^2\hat{w}}{dz_K^{\,2}} \\ \frac{1}{\zeta_K^{\,3}} \frac{d^3\hat{w}}{dz_K^{\,3}} \end{bmatrix} ; \quad \underline{C} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} \tag{5.50}$$

The flexural primitive vector at the beginning of the shaft segment, $(\underline{Q}_f)_K$, is related to $\underline{w}$ $(z_K = 0)$ through scale factors which are associated with (E, I_t, ζ_K). At the same time, allowance for rotary inertias of one-half the shaft segment can be made in a manner similar to the treatment of lumped inertias at a station. Thus

$$(\underline{\underline{T}}_{f2})_K \cdot (\hat{\underline{Q}}_f)_K = \underline{\underline{g}}_f \cdot \underline{\underline{S}}_f(0) \cdot \underline{C} \tag{5.51}$$

where

$$(\underline{\underline{T}}_{f2})_K = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{g}\left(\frac{\nu^2}{2} \mp \nu\omega\right)(\rho J_p \ell)_K & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{\underline{\sigma}}_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \zeta_K & 0 & 0 \\ 0 & 0 & \zeta_K{}^2 (EI_t)_K & 0 \\ 0 & 0 & 0 & \zeta_K{}^3 (EI_t)_K \end{bmatrix} \quad (5.52)$$

Similarly,

$$(\hat{\underline{Q}}_f')_K = (\underline{\underline{T}}_{f2})_K \cdot \underline{\underline{\sigma}}_f \cdot \underline{\underline{S}}_f(\ell_K) \cdot \underline{C} \quad (5.53)$$

Eliminating $\underline{C}$ between Equations (5.51) and (5.53), one obtains finally

$$(\hat{\underline{Q}}_f')_K = \left[\underline{\underline{T}}_{f2} \cdot \underline{\underline{\sigma}}_f \cdot \underline{\underline{S}}_f(\ell_K) \cdot \underline{\underline{S}}_f(0)^{-1} \cdot \underline{\underline{\sigma}}_f{}^{-1} \cdot \underline{\underline{T}}_{f2}\right]_K \cdot (\hat{\underline{Q}}_f)_K \quad (5.54)$$

5.2.3 Lateral Mobility Matrix

Combining Equations (5.43) and (5.54), one can write

$$(\hat{\underline{Q}}_f')_K = (\underline{\underline{C}}_f)_K \cdot (\hat{\underline{Q}}_f')_{K-1} \quad (5.55)$$

where

$$(\underline{\underline{C}}_f)_K = \left[\underline{\underline{T}}_{f2} \cdot \underline{\underline{\sigma}}_f \cdot \underline{\underline{S}}_f(\ell_K) \cdot \underline{\underline{S}}_f(0)^{-1} \cdot \underline{\underline{\sigma}}_f^{-1} \cdot \underline{\underline{T}}_{f2} \cdot \underline{\underline{T}}_{f1}\right]_K \tag{5.56}$$

which is the flexural connection matrix of the primitive vector from the left side of station K to the right end of segment K as written in whirl coordinates.

The left free end condition is given by

$$(\hat{\underline{Q}}_f')_0 = \hat{w}_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \hat{\theta}_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \tag{5.57}$$

Thus the homogeneous solution for the flexural primitive vector at the left of station number N+1 is

$$(\hat{\underline{Q}}_f')_N \text{ homogeneous}$$

$$= \prod_{K=1}^{N} (\underline{\underline{C}}_f)_{N+1-K} \begin{bmatrix} \hat{w}_1 \\ \hat{\theta}_1 \\ 0 \\ 0 \end{bmatrix} \tag{5.58}$$

If a unit transverse force (in whirl coordinates) is applied at station J, the corresponding particular solution for $N \geq J$ is

$$(\hat{\underline{Q}}_f')_{N,\text{ particular, force at J}}$$

$$= \prod_{K=1}^{N+1-J} (\underline{\underline{C}}_f)_{N+1-K} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5.59}$$

According.. for the right end, on the right of station M+1, the flexural primitive vector is

$$(\hat{\underline{Q}}_f)_{M+1}, \text{ force at J}$$

$$= (\underline{\underline{T}}_{f1})_{M+1} \cdot \left\{ \prod_{K=1}^{M} (\underline{\underline{C}}_f)_{M+1-K} \begin{bmatrix} \hat{w}_1 \\ \hat{\theta}_1 \\ 0 \\ 0 \end{bmatrix} + \prod_{K=1}^{M+1-J} (\underline{\underline{C}}_f)_{M+1-K} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\} \tag{5.60}$$

One may write

$$(\underline{\underline{T}}_{f1})_{M+1} \cdot \prod_{K=1}^{M} (\underline{\underline{C}}_f)_{M+1-K} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}_{\text{flexure}}$$

$$(\underline{\underline{T}}_{f1})_{M+1} \cdot \prod_{K=1}^{M+1-J} (\underline{\underline{C}}_f)_{M+1-K} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix}_{J,\ \text{flexure}} \tag{5.61}$$

Note, that notations conflicting with Equation (5.22) are used here, therefore annotation "flexure" is used to emphasize the distinction. Since the right end is free,

$$\begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}_{\text{flexure}} \cdot \begin{bmatrix} \hat{w}_1 \\ \hat{\theta}_1 \end{bmatrix}_{\text{force at J}} + \begin{bmatrix} B_{34} \\ B_{44} \end{bmatrix}_{\text{J, flexure}} = 0$$

Or,

$$\begin{bmatrix} \hat{w}_1 \\ \hat{\theta}_1 \end{bmatrix}_{\text{force at J}} = - \begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}^{-1}_{\text{flexure}} \cdot \begin{bmatrix} B_{34} \\ B_{44} \end{bmatrix}_{\text{J, flexure}} \tag{5.62}$$

Substituting back into Equation (5.58), for N<J,

$$(\underset{=}{C}_f')_{N,\text{ force at J}} = - \prod_{K=1}^{N} (\underset{=}{C}_f)_{N+1-K} \begin{bmatrix} \begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}^{-1}_{\text{flexure}} \begin{bmatrix} B_{34} \\ B_{44} \end{bmatrix}_{\text{J, flexure}} \\ \underset{=}{0} \end{bmatrix} \tag{5.63a}$$

and for $N \geq J$,

$$(\hat{\underline{Q}}_f')_{N,\ \text{force at } J}$$

$$= - \prod_{K=1}^{N} (\underline{\underline{C}}_f)_{N+1-K} \begin{bmatrix} \begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}^{-1}_{\text{flexure}} \\ \\ \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} B_{34} \\ B_{44} \end{bmatrix}_{J,\ \text{flexure}}$$

$$+ \prod_{K=1}^{N+1-J} (\underline{\underline{C}}_f)_{N+1-K} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad (5.63b)$$

Similarly, if a unit moment is applied at J, then for $N<J$

$$(\hat{\underline{Q}}_f')_{N,\ \text{moment at } J}$$

$$= \prod_{K=1}^{N} (\underline{\underline{C}}_f)_{N+1-K} \begin{bmatrix} \begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}^{-1}_{\text{flexure}} \\ \\ \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} B_{33} \\ B_{43} \end{bmatrix}_{J,\ \text{flexure}} \qquad (5.64a)$$

and for $N \geq J$

$$(\hat{\underline{Q}}_f')_{N,\ \text{moment at } J}$$

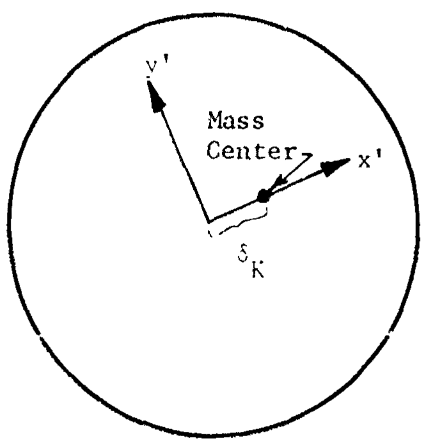
$$= \prod_{K=1}^{N} (\underline{\underline{C}}_f)_{N+1-K} \begin{bmatrix} \begin{bmatrix} A_{31} & A_{32} \\ A_{41} & A_{42} \end{bmatrix}^{-1}_{\text{flexure}} \\ \\ \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} B_{33} \\ B_{43} \end{bmatrix}_{J,\ \text{flexure}}$$

$$- \prod_{K=1}^{N+1-J} (\underline{\underline{C}}_f)_{N+1-K} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad (5.64b)$$

In a flexural system each station may provide either one (w or θ) or two (w and θ) degrees of freedom. Thus, to construct the flexural mobility matrix, one must first identify, S_f, the set of active flexural degrees of freedom to which (w and/or $\theta)_J$ belong. Equations (5.63 and/or 5.64) are then used to calculate $(\hat{Q}_f)_N$ for all N which belong to S_f. The array of (w and/or $\theta)_N$ thus obtained is the column(s) of the flexural mobility matrix corresponding to the particular degree of freedom at J.

5.2.4 Unbalance Excitation

A common problem in rotor dynamics is the unbalance response. The unbalance problem arises when the center of gravity of a rotor section is not coincident with the axis of rotation. One may elect to describe the situation in terms of a mass moment at the station K, $(M\delta)_K$. The phenomenon of mass unbalance can be understood by studying the rigid-body d'Alembert effects of a rotor section which contains the unbalance mass moment. The chosen coordinate system (x', y', z') is fixed to the rotor section with the x'-axis passing through both the cross-sectional reference center, which is the intersection of the cross-section with the nominal rotor axis under a reference static condition, and the mass center.



The instantaneous position vector of the mass center can be expressed according to the parallelogram law as the sum of the position vector pointing to the origin of the body fixed coordinate system and the relative position vector of the mass center in the body-fixed coordinate system; thus

$$\vec{R} = \vec{R}_0 + \delta\vec{R}' \tag{5.65}$$

δ_K measures the shift of the mass center from the cross-sectional <u>reference</u> center

where,

$$\vec{R}_0 = \vec{i}_1 x_1 + \vec{i}_2 x_2 + \vec{i}_3 x_3$$

$$\delta\vec{R}' = \vec{i}_1{}' \; \delta_K \tag{5.66}$$

$(\vec{i}_1{}', \vec{i}_2{}', \vec{i}_3{}')$ are body-fixed unit vectors while $(\vec{i}_1, \vec{i}_2, \vec{i}_3)$ are earth-bound stationary unit vectors. (x,y,z) are components of the instantaneous displacement of the rotor axis. These two sets of unit triads are linearly related to each other through three successive rotational transformations (translational transformation has already been included in $\vec{R}_0$). In matrix notation,

$$\underline{\vec{i}'} = \underline{\underline{\Psi}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Theta}} \cdot \underline{\vec{i}} \tag{5.67}$$

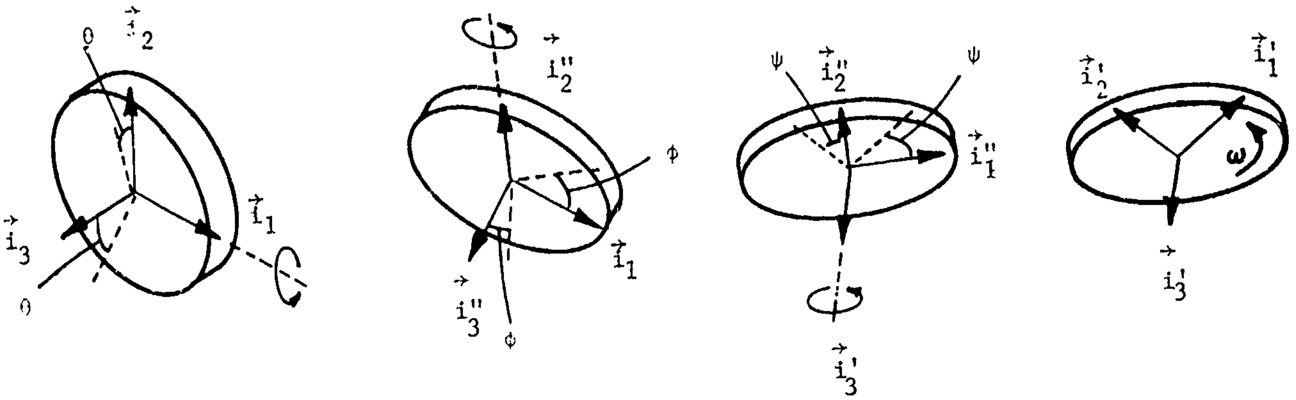
where

$$\underline{\vec{i}} = \begin{bmatrix} \vec{i}_1 \\ \vec{i}_2 \\ \vec{i}_3 \end{bmatrix}; \quad \underline{\vec{i}'} = \begin{bmatrix} \vec{i}'_1 \\ \vec{i}'_2 \\ \vec{i}'_3 \end{bmatrix} \tag{5.68}$$

$$\underline{\underline{\Theta}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$\underline{\underline{\Phi}} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix}$$

$$\underline{\underline{\Psi}} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5.69}$$



(θ,φ,ψ) are the Euler angles. The geometrical interpretation of Equation (5.67) is illustrated above showing the three steps of rotation transformations:

$$\begin{bmatrix} \vec{i}_1 \\ \vec{i}''_2 \\ \vec{i}''_3 \end{bmatrix} = \underline{\underline{\Theta}} \cdot \underline{\vec{i}} \; ; \qquad \begin{bmatrix} \vec{i}''_1 \\ \vec{i}''_2 \\ \vec{i}'_3 \end{bmatrix} = \underline{\underline{\Phi}} \cdot \begin{bmatrix} \vec{i}_1 \\ \vec{i}''_2 \\ \vec{i}''_3 \end{bmatrix} ; \qquad \underline{\vec{i}}' = \underline{\underline{\Psi}} \cdot \begin{bmatrix} \vec{i}''_1 \\ \vec{i}''_2 \\ \vec{i}'_3 \end{bmatrix} \tag{5.70}$$

For small flexural deflections, (θ,ϕ) are regarded as infinitesimal angles. Time-derivatives of $\vec{i}$ can be expressed in terms of time-derivatives of (θ,ϕ,ψ); that is

$$\underline{\dot{\vec{i}}'} = (\dot{\underline{\underline{\Psi}}} \cdot \underline{\underline{\Phi}} \cdot \underline{\underline{\Theta}} + \underline{\underline{\Psi}} \cdot \dot{\underline{\underline{\Phi}}} \cdot \underline{\underline{\Theta}} + \underline{\underline{\Psi}} \cdot \underline{\underline{\Phi}} \cdot \dot{\underline{\underline{\Theta}}}) \cdot \underline{\vec{i}} \tag{5.71}$$

where

$$\dot{\underline{\underline{\Theta}}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin\theta & \cos\theta \\ 0 & -\cos\theta & -\sin\theta \end{bmatrix} \dot{\theta}$$

$$\dot{\underline{\underline{\Phi}}} = \begin{bmatrix} -\sin\phi & 0 & -\cos\phi \\ 0 & 0 & 0 \\ \cos\phi & 0 & -\sin\phi \end{bmatrix} \dot{\phi}$$

$$\dot{\underline{\underline{\Psi}}} = \begin{bmatrix} -\sin\psi & \cos\psi & 0 \\ -\cos\psi & -\sin\psi & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\psi} \tag{5.72}$$

Equation (5.67) can be inverted:

$$\vec{i} = \underline{\underline{\Theta}}^{-1} \cdot \underline{\underline{\Phi}}^{-1} \cdot \underline{\underline{\Psi}}^{-1} \cdot \vec{i}', \tag{5.73}$$

Substituting into Equation (5.71), one obtains

$$\begin{aligned} \underline{\dot{\vec{i}}'} &= (\dot{\underline{\underline{\Psi}}} \cdot \underline{\underline{\Psi}}^{-1} + \underline{\underline{\Psi}} \cdot \dot{\underline{\underline{\Phi}}} \cdot \underline{\underline{\Phi}}^{-1} \underline{\underline{\Psi}}^{-1} + \underline{\underline{\Psi}} \cdot \underline{\underline{\Phi}} \cdot \dot{\underline{\underline{\Theta}}} \cdot \underline{\underline{\Theta}}^{-1} \cdot \underline{\underline{\Phi}}^{-1} \cdot \underline{\underline{\Psi}}^{-1}) \cdot \vec{i}' \\ &= \begin{bmatrix} 0 & \omega_3' & -\omega_2' \\ -\omega_3' & 0 & \omega_1' \\ \omega_2' & -\omega_1' & 0 \end{bmatrix} \cdot \underline{\vec{i}'} \end{aligned} \tag{5.74}$$

where,

$$
\begin{aligned}
\omega_1' &= \dot{\theta}\cos\phi\cos\psi + \dot{\phi}\sin\psi \\
\omega_2' &= -\dot{\theta}\cos\phi\sin\psi + \dot{\phi}\cos\psi \\
\omega_3' &= \dot{\theta}\sin\phi + \dot{\psi}
\end{aligned}
\tag{5.75}
$$

and

$$
\ddot{\vec{\underline{i}}}' = \left\{ \begin{bmatrix} 0 & \dot{\omega}_3' & -\dot{\omega}_2' \\ -\dot{\omega}_3' & 0 & \dot{\omega}_1' \\ \dot{\omega}_2' & -\dot{\omega}_1' & 0 \end{bmatrix} + \begin{bmatrix} -\omega_3'^2-\omega_2'^2 & \omega_1'\omega_2' & \omega_3'\omega_1' \\ \omega_1'\omega_2' & -\omega_3'^2-\omega_1'^2 & \omega_2'\omega_3' \\ \omega_3'\omega_1' & \omega_2'\omega_3' & -\omega_1'^2-\omega_2'^2 \end{bmatrix} \right\} \cdot \vec{\underline{i}}'
\tag{5.76}
$$

where,

$$
\begin{aligned}
\dot{\omega}_1' &= \ddot{\theta}\cos\phi\cos\psi + \ddot{\phi}\sin\psi - \dot{\theta}\dot{\phi}\sin\phi\cos\psi + \dot{\phi}\dot{\psi}\cos\psi - \dot{\psi}\dot{\theta}\cos\phi\sin\psi \\
\dot{\omega}_2' &= -\ddot{\theta}\cos\phi\sin\psi + \ddot{\phi}\cos\psi + \dot{\theta}\dot{\phi}\sin\phi\sin\psi - \dot{\phi}\dot{\psi}\sin\psi - \dot{\psi}\dot{\theta}\cos\phi\cos\psi \\
\dot{\omega}_3' &= \ddot{\theta}\sin\phi + \ddot{\psi} + \dot{\theta}\dot{\phi}\cos\phi
\end{aligned}
\tag{5.77}
$$

If flexural displacements are small, (θ,ϕ) are regarded to be infinitesimal. Then Equations (5.75) are reduced to:

$$
\begin{aligned}
\omega_1' &\simeq \dot{\theta}\cos\psi + \dot{\phi}\sin\psi \\
\omega_2' &\simeq -\dot{\theta}\sin\psi + \dot{\phi}\cos\psi \\
\omega_3' &\simeq \dot{\psi}
\end{aligned}
\tag{5.78}
$$

Accordingly, neglecting second order infinitesimal terms,

$$
\ddot{\vec{\underline{i}}}' \simeq \left\{ \begin{bmatrix} 0 & \dot{\omega}_3' & -\dot{\omega}_2' \\ -\dot{\omega}_3' & 0 & \dot{\omega}_1' \\ \dot{\omega}_2' & -\dot{\omega}_1' & 0 \end{bmatrix} + \begin{bmatrix} -\omega_3'^2 & 0 & \omega_3'\omega_1' \\ 0 & -\omega_3'^2 & \omega_2'\omega_3' \\ \omega_3'\omega_1' & \omega_2'\omega_3' & 0 \end{bmatrix} \right\} \cdot \vec{i}'
\tag{5.79}
$$

and

$$\dot{\omega}'_1 = \ddot{\theta}\cos\psi + \ddot{\phi}\sin\psi + \dot{\phi}\dot{\psi}\cos\psi$$

$$\dot{\omega}'_2 = -\ddot{\theta}\sin\psi + \ddot{\phi}\cos\psi - \dot{\psi}\dot{\theta}\cos\psi \qquad (5.80)$$

$$\dot{\omega}'_3 = \ddot{\psi}$$

The d'Alembert force can thus be obtained by taking the second time derivatives of Equations (5.65) and (5.66):

$$-M\ddot{\vec{R}} = -M(\ddot{\vec{R}}_o + \ddot{\vec{S}}\vec{R}')$$

$$= -M\ddot{\vec{R}}_o - \ddot{\vec{i}}'_1 \; M\delta_k$$

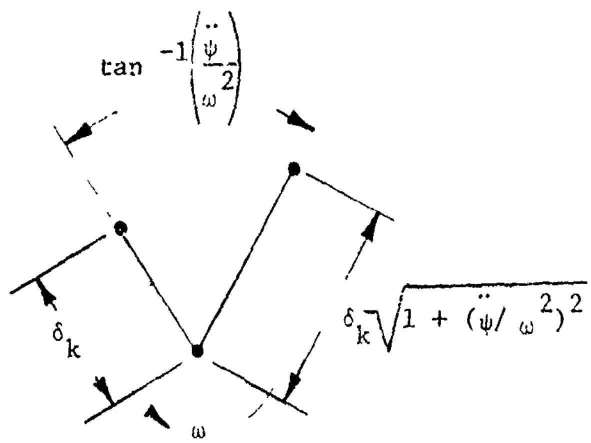
And, making use of Equations (5.79) and (5.80),

$$-M\ddot{\vec{R}} = -M\ddot{\vec{R}}_o - M\delta_k \{-\vec{i}'_1 (\dot{\psi})^2 + \vec{i}'_2 \ddot{\psi} + \vec{i}'_3 (\ddot{\theta}\sin\psi - \ddot{\phi}\cos\psi$$
$$+ 2\dot{\psi}\dot{\theta}\cos\psi + 2\dot{\psi}\dot{\phi}\sin\psi)\} \qquad (5.81)$$

In an ideal situation, all sections of the rotor would be perfectly balanced so that $\delta_k = 0$. Furthermore, in the absence of other forms of vibrational excitation, $(|\vec{R}_o|, \theta, \phi) = 0$ and $\dot{\psi} = \omega$ = constant is the rotational angular speed. The dominant effect of the unbalance response problem is measured by the magnitude $M\delta_k\omega^2$. Constancy of $\dot{\psi}$ would be perturbed by rotor acceleration and/or torsional vibration. Ordinarily, both are secondary effects. The rate of angular acceleration usually is small in comparison with the rate of rotation and the torsional twist is generally a small quantity. Thus, one may write $\dot{\psi} = \omega + \delta\dot{\psi}$, $\delta\dot{\psi}$ being the "rotational rate perturbation". $(\delta\psi, \delta\theta, \delta\phi)$ are regarded as first order perturbations. Equation (5.81) can thus be reduced to

$$-M\ddot{\vec{R}} \simeq -M\ddot{\vec{R}}_o + \vec{i}'_1 \; M\delta_k \, \omega^2 \qquad (5.82)$$

$-M\ddot{\vec{R}}_o$ is the d'Alembert force which is already included in the lateral vibration analysis. Since $\vec{i}_1'$ is fixed with the rotor cross section, $\vec{i}_1'\ M\delta_k\ \omega^2$ represents the unbalance excitation in the form of a centrifugal force, which has a constant amplitude and is revolving synchronously. Thus, one may conveniently use the whirl coordinate system as described in Section 5.2.1. The unbalance excitation would be represented by the constant $M\delta_k\ \omega^2$ as a forward whirling force. If unbalances at different axial locations are in various radial planes, a complex number would be used; the amplitude would be the magnitude of the centrifugal force, and the argument would indicate the angular orientation of the unbalance measured from a suitable reference in the same sense as the rotation vector. One may observe from Equation (5.81) that acceleration of the rotor, $(-\vec{i}_2'\ M\delta_k\ \ddot{\psi})$, would appear as a lag component as illustrated below.



With acceleration, the apparent phase angle is

$$-\tan^{-1}\left(\frac{\ddot{\psi}}{\omega^2}\right) \tag{5.83}$$

and the apparent amplitude is enlarged by the factor

$$\{(\ddot{\psi})^2 + \omega^4\}^{1/2}/\omega^2 \tag{5.84}$$

When processing unbalance response data during acceleration or deceleration, compensation for these effects may be necessary.

5.3 Impedance Matrix

5.3.1 Combined Isotropic Matrices

In Section 5.1 it is shown that a linear relationship exists between the angle of twist at station m, ϕ_m, and an externally applied torsional moment at station n, $M_{z,n}$, at any oscillation frequency ν. This can be written in the matrix notation as

$$\underline{\phi} = \underline{\underline{A}}_{torsion} \cdot \underline{M}_z \tag{5.85}$$

$\underline{\underline{A}}_{torsion}$ is a square matrix, the elements of which may be dependent on ν. Since the torsional vibration problem concerns a conservative system, $\underline{\underline{A}}_{torsion}$ is symmetrical in accordance with the classical theory of mechanics. $\underline{\phi}$ and $\underline{M}_z$ are column vectors, the elements of which respectively designate the angles of twist and applied torsional moments at various stations of interest.

The analogous flexure problem was described in Section 5.2. The motion at each rotor station for a flexure problem consists of four degrees of freedom. The flexure motion may have components in either of two lateral orthogonal planes. As shown in Section 5.2.1, due to the gyroscopic effect, it is more convenient to use whirl coordinates to represent the flexural motion instead of the more conventional Cartesian coordinates associated with the lateral planes. For either the forward or the backward whirl component, the flexural motion at station m is described by both a deflection $\hat{w}_m$ and an inclination, $\hat{\theta}_m$. External flexural excitation at station n can be similarly represented in whirl coordinates and consists of both a lateral force, $\hat{F}_n$ and a lateral moment, $\hat{M}_n$. The applicable matrix equation is

$$\underline{W} = \underline{\underline{A}}_{flexure} \cdot \underline{F} \tag{5.86}$$

$\underline{W}$, which describes the flexural motion, is a column vector of an even rank. The upper and lower halves of $\underline{W}$ respectively represent the forward and backward whirl components. At each station, for either the forward or the backward whirl component, the deflection and the inclination make up a pair of elements of the $\underline{W}$ vector. Thus the contents of $\underline{W}$ are $(\acute{w}_1, \acute{\theta}_1, \ldots; \grave{w}_2, \grave{\theta}_2, \ldots)$. $\underline{F}$ is a column vector describing the flexural excitations. The elements of $\underline{F}$ are the lateral force and moment of either whirl component. Since the rotor structure is symmetrical and bearing supports are approximated by isotropic springs, the forward and backward whirl motions are uncoupled. Thus, $\underline{\underline{A}}_{flexure}$ is empty in the off-diagonal half partitions and can be expressed as

$$\underline{\underline{A}}_{flexure} = \begin{bmatrix} \acute{\underline{\underline{A}}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \grave{\underline{\underline{A}}} \end{bmatrix} \tag{5.87}$$

$\acute{\underline{\underline{A}}}$ and $\grave{\underline{\underline{A}}}$ are of half rank. Since the flexure problem (with bearing supports approximated by isotropic springs) is also conservative, $\acute{\underline{\underline{A}}}$ and $\grave{\underline{\underline{A}}}$ are both symmetrical. $\underline{\phi}$ and $\underline{W}$ can be put together to form a generalized displacement vector $\underline{x}$, and correspondingly, $\underline{M}_z$ and $\underline{F}$ would be put together to form the generalized force vector $\underline{P}$.

If $\underline{\phi}$ and $\underline{W}$ contain only elements which are identified with stations of the rotor, then Equations (5.85) and (5.86) are actually truncated representations of the physical system as computed by the procedures outlined in Sections 5.1 and 5.2. Suppose there are N shaft segments, then there are N+1 stations. The rank of $\underline{\phi}$ is N+1 while that of $\underline{W}$ is 4(N+1). Due to the distributed method of analysis of the shaft segments, the number of free modes which can be calculated accurately is unlimited. In contrast, with a lumped parameter analysis [22], mass effects are assigned as rigid bodies at the stations while shaft segments are regarded as massless torsion/flexure springs; accordingly, one can only calculate N+1 torsional modes and 4(N+1) flexural modes. Actually, the accuracy of the higher modes as computed by the latter

method is questionable; one ordinarily would accept only about the lower 1/3 modes as accurately calculated.

Truncation can be carried out further by omitting some stations altogether, and/or by leaving out one of the two flexure motion parameters (deflection and inclination), and/or by disregarding the backward whirl motion if the rotor system is known to be isotropic while the excitation is a pure forward whirl (e.g., unbalance). In addition, torsional and flexural problems are usually separately treated. For example, suppose there are 10 rotor stations but only the following degrees of freedom are considered to be active,

	Active Station Number
Torsion	2,9
Deflection	1,2,6
Inclination	2,9

and that whirls in both senses are of interest; then the generalized displacement vector is

$$\underline{x} = \begin{bmatrix} \phi_2 \\ \phi_9 \\ \acute{w}_1 \\ \acute{w}_2 \\ \acute{\theta}_2 \\ \acute{w}_6 \\ \acute{\theta}_9 \\ \grave{w}_1 \\ \grave{w}_2 \\ \grave{\theta}_2 \\ \grave{w}_6 \\ \grave{\theta}_9 \end{bmatrix} \tag{5.88}$$

The total rank of this formulation is reduced to 12 from the maximum possible rank of 50. The major motivation for rank reduction by truncation is to achieve computation economy. One should retain only sufficient active stations and/or degrees of freedom in order to describe the mode shapes in sufficient detail and to allow consideration of various engineering considerations such as:

- Balancing corrections,
- Bearing support variations,
- Bearing relocations,
- Aerodynamic excitation,
- Wheel shroud rubbing, and
- Geared coupling of rotors.

Regardless of the extent of truncation, a conservative isotropic rotor system can be represented by the combined matrix equation

$$\underline{X} = \underline{\underline{A}} \cdot \underline{P} \qquad (5.89)$$

where,

$$\underline{X} = \begin{bmatrix} \underline{\phi} \\ \underline{\acute{W}} \\ \underline{\grave{W}} \end{bmatrix} ; \quad P = \begin{bmatrix} \underline{M_z} \\ \underline{\acute{F}} \\ \underline{\grave{F}} \end{bmatrix} \qquad (5.90)$$

And the $\underline{\underline{A}}$ matrix (commonly known as influence or mobility matrix) is quasi-diagonal; that is,

$$\underline{\underline{A}} = \begin{bmatrix} \underline{\underline{A}}_{torsion} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{\acute{A}}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{0}} & \underline{\underline{\grave{A}}} \end{bmatrix} \qquad (5.91)$$

$\underline{\underline{A}}_{tors}$, $\underline{\underline{\dot{A}}}$ and $\underline{\underline{\grave{A}}}$ are all symmetrical. Upon inversion, one obtains

$$\underline{P} = \underline{\underline{K}} \cdot X \qquad (5.92)$$

where,

$$\underline{\underline{K}} = \begin{bmatrix} \underline{\underline{K}}_{torsion} & \underline{\underline{0}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \acute{\underline{\underline{K}}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \underline{\underline{0}} & \grave{\underline{\underline{K}}} \end{bmatrix}$$

$$\underline{\underline{K}}_{torsion} = \underline{\underline{A}}^{-1}_{torsion}; \quad \acute{\underline{\underline{K}}} = \acute{\underline{\underline{A}}}^{-1}; \quad \grave{\underline{\underline{K}}} = \grave{\underline{\underline{A}}}^{-1} \tag{5.93}$$

$\underline{\underline{K}}$ is known as the stiffness or impedance matrix.

5.3.2 Study of Bearing Effects - Impedance Alteration

Representation of a bearing support by an isotropic spring, together with the use of whirl coordinates, allows considerable simplification in the computation of $\underline{\underline{A}}_{flexure}$ and its inversion to obtain $\underline{\underline{K}}_{flexure}$ in two ways:

- All elements of $\underline{\underline{A}}_{flexure}$ are real.
- $\underline{\underline{A}}_{flexure}$ is empty in its off-diagonal half partitions.

However, fluid film bearings, which are in increasing use in modern rotating machines, are commonly described by eight coefficients*. Thus, in Cartesian coordinates, reactions of a fluid film bearing to lateral displacements are expressed as

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix}_{brg} = - \begin{bmatrix} K_{xx} + i\nu B_{xx} & K_{xy} + i\nu B_{xy} \\ K_{yx} + i\nu B_{yx} & K_{yy} + i\nu B_{yy} \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix} \tag{5.94}$$

---

* The equation governing fluid film journal bearings is self-adjoint with respect to squeeze-film motion. Therefore, $B_{xy} = B_{yx}$ generally holds.

Upon transforming to whirl coordinates according to the procedure outlined in Section 5.2.1, this becomes

$$\begin{bmatrix} \acute{F} \\ \grave{F} \end{bmatrix}_{brg} = - \begin{bmatrix} Z_{||} - iZ_{\perp} & \Delta Z_{||} + i\Delta Z_{\perp} \\ \Delta Z_{||} - i\Delta Z_{\perp} & Z_{||} + iZ_{\perp} \end{bmatrix} \begin{bmatrix} \acute{W} \\ \grave{W} \end{bmatrix} \tag{5.95}$$

where

$$Z_{||} = \frac{1}{2} \{(K_{xx} + K_{yy}) + i\nu(B_{xx} + B_{yy})\}$$

$$\Delta Z_{||} = \frac{1}{2} \{(K_{xx} - K_{yy}) + i\nu(B_{xx} - B_{yy})\}$$

$$Z_{\perp} = \frac{1}{2} \{(K_{xy} - K_{yx}) + i\nu(B_{xy} - B_{yx})\}$$

$$\Delta Z_{\perp} = \frac{1}{2} \{(K_{xy} + K_{yx}) + i\nu(B_{xy} + B_{yx})\} \tag{5.96}$$

If one recognizes that the matrix of Equation (5.95) should replace the stiffness coefficients of an isotropic spring, then one can write, for the particular station in concern

$$\begin{bmatrix} \acute{F} \\ \grave{F} \end{bmatrix}_{\substack{isotropic \\ spring}}$$

$$= \begin{bmatrix} \acute{F} \\ \grave{F} \end{bmatrix}_{actual} - \begin{bmatrix} Z_{||} - k - iZ_{\perp} & \Delta Z_{||} + i\Delta Z_{\perp} \\ \Delta Z_{||} - i\Delta Z_{\perp} & Z_{||} - k + iZ_{\perp} \end{bmatrix} \begin{bmatrix} \acute{W} \\ \grave{W} \end{bmatrix} \tag{5.97}$$

Upon substituting this relation into Equation (5.92) and regrouping, one finds

$$\underline{\underline{K}}_{flexure} = \begin{bmatrix} \acute{\underline{\underline{K}}} & \underline{\underline{0}} \\ \underline{\underline{0}} & \grave{\underline{\underline{K}}} \end{bmatrix} + \underline{\underline{K}}_{alteration} \tag{5.98}$$

to be the flexure impedance matrix with the actual bearing characteristics.

If, for instance, such an alteration is applicable to station 2 of Equation (5.88), then

$$\underline{\underline{K}}_{alteration}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{||}-k-iZ_{\perp} & 0 & 0 & 0 & 0 & \Delta Z_{||}+i\Delta Z_{\perp} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Delta Z_{||}-i\Delta Z_{\perp} & 0 & 0 & 0 & 0 & Z_{||}-k+iZ_{\perp} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5.99)$$

By such a procedure, the influence of varying bearing characteristics can be studied without repeating the computations described in Section 5.2.

It should be clear that since the alteration procedure can subtract from one station the stiffness which was originally assigned to it in computing the original isotropic mobility matrix and add some stiffness to another station, so long as both stations have been identified as active stations, one can evaluate the effects of relocating bearings quite conveniently.

5.3.3 Scaling of Impedance Matrix

Each diagonal element of the impedance matrix is roughly the linear combination of stiffness, inertia, and damping effects associated with the portion of rotor near the junction represented by the matrix element. In typical problems, the numerical value of the matrix element can be quite large, it is not uncommon to be in excess of 10^6. The determinant of the impedance matrix is then of the order of $(10^6)^N$, N being the rank of the matrix. Even in a truncated representation, several junctions would be identified as active degrees of freedom; for the lateral vibration analysis of an anisotropic rotor the rank of the matrix would be twice the number of active junctions. For instance, if four junctions are active, N would be eight. Thus, computation of the determinant of the impedance matrix, which is done repeatedly in treating the eigenvalue problems, very large numbers, in excess of 10^{48}, are encountered. This situation readily causes difficulty of exponent overflow in numerical computation. The problem is proportionally more serious when higher modes are treated. Rescaling of the impedance matrix is therefore necessary to ensure trouble-free computation. A suitable rescaling procedure is to divide each element of the impedance matrix by the square of the excitation frequency (frequency is measured in radians/sec here). By so doing, a diagonal element is converted to a mass from inertia. Its range of numerical value then becomes bounded within very reasonable limits.

SECTION VI
EIGENVALUE PROBLEMS

6.1 Diagonalization of the Impedance Matrix

Given the impedance matrix $\underline{\underline{K}}$, which may contain the actual bearing characteristics, one may pose the question whether or not one can find the condition

$$\underline{\underline{K}} \cdot \underline{x} = \kappa \, \underline{x} \tag{6.1}$$

This equation indicates that premultiplication of the displacement vector, $\underline{x}$, results in the uniform "complication" of all elements by the same factor κ. It can be rewritten as

$$(\underline{\underline{K}} - \kappa \, \underline{\underline{I}}) \cdot \underline{x} = 0 \tag{6.2}$$

where $\underline{\underline{I}}$ is the identity matrix. In the last form, one is confronted with a linear homogeneous system for which the only non-trivial solution is satisfied by the condition

$$|\underline{\underline{K}} - \kappa \, \underline{\underline{I}}| = 0 \tag{6.3}$$

Only discrete values of $\kappa = \kappa_\alpha$ can satisfy Equation (6.3). Associated with each κ_α is the modal vector $\underline{x}_\alpha$, which together with κ_α satisfies Equation (6.1). The homogeneous nature of Equation (6.2) makes the amplitude of $\underline{x}_\alpha$ an indeterminate quantity. A common normalization condition is to equate the largest element of $\underline{x}_\alpha$ to unity. Since $\underline{\underline{K}}$ is dependent on the frequency of motion, ν, κ_α and $\underline{x}_\alpha$ are thus also frequency dependent. The mathematical problem outlined above is known as an eigenvalue problem. Equation (6.3) is the eigenvalue or characteristic value equation; the roots of which, κ_i, are the eigenvalues, and $\underline{x}_i$ are the eigenvectors or modal vectors.

6.2 Pseudo Single-Degree-of-Freedom Concepts

κ_α in general can be a complex quantity, hence one can write

$$\kappa_\alpha(\nu) = u_\alpha(\nu) + i \, v_\alpha(\nu) \tag{6.4}$$

Substituting Equations (6.1) and (6.4) into Equation (6.2), one obtains

$$\underline{P}_\alpha = (u_\alpha + iv_\alpha)\,\underline{x}_\alpha \tag{6.5}$$

which may be regarded as the modal force. Of special interest is the condition

$$u_\alpha(\nu_\alpha) = 0 \tag{6.6}$$

which defines the natural frequency ν_α at which $\underline{x}_\alpha$ could be self-sustained provided $v_\alpha(\nu_\alpha)$ also vanishes. If $v_\alpha(\nu_\alpha)$ does not vanish, then its sign indicates whether $\underline{P}_\alpha$ would be providing energy to the rotor system if $v_\alpha > 0$, or $\underline{P}_\alpha$ would be removing energy from the rotor system if $v_\alpha < 0$. Drawing analogy with a single-degree-of-freedom mass point system, the above reasoning suggests that Equation (6.3) may be expressed as

$$W = |\underline{\underline{K}} - \kappa\,\underline{\underline{I}}| = [k_\alpha - m_\alpha \nu^2 + i(\nu_\alpha c_\alpha - \nu c)]\, H_\alpha\,(\nu, c) \tag{6.7}$$

m_α may be thought of as the modal mass, while c_α would be the modal damping coefficient. Extending the analogy one step further, the modal critical damping ratio may be defined as (6.1):

$$\beta_\alpha = \frac{c_\alpha}{2m_\alpha \nu_\alpha} \tag{6.8}$$

Computation of $(\nu_\alpha, \beta_\alpha)$ is of great importance since ν_α indicates the frequency at which a natural mode tends to oscillate and $1/\beta_\alpha$ is an indication of the excitability (amplification factor) of the natural mode. The condition $\beta<0$ signifies an unstable condition such that $\underline{x}_\alpha$ would require external dissipative restraint in order not to become unbounded in amplitude.

In the study of rotor dynamics, it is usually desirable to solve the eigenvalue problem within a specified range of ν. Since the elements

of $\underline{\underline{K}}$ are smooth functions of ν, it is feasible to adopt an interpolation strategy. The isotropic rotor problem would be solved at suitable intervals of ν through the desired range. Since the rank of $\underline{\underline{K}}$ is reduced to a minimum by truncation, an accurate interpolation scheme can be systematically implemented very economically.

6.3 Computation of the Conservative System

Since the rotor structure as modelled in Sections 5.1 and 5.2 is non-dissipative, and if the actual bearing characteristics are also conservative, then one can expect c_α, $\nu_\alpha = 0$ so that Equation (6.7) becomes

$$W = (k_\alpha - m_\alpha \nu^2)\, H_\alpha(\nu)$$

In fact, if up to N natural modes are of interest, then one can write

$$W = \prod_{\alpha=1}^{N} (k_\alpha - m_\alpha \nu^2)\, H_N(\nu) \tag{6.9}$$

It is relatively straightforward to determine the roots of such an equation by numerical interpolation. However, for rotors of common interest and in engineering units, ν would be a relatively large number, while m would be a number of moderate magnitude. Thus Equation (6.9) tends to behave like an undulating function which has an ever-increasing envelope. Numerical interpolation of such a function may not be accurate if it is to be performed over a wide range of ν. It has been determined through fairly extensive testing that a cubic spline interpolation procedure is quite satisfactory, provided each element of K (therefore κ also) is first divided by ν^2 and a uniform geometric progression is adopted with consecutive increments not to exceed 10 percent. The cubic spline interpolation algorithm is discussed in Appendix B.

6.4 Computation of the Non-Conservative System

If it is not known in advance that the rotor system is conservative,

v_α is an unknown quantity. The eigenvalue problem involves the solution of

$$W(\nu_\alpha, \kappa_\alpha = i\nu_\alpha) = |\underline{\underline{K}} - i\nu_\alpha c_\alpha \underline{\underline{I}}|$$
$$= U(\nu_\alpha, c_\alpha) + iV(\nu_\alpha, c_\alpha)$$
$$= 0 \tag{6.10}$$

(U,V) are required to vanish simultaneously, yielding two conditions to determine the values of (ν_α, c_α). This can be done using a two-parameter Newton iterative scheme. Assuming that (ν'_α, c'_α) are fairly good estimates for (ν_α, c_α) so that

$$\delta\nu = \nu_\alpha - \nu'_\alpha; \quad \delta c = c_\alpha - c'_\alpha \tag{6.11}$$

are respectively small quantities in comparison to (ν_α, c_α); then one can approximate Equation (6.10) by its Taylor series expansion:

$$0 = U' + \delta\nu \frac{\partial U'}{\partial \nu'_\alpha} + \delta c \frac{\partial U'}{\partial c'_\alpha} + \ldots$$

$$0 = V' + \delta\nu \frac{\partial V'}{\partial \nu'_\alpha} + \delta v \frac{\partial V'}{\partial c'_\alpha} + \ldots \tag{6.12}$$

The necessary improvements are found by truncating Equation (6.12) beyond the first order expansions and making use of the Kramer's rule:

$$\begin{bmatrix} \delta\nu \\ \\ \delta c \end{bmatrix} = \left\{ \frac{\partial U'}{\partial \nu'_\alpha} \frac{\partial V'}{\partial c'_\alpha} - \frac{\partial U'}{\partial c'_\alpha} \frac{\partial V'}{\partial \nu'_\alpha} \right\}^{-1} \begin{bmatrix} V' \dfrac{\partial U'}{\partial c'_\alpha} - U' \dfrac{\partial V'}{\partial c'_\alpha} \\ \\ U' \dfrac{\partial V'}{\partial \nu'_\alpha} - V' \dfrac{\partial U'}{\partial \nu'_\alpha} \end{bmatrix} \tag{6.13}$$

where

(U', V') are values of (U, V) at (ν'_α, c'_α).

In principle, the above formulae can be used repeatedly until $(\delta\nu, \delta c)$ become negligibly small. In practice, a number of algorithmic difficulties must be resolved. They are enumerated as follows:

- A logical procedure is needed to initiate the iteration process; what should the first values of (ν'_α, c'_α) be?
- If (ν'_α, c'_α) are not sufficiently close to (ν_α, c_α), an excessively large value of $\delta\nu$ would be found so that the improved value $(\nu'_\alpha + \delta\nu)$ would be outside the frequency range for which $\underset{\approx}{K}$ is available; how should the iteration process continue?
- Suppose one set of (ν_α, c_α) has been established, how can one avoid repeated convergence to the same set in subsequent iterations?
- Since ν_α and c_α have different units, how should c_α be scaled?

<u>First Estimates for (ν'_α, c'_α)</u>

There is no simple way to estimate c'_α for the first time. In fact, one does not even know whether or not the system is conservative. Under the circumstances, $c'_\alpha = 0$ may be assumed for the first time. This has been found satisfactory even for cases where the final c_α ultimately approaches a relatively large magnitude. With such an assumed c'_α, one wishes to determine ν'_α such that (U', V') are both as close to null values as possible. This is equivalent to seeking the minimum condition of $U'^2 + V'^2$; or

$$U' \frac{\partial U'}{\partial \nu'_\alpha} + V' \frac{\partial V'}{\partial \nu'_\alpha} = 0; \quad \left(\frac{\partial U'}{\partial \nu'_\alpha}\right)^2 + \left(\frac{\partial V'}{\partial \nu'_\alpha}\right)^2 + U' \frac{\partial^2 U'}{\partial \nu'^2_\alpha} + V' \frac{\partial^2 V'}{\partial \nu'^2_\alpha} > 0 \tag{6.14}$$

within the assigned frequency range. This can be readily accomplished with the cubic spline interpolation method which is described in Appendix B.

It is quite possible that $U'^2 + V'^2$ does not have a minimum in the

specified range of ν, then the geometrical mean of the extreme limits can be used as the first guess for ν'_α.

A provision is made to allow the user to enter first estimates for (ν'_α, c'_α) of his choice. This is done by assigning the value -1 to the parameter IBEG in input card 14 and then using input card 17 to enter the desired first estimates.

Excessive $\delta\nu$

If the current values of (ν'_α, c'_α) are not sufficiently close to the eigenvalue set, $\delta\nu$ would be excessively large in magnitude to project the improved ν'_α beyond either extreme of the specified frequency range. Since an extrapolation scheme would not be dependable, it is necessary to keep trial values of ν'_α within the limits at all times. Thus, upon detecting the trend for ν'_α to pass beyond either limit while the current ν'_α is not at the same limit, $\delta\nu$ is scaled down to place ν'_α at the limit while δc is scaled down by the same proportion. If the current ν'_α is already at the same limit, then the geometrical mean of the extreme limits of ν would be used as the next trial value of ν'_α while c'_α would not be changed.

If the specified range of ν actually does not contain an eigenvalue set, the above situation would be cycled indefinitely. A limit on the maximum number of iterations is imposed to terminate the search. Experience has indicated that a maximum limit of 20 iteration steps for each set of (ν_α, c_α) is satisfactory. In most test cases, convergence is reached in about six iterations; only very seldom it was found necessary to exceed 10 steps.

If the iteration progression should attempt to exceed either limit of the frequency range repeatedly, the value of the iteration residue $(U')^2 + (V')^2$ would accordingly oscillate. Such oscillations are allowed to occur three times. When the fourth time is detected, the computation will be terminated and a message is printed out to indicate

the direction of shifting the frequency range for a subsequent trial, if so desired, as suggested by the iteration history.

Computation of (m_α, β_α)

Upon satisfying Equation (6.10) by repeated iterations with the aid of Equation (6.13), (ν_α, c_α) become known. Since the iterative steps also involves the computations of

$$\left.\frac{\partial W}{\partial \nu}\right|_{\nu_\alpha, c_\alpha} = \left.\frac{\partial U}{\partial \nu}\right|_{\nu_\alpha, c_\alpha} + i \left.\frac{\partial V}{\partial \nu}\right|_{\nu_\alpha, c_\alpha}$$

and

$$\left.\frac{\partial W}{\partial c}\right|_{\nu_\alpha, c_\alpha} = \left.\frac{\partial U}{\partial c}\right|_{\nu_\alpha, c_\alpha} + i \left.\frac{\partial V}{\partial c}\right|_{\nu_\alpha, c_\alpha}$$

one can subsequently compute (m_α, β_α).

If one formally differentiates Equation (6.7) with respect to ν and c, it is found

$$\frac{\partial W}{\partial \nu} = (-2m_\alpha \nu - ic) \ H_\alpha + \{(k_\alpha - m_\alpha \nu^2) + i(\nu_\alpha c_\alpha - \nu c)\} \frac{\partial H_\alpha}{\partial \nu}$$

$$\frac{\partial W}{\partial c} = -i\nu H_\alpha + \{(k_\alpha - m_\alpha \nu^2) + i(\nu_\alpha c_\alpha - \nu c)\} \frac{\partial H_\alpha}{\partial c}$$

Allowing the limiting process $(\nu \to \nu_\alpha, c \to c_\alpha)$

$$\left.\frac{\partial W}{\partial \nu}\right|_{\nu_\alpha, c_\alpha} = (-2m_\alpha \nu_\alpha \cdot ic_\alpha) \ H_\alpha \ (\nu_\alpha, c_\alpha)$$

$$\left.\frac{\partial W}{\partial c}\right|_{\nu_\alpha, c_\alpha} = -i\nu_\alpha H_\alpha(\nu_\alpha, c_\alpha)$$

H_α can be eliminated, yielding

$$\frac{(\partial W/\partial \nu)}{(\partial W/\partial c)}\Big|_{\nu_\alpha,\ c_\alpha} = \frac{c_\alpha}{\nu_\alpha} - i(2m_\alpha) \tag{6.15}$$

Therefore

$$m_\alpha = -\frac{1}{2}\ \mathrm{Im}\{\frac{(\partial W/\partial \nu)}{(\partial W/\partial c)}\}\Big|_{\nu_\alpha,\ c_\alpha} \tag{6.16}$$

and

$$\beta_\alpha = \frac{\mathrm{Re}\ \{\frac{(\partial W/\partial \nu)}{(\partial W/\partial c)}\}}{\mathrm{Im}\ \{\frac{(\partial W/\partial \nu)}{(\partial W/\partial c)}\}}\Bigg|_{\nu_\alpha,\ c_\alpha} \tag{6.17}$$

The real part of Equation (6.15) yields the redundant condition

$$c_\alpha = \nu_\alpha\ \mathrm{Re}\{\frac{(\partial W/\partial \nu)}{(\partial W/\partial c)}\}\Big|_{\nu_\alpha,\ c_\alpha}$$

since c_α is directly calculated in the iterative process. Accordingly,

$$\varepsilon_\alpha = \frac{c_\alpha - \nu_\alpha \mathrm{Re}\ \{\frac{(\partial W/\partial \nu)}{(\partial W/\partial c)}\}\Big|_{\nu_\alpha,\ c_\alpha}}{\nu_\alpha \cdot \mathrm{Im}\{\frac{(\partial W/\partial \nu)}{(\partial W/\partial c)}\}\Big|_{\nu_\alpha,\ c_\alpha}} \tag{6.18}$$

is an indicator of the precision of the numerical computation of the complex derivatives $(\partial W/\partial \nu,\ \partial W/\partial c)$.

<u>Avoidance of Previously Determined (ν_α, c_α)</u>

Equation (6.7) can be extended to include all eigenvalue sets within the frequency range. Suppose N eigenvalue sets are in the range, then one can write

$$W = \prod_{\alpha=1}^{N} \{(k_\alpha - m_\alpha \nu^2) + i(\nu_\alpha c_\alpha - \nu c)\} H_N(\nu, c) \tag{6.19}$$

H_N does not vanish in the range for any value of α. Upon finding the first set (k_1, m_1, c_1), one can divide out the factor

$$(k_1 - m_1\nu^2) + i(\nu_1 c_1 - \nu c)$$

to obtain

$$W_1 = \frac{W}{(k_1 - m_1\nu^2) + i(\nu_1 c_1 - \nu c)}$$

$$= \prod_{\alpha=2}^{N} \{(k_\alpha - m_\alpha\nu^2) + i\,(\nu_\alpha c_\alpha - \nu c)\}\, H_N(\nu, c) \tag{6.20}$$

(U_1, V_1) and their derivatives are used in subsequent search for (ν_2, c_2). The process is repeated every time when a new set of (ν_α, c_α) is found.

Scaling of c_α

The question of scaling c_α comes up when one introduces the concept of modal critical damping ratio, Equation (6-8), in which the modal mass m appears. Actually, without an appropriate scale for c_α, $(\partial U'/\partial c_\alpha', \partial V'/\partial c_\alpha')$ which are needed during the iterative search for c_α, cannot be computed by finite difference.

Fortunately, these derivatives can be calculated by an analytical method. This derivation is as follows:

$$\frac{\partial W}{\partial c} = \sum_{m=1}^{M} \left[- i \frac{\partial W}{\partial (K_{mm} - i\nu c)}\right]$$

where M is the rank of the system and where $K_{mm}-i\nu c$ is the mth diagonal element of the matrix $[\underline{\underline{K}} - i\nu c\underline{\underline{I}}]$. The derivative $\frac{\partial W}{\partial (K_{mm}-i\nu c)}$ can be computed by using

two matrices. The first matrix, $\underline{K}_{(1)m}$, is that formed by substituting unity for the mth diagonal element in $[\underline{K} - i\nu c\underline{I}]$ The second matrix, $\underline{K}_{(0)m}$, is that formed by substituting zero for the mth diagonal element in $[\underline{K} - i\nu c\underline{I}]$.
From these

$$\frac{\partial W}{\partial (K_{mm} - i\nu c)} = |\underline{K}_{(1)m}| - |\underline{K}_{(0)m}|$$

so that

$$\frac{\partial W}{\partial c} = - i\nu \cdot \sum_{m=1}^{M} \{ |\underline{K}_{(1)m}| - |\underline{K}_{(0)m}| \} \tag{6 21}$$

APPENDIX A

GYROSCOPIC INERTIA FOR SMALL LATERAL ANGULAR OSCILLATIONS

In the analysis of lateral vibrations of a high speed rotor, gyroscopic inertia must be considered if the motion involves an inclination of the rotor axis. The problem is essentially a straightforward application of Euler's theorem of angular momentum. However in the conventional presentation of Euler's theorem, the Euler angles are so chosen that a quadratic combination of their time derivatives appear in the angular inertia. Since the lateral inclinations are always small in practical problems, it is convenient to identify two of the Euler angles with them and hence the angular inertia can be linearly related to the time derivatives of the lateral inclinations.

The treatment to be presented applies either to a portion of a rotor, which can be regarded as a rigid body (see Section 5.2.1), or to an infinitesimal element of a flexible shaft segment, which is regarded as a rigid disk (see Section 5.2.2).

The angular momentum of a rigid body is

$$\vec{H} = \vec{\omega} \cdot I \tag{A-1}$$

$\vec{\omega}$ is the angular velocity; I is the second order moment of inertia tensor and is an invariant in body-fixed coordinates. Let $(\vec{i}_1', \vec{i}_2', \vec{i}_3')$ be a triad of unit Cartesian base vectors attached to the rigid body at its mass center, then

$$I = \vec{i}_k' \, (I_{mm} \, \delta_{kl} - I_{kl}) \, \vec{i}_l' \tag{A-2}$$

where I_{kl} is an element of the matrix

$$\underline{\underline{I}} = \int \begin{bmatrix} \xi^2 & \xi\eta & \xi\zeta \\ \eta\xi & \eta^2 & \eta\zeta \\ \zeta\xi & \zeta\eta & \zeta^2 \end{bmatrix} dM \tag{A-3}$$

dM is a mass element of the rigid body. (ξ,η,ζ) are coordinates of the location of dM with respect to the body fixed base vectors. δ_{kl} is the Kronecker delta. The summation convention applies in Equation (A-2). Since the origin coincides with the mass center, all off-diagonal terms of Equation (A-3) vanish. Thus,

$$I = \vec{i}_1' \vec{i}_1' I_1 + \vec{i}_2' \vec{i}_2' I_2 + \vec{i}_3' \vec{i}_3' I_3 \tag{A-4}$$

where,

$$I_1 = \int (\eta^2 + \zeta^2) dM$$

$$I_2 = \int (\zeta^2 + \xi^2) dM$$

$$I_3 = \int (\xi^2 + \eta^2) dM \tag{A-5}$$

(I_1, I_2) are the two transverse moments of inertia, I_3 is the polar moment of inertia.

The body fixed base vectors $(\vec{i}_1', \vec{i}_2', \vec{i}_3')$ may be related to a set of ground fixed base vectors $(\vec{i}_1, \vec{i}_2, \vec{i}_3)$. Let $\vec{i}_3'$ be coincident with the rotor axis and $\vec{i}_3$ be coincident with the bearing axis. $(\vec{i}_1, \vec{i}_2, \vec{i}_3)$ can be mapped onto $(\vec{i}_1', \vec{i}_2', \vec{i}_3')$ through three successive rotations through the Euler angles (θ,ϕ,ψ) as illustrated in Figure 3.

If a rotation about $\vec{i}_1$ through an angle θ is performed so that $\vec{i}_3$ is made to fall on $\vec{i}_3''$. $\vec{i}_2$ is accordingly moved to $\vec{i}_2''$. In matrix notation, this is represented by

$$\begin{bmatrix} \vec{i}_1 \\ \vec{i}_2'' \\ \vec{i}_3'' \end{bmatrix} = \underline{\theta} \cdot \begin{bmatrix} \vec{i}_1 \\ \vec{i}_2 \\ \vec{i}_3 \end{bmatrix} ; \quad \underline{\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{A-6}$$

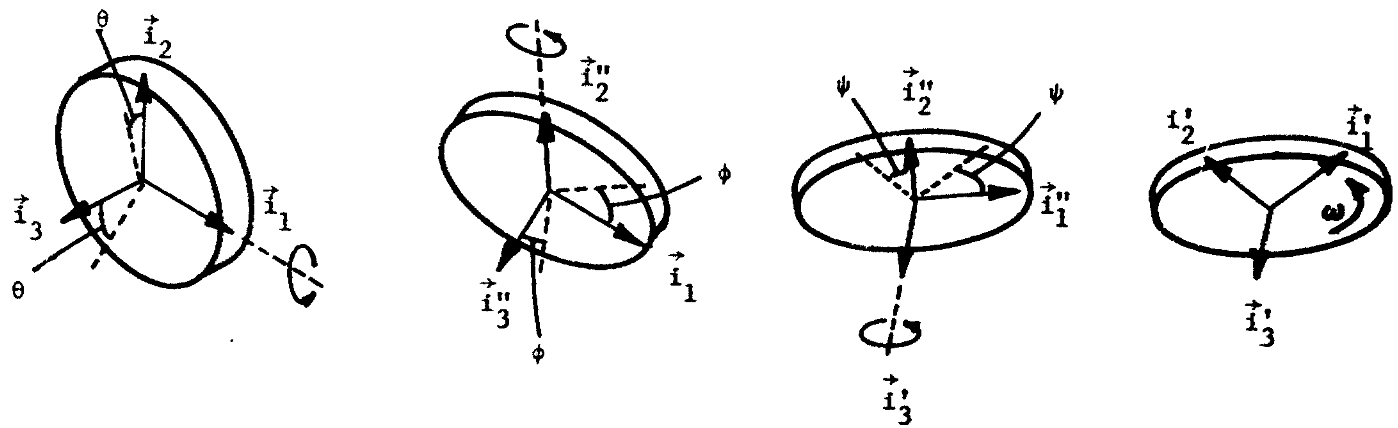


Figure 3   Transformation of Base Vectors

The second rotion is about $\vec{i}''_2$ through the angle ϕ

$$\begin{bmatrix} \vec{i}''_1 \\ \vec{i}''_2 \\ \vec{i}'_3 \end{bmatrix} = \underline{\underline{\Phi}} \cdot \begin{bmatrix} \vec{i}_1 \\ \vec{i}''_2 \\ \vec{i}''_3 \end{bmatrix} ; \quad \underline{\underline{\Phi}} = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \qquad \text{(A-7)}$$

Finally, rotating about $\vec{i}'_3$ through Ψ:

$$\begin{bmatrix} \vec{i}'_1 \\ \vec{i}'_2 \\ \vec{i}'_3 \end{bmatrix} = \underline{\underline{\Psi}} \cdot \begin{bmatrix} \vec{i}''_1 \\ \vec{i}''_2 \\ \vec{i}'_3 \end{bmatrix} ; \quad \underline{\underline{\Psi}} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{(A-8)}$$

Note the (θ,ϕ) are actually the lateral inclinations and would be regarded as infinitesimal.

Time-derivatives of the Euler angles contribute to the angular velocity of the rigid body. Writing

$$\vec{\omega} = \vec{i}'_1\, \omega'_1 + \vec{i}'_2\, \omega'_2 + \vec{i}'_3\, \omega'_3 \qquad \text{(A-9)}$$

then

$$\omega'_1 = \dot{\theta}\cos\phi\, \cos\psi + \dot{\phi}\, \sin\psi$$

$$\omega'_2 = \dot{\theta}\cos\phi\, \sin\psi + \dot{\phi}\, \cos\psi$$

$$\omega'_3 = \dot{\psi} + \dot{\theta}\, \sin\phi \qquad \text{(A-10)}$$

Alternatively, the angular velocity may be written in terms of components with respect to the ground fixed base vectors:

$$\vec{\omega} = \vec{i}_1\,\omega_1 + \vec{i}_2\,\omega_2 + \vec{i}_3\,\omega_3 \qquad \text{(A-11)}$$

$$\omega_1 = \dot{\theta} + \dot{\psi}\sin\phi$$

$$\omega_2 = \dot{\phi}\cos\theta - \dot{\psi}\cos\phi\sin\theta$$

$$\omega_3 = \dot{\psi}\cos\phi\cos\theta + \dot{\phi}\sin\theta \qquad \text{(A-12)}$$

Substituting Equations (A-4) and (A-9) into Equation (A-1), one obtains

$$\vec{H} = \vec{i}_1'\,\omega_1' I_1 + \vec{i}_2'\,\omega_2' I_2 + \vec{i}_3'\,\omega_3' I_3 \qquad \text{(A-13)}$$

Differentiating with time, it is found that the angular inertia is

$$\dot{\vec{H}} = \vec{i}_1'\,\dot{\omega}_1' I_1 + \vec{i}_2'\,\dot{\omega}_2' I_2 + \vec{i}_3'\,\dot{\omega}_3' I_3$$

$$+ \omega_1' I_1\,\dot{\vec{i}}_1' + \omega_2' I_2\,\dot{\vec{i}}_2' + \omega_3' I_3\,\dot{\vec{i}}_3'$$

but since

$$\dot{\vec{i}}_1' = \vec{i}_2'\omega_3' - \vec{i}_3'\omega_2';\ \dot{\vec{i}}_2' = -\vec{i}_1'\omega_3' + \vec{i}_3'\omega_1';\ \dot{\vec{i}}_3' = \vec{i}_1\omega_2' - \vec{i}_2'\omega_1'$$

therefore,

$$\dot{\vec{H}} = \vec{i}_1\left[\dot{\omega}_1' I_1 + \omega_2'\omega_3'\,(I_3 - I_2)\right] + \vec{i}_2'\left[\dot{\omega}_2' I_2 + \omega_3'\omega_1'\,(I_1 - I_3)\right]$$

$$+ \vec{i}_3'\left[\dot{\omega}_3' I_3 + \omega_1'\omega_2'\,(I_2 - I_1)\right] \qquad \text{(A-14)}$$

To obtain a ground fixed representation, the transformations given by Equations (A-6), (A-7), and (A-8) are applied in reverse:

$$\dot{\vec{H}} = \vec{i}_1 \;\{\cos\phi\left[\dot{H}_1'' + \omega_3'(\omega_2'' I_3 - H_2'')\right] + \sin\phi\left[\dot{H}_3'' - \omega_1'\omega_2'(I_1 - I_2)\right]\}$$

$$+ \vec{i}_2 \;\{\cos\theta\left[\dot{H}_2'' + \omega_3'(H_1'' - \omega_1'' I_3)\right] + \sin\theta\sin\phi\left[\dot{H}_1'' + \omega_3'(\omega_2'' I_3 - H_2'')\right]$$

$$-\sin\theta\cos\phi\left[\dot{H}_3'' - \omega_1'\omega_2'(I_1 - I_2)\right]\}$$

$$+ \vec{i}_3 \;\{\cos\theta\cos\phi\left[\dot{H}_3'' - \omega_1'\omega_2'(I_1 - I_2)\right] - \cos\theta\sin\phi\left[\dot{H}_1'' + \omega_3'(\omega_2'' I_3 - H_2'')\right]$$

$$+ \sin\theta\left[\dot{H}_2'' + \omega_3'(H_1'' - \omega_1'' I_3)\right]\} \qquad \text{(A-15)}$$

$$\omega_1'' = \omega_1' \cos\psi - \omega_2' \sin\psi; \quad \omega_2'' = \omega_1' \sin\psi + \omega_2' \cos\psi;$$

$$H_1'' = \omega_1' \cos\psi I_1 - \omega_2' \sin\psi I_2; \quad H_2'' = \omega_1' \sin\psi I_1 + \omega_2' \cos\psi I_2;$$

$$\dot{H}_1'' = \dot{\omega}_1' \cos\psi I_1 - \dot{\omega}_2' \sin\psi I_2; \quad \dot{H}_2'' = \dot{\omega}_1' \sin\psi I_1 + \dot{\omega}_2' \cos\psi I_2;$$

$$\dot{H}_3'' = \dot{\omega}_3' I_3 \qquad \text{(A-16)}$$

Making use of Equation (A-10), various groups in the above relations are found to be

$$\dot{H}_1'' + \omega_3'(\omega_2'' I_3 - H_2'') =$$

$$I_o \frac{d}{dt}(\cos\phi\,\dot{\theta}) + \Delta I \frac{d}{dt}(\cos\phi\cos 2\psi\dot{\theta} + \sin 2\psi\dot{\phi}) + I_3\dot{\phi}(\dot{\psi} + \sin\phi\dot{\theta})$$

$$-I_o \sin\phi\dot{\theta}\dot{\phi} - \Delta I(\sin\phi\cos\phi\sin 2\psi\dot{\theta}^2 - \sin\phi\cos 2\psi\dot{\theta}\dot{\phi})$$

$$\dot{H}_2'' + \omega_3'(H_1''-\omega_1''I_3)$$

$$= I_o\ddot{\phi} + \Delta I \frac{d}{dt}(\cos\phi\sin2\psi\dot{\theta} - \cos2\psi\dot{\phi}) - I_3\cos\phi(\dot{\psi} + \sin\phi\dot{\theta})\dot{\theta}$$

$$+ I_o\sin\phi\cos\phi\dot{\theta}^2 + \Delta I\sin\phi(\cos\phi\cos2\psi\dot{\theta} + \sin2\psi\dot{\phi})\dot{\theta}$$

$$\dot{H}_3'' - \omega_1'\omega_2'(I_1-I_2)$$

$$= I_3 \frac{d}{dt}(\dot{\psi} + \sin\phi\dot{\theta}) + \Delta I\{\sin2\psi\left[(\cos\phi\dot{\theta})^2 - \dot{\phi}^2\right] - 2\cos2\psi(\cos\phi\dot{\theta})\dot{\phi}\} \tag{A-17}$$

where $I_o = \frac{1}{2}(I_1 + I_2)$ is the mean transverse inertia and $\Delta I = \frac{1}{2}(I_1 - I_2)$ is the transverse inertia differential. Consolidating these expressions, one finds

$$\begin{aligned}
\dot{\vec{H}} = \vec{i}_1\Bigg[& I_o \frac{d}{dt}(\cos^2\phi\dot{\theta}) + \Delta I \frac{d}{dt}\left\{\cos\phi\left[\cos2\psi(\cos\phi\dot{\theta}) + \sin2\psi\dot{\phi}\right]\right\} \\
& + I_3 \frac{d}{dt}\left[\sin\phi(\dot{\psi} + \sin\phi\dot{\theta})\right]\Bigg] \\
+ \vec{i}_2\Bigg[& I_o \frac{d}{dt}\left\{\cos\theta\dot{\phi} + \sin\theta\sin\phi(\cos\phi\dot{\theta})\right\} \\
& + \Delta I \frac{d}{dt}\Big\{\sin\theta\sin\phi\left[\cos2\psi(\cos\phi\dot{\theta}) + \sin2\psi\dot{\phi}\right] \\
& + \cos\theta\left[\sin2\psi(\cos\phi\dot{\theta}) - 2\dot{\psi}\dot{\phi}\right]\Big\} \\
& - I_3 \frac{d}{dt}\left[\sin\theta\cos\phi(\dot{\psi} + \sin\phi\dot{\theta})\right]\Bigg] \\
+ \vec{i}_3\Bigg[& I_3 \frac{d}{dt}\left[\cos\theta\cos\phi(\dot{\psi} + \sin\phi\dot{\theta})\right] \\
& - I_o \frac{d}{dt}\left[\cos\theta\sin\phi(\cos\phi\dot{\theta}) - \sin\theta\dot{\phi}\right] \\
& + \Delta I \frac{d}{dt}\Big\{\sin\theta\left[\sin2\psi(\cos\phi\dot{\theta}) - \cos2\psi\dot{\phi}\right] \\
& - \cos\theta\sin\phi\left[\cos2\psi(\cos\phi\dot{\theta}) + \sin2\psi\dot{\phi}\right]\Big\}\Bigg]
\end{aligned} \tag{A-18}$$

For a small deflection analysis, $(\theta,\phi,\psi-\omega t)$ may be regarded to be infinitesimal, ω being the shaft rotation speed; then one obtains

$$\dot{\vec{H}} \simeq \vec{i}_1 \left\{ I_o \frac{d^2\theta}{dt^2} + \Delta I \frac{d}{dt}\left(\cos 2\omega t \frac{d\theta}{dt} + \sin 2\omega t \frac{d\phi}{dt}\right) + \omega I_3 \frac{d\phi}{dt} \right\}$$

$$+ \vec{i}_2 \left\{ -\Delta I \frac{d}{dt}\left(\cos 2\omega t \frac{d\theta}{dt}\right) - \omega I_3 \frac{d\theta}{dt} + I_o \frac{d^2\phi}{dt^2} + \Delta I \frac{d}{dt}\left(\sin 2\omega t \frac{d\phi}{dt}\right) \right\}$$

$$+ \vec{i}_3 \left\{ I_3 \frac{d^2\delta\psi}{dt^2} \right\}$$

where $\delta\psi = \psi - \omega t$ is the infinitesimal torsional angular displacement.

APPENDIX B

CUBIC SPLINE INTERPOLATION

B.1 Interpolation Requirements

Use of polynomials to perform curve fitting or interpolation is a common practice in engineering work. In each of the five major modes of analysis of flexible rotor dynamics, namely:

- o critical speeds,
- o damped unbalance response,
- o asynchronous resonance,
- o damped asynchronous response, and
- o stability analysis,

an accurate interpolation on the determinant of the system impedance matrix and/or its derivatives is performed.

When seeking critical speeds, rotor spin rate and vibration frequency are equal. The system is conservative and isotropic. Only the forward whirl motion is of interest. The determinant of the impedance matrix is real and presumably a smooth function of frequency. Critical speeds are the roots of the real determinant. This calls for a root searching algorithm. An asynchronous resonance analysis seeks all the natural frequencies within a frequency range of interest at a fixed rotor speed. The impedance matrix, again, has a real and presumably a smoothly varying determinant. Similarly as in the critical speed analysis, a root searching algorithm is needed.

When performing an unbalance response calculation, the frequencies of interest can be specifically specified. However, the most interesting may not be known beforehand. Therefore it is useful to locate first the "damped" critical speeds, then perform corresponding response calculations. A similar situation exists when the damped asynchronous response problem is solved. The "damped" resonance problem, either synchronous or asynchronous, requires an algorithm for the complex eigenvalue problem

discussed in Appendix A. The explicit parameter is directly related to the system damping. An implicit parameter is the "damped" natural frequency. A pair of these parameters is determined for every natural mode by requiring both the real and the imaginary parts of the complex determinant of the characteristic matrix to vanish. The complex eigenvalue problem is also involved in the stability analysis. In the latter case, a negative sign for the system damping indicates an unstable mode. The system damping can further be expressed in terms of a critical damping ratio. A numerically small, albeit positive, critical damping ratio indicates a high "Q" or sensitive natural mode. In each of these three modes of analysis, solution of the complex eigenvalue problem is a central issue. The procedure for solving the complex eigenvalue problem, in turn, requires repeated use of an efficient numerical interpolation scheme.

To understand the requirements of a numerical interpolation scheme to be used in the complex eigenvalue algorithm, it is useful to outline briefly the major computation steps. The method of computation is iterative in character. It is necessary to begin with an initial guess for the two parameters, system damping coefficient and natural frequency. The system damping coefficient is initally assumed to be zero, but one is still left with the question of finding the best starting frequency. Since both real and imaginary parts of the characteristic determinant should vanish upon convergence to a solution, a positive minimal of the square of its magnitude should represent a suitable criterion for the starting frequency. Two specific computations are required:

- Since a minimum must have a vanishing first derivative, locate roots of the first derivative.
- Select that minimum which has a positive second derivative.

Upon establishing a starting condition, it is necessary to determine the "present error" and the best "corrections" to the two parameters in order to continue the iterative process. These require interpolation of the complex determinant and its derivatives at the current value of frequency.

B.2 Description of the Spline Curve-fitting Concept

The spline interpolation allows one to curve fit a set of data points precisely and to maintain a certain degree of smoothness. This method utilizes an odd order (2n+1) polynomial ($n \geq 1$) segmentally in consecutive intervals, maintaining continuity up to the 2n-th derivative at all internal points while permitting n constraints at each end. It turned out that the cubic spline (n=1) would be the simplest form to satisfy various requirements indicated previously.

Within each interval, a cubic polynomial allows the presence of up to two extrema, of which only one may be a minimum. The locations of these extrema are explicitly given by the quadratic formula. The determination of roots is somewhat tedious. The procedures for these calculations will be given later following a description for the determination of the polynomial coefficients in each interval. A common choice of the constraint at either end is a vanishing second derivative.

B.3 The Cubic Spline

Suppose N data points, $y_n = y(x_n)$ for n = 1 through N, are known. The N-1 intervals, $\Delta x_n = x_{n+1} - x_n$ for n = 1 through N-1, do not have to be uniform. A cubic polynomial applicable in the interval $x_n \leq x \leq x_{n+1}$ is

$$y = \left(\frac{x-x_n}{\Delta x_n}\right) y_{n+1} + \left(\frac{x_{n+1}-x}{\Delta x_n}\right) \left\{ y_n + (x-x_n) \left[1 + \frac{x-x_n}{\Delta x_n}\right] \left(\theta_n - \frac{\Delta y_n}{\Delta x_n}\right) + \left(\frac{x-x_n}{2}\right)^2 K_n \right\} \tag{B-1}$$

where, (θ_n, K_n) are respectively first and second derivatives of y at x_n while $\Delta y_n = y_{n+1} - y_n$. Differentiating three times successively,

$$y' = \frac{\Delta y_n}{\Delta x_n} - \frac{(x-x_n)}{\Delta x_n}\left[1 + \frac{(x-x_n)}{\Delta x_n}\right](\theta_n - \frac{\Delta y_n}{\Delta x_n}) - \frac{(x-x_n)^2}{2\Delta x_n} K_n$$

$$+ (\frac{x_{n+1}-x}{\Delta x_n})\left\{\left[1 + \frac{2(x-x_n)}{\Delta x_n}\right](\theta_n - \frac{\Delta y_n}{\Delta x_n}) + (x-x_n) K_n\right\} \quad \text{(B-2)}$$

$$y'' = \frac{2}{\Delta x_n}\left[1 + \frac{2(x-x_n)}{\Delta x_n}\right](\theta_n - \frac{\Delta y_n}{\Delta x_n}) - \frac{2(x-x_n)}{\Delta x_n} K_n$$

$$+ \frac{(x_{n+1}-x)}{\Delta x_n}\left\{\frac{2}{\Delta x_n}(\theta_n - \frac{\Delta y_n}{\Delta x_n}) + K_n\right\} \quad \text{(B-3)}$$

$$y''' = -\frac{3}{\Delta x_n}\left\{\frac{2}{\Delta x_n}(\theta_n - \frac{\Delta y_n}{\Delta x_n}) + K_n\right\} = \text{const} \quad \text{(B-4)}$$

Substituting $x = x_{n+1}$ into above, since continuity up to the second derivative is maintained

$$\theta_{n+1} = y'_{n+1} = 3(\frac{\Delta y_n}{\Delta x_n}) - 2\,\theta_n - \frac{\Delta x_n}{2} K_n \quad \text{(B-5)}$$

$$K_{n+1} = y''_{n+1} = \frac{6}{\Delta x_n}(\frac{\Delta y_n - \theta_n}{\Delta x_n}) - 2\,K_n \quad \text{(B-6)}$$

A recurrence relationship can be developed by writing

$$K_n = a_n\theta_n + b_n; \quad K_{n+1} = a_{n+1}\theta_{n+1} + b_{n+1} \quad \text{(B-7)}$$

Eliminating (θ_n, K_n) among above relationships, one obtains

$$K_{n+1} = (2 + \frac{\Delta x_n}{2} a_n)^{-1}\left\{\frac{2}{\Delta x_n}(3+\Delta x_n a_n)\theta_{n+1} - \frac{3}{\Delta x_n}(2+\Delta x_n a_n)\frac{\Delta y_n}{\Delta x_n} - b_n\right\} \quad \text{(B-8)}$$

Comparing with Equation (B-7), with n replaced by n+1, one finds

$$a_{n+1} = \frac{2(3+\Delta x_n a_n)}{\Delta x_n(2+\frac{1}{2}\Delta x_n a_n)}$$

$$b_{n+1} = \frac{\left\{b_n + \frac{3}{\Delta x_n}(2+\Delta x_n a_n)\frac{\Delta y_n}{\Delta x_n}\right\}}{(2+\frac{1}{2}\Delta x_n a_n)} \qquad \text{(B-9)}$$

These are the recurrence formulae for (a_n,b_n). Starting with the condition $K_1 = 0$ such that $a_1 = b_1 = 0$, Equations (B-9) can be applied repeatedly to generate the entire set of (a_n,b_n) for n = 2 through N. Now, substituting $K_n = a_n\theta_n+b_n$ into Equation (B-6) and solving for θ_n:

$$\theta_n = \frac{3\frac{y_n}{x_n} - x_n b_n - \frac{x_n}{2}K_{n+1}}{(3 + \Delta x_n a_n)} \qquad \text{(B-10)}$$

For n = N, since $K_{N+1} = 0$, therefore

$$\theta_N = \frac{3\frac{\Delta y_N}{\Delta x_N} - \Delta x_N b_N}{3 + \Delta x_N a_N} \qquad \text{(B-11)}$$

where upon, Equation (B-7) can be used to calculate K_N, and the process can be repeated to obtain (θ_n,K_n) for successively smaller n.

To summarize, the cubic-spline curve-fitting procedure includes the following:

1. Set $(n = 1;\ a_n = b_n = 0)$
2. Use Equations (B-9) to obtain (a_{n+1},b_{n+1}) repeatedly until (a_n,b_n) are determined for the entire set n = 1 through N.
3. Set $(n = N,\ K_{n+1} = 0)$
4. Use Equation (B-10) to obtain θ_n. If current n is 1, the curve-fitting procedure has completed; otherwise continue.

5. Calculate K_n using Equation (B-7).

6. Decrease n by 1; go back to Step 4.

The above procedure can be used for any $n \geq 2$.

Upon completing the curve-fitting calculations, (θ_n, K_n) are known for every interval of n = 1 through N. Within the interval, the interpolation formula and its derivatives are

$$y = y_n + \theta_n(x-x_n) + \frac{K_n}{2}(x-x_n)^2$$

$$- \frac{1}{\Delta x_n}\left[\frac{1}{2} K_n + \frac{1}{\Delta x_n}\left(\theta_n - \frac{\Delta y_n}{\Delta x_n}\right)\right] (x-x_n)^3$$

$$y' = \theta_n + K_n(x-x_n) - \frac{3}{\Delta x_n}\left[\frac{1}{2} K_n + \frac{1}{\Delta x_n}\left(\theta_n - \frac{\Delta y_n}{\Delta x_n}\right)\right] (x-x_n)^2$$

$$y'' = K_n - \frac{3}{\Delta x_n}\left[K_n + \frac{2}{\Delta x_n}\left(\theta_n - \frac{\Delta y_n}{\Delta x_n}\right)\right] (x-x_n) \qquad \text{(B-12)}$$

B.4 Determination of Stationary Values

The stationary values are found by solving for

$$y' = 0$$

or

$$\frac{x_{(\alpha)}-x_n}{\Delta x_n} = \frac{K_n \pm \sqrt{K_n^2 + \frac{6\theta_n}{\Delta x_n}\left[K_n + \frac{2}{\Delta x_n}\left(\theta_n - \frac{\Delta y_n}{\Delta x_n}\right)\right]}}{\left[K_n + \frac{2}{\Delta x_n}\left(\theta_n - \frac{\Delta y_n}{\Delta x_n}\right)\right]} \qquad \text{(B-13)}$$

for $\alpha = 1, 2$

For an extreme value to be within the interval, the following constraint must be satisfied

$$0 \leq \frac{x_{(\alpha)}-x_n}{\Delta x_n} < 1 \qquad \text{(B-14)}$$

The righthand bound is left open because x_{n+1} may be regarded to belong to the next interval (except when $n = N$). α is a positive integer not greater than 2. The following convention is adopted:

$$x_n \leq x_{(1)} \leq x_{(2)} < x_{n+1} \qquad \text{(B-15)}$$

Upon finding an extreme value, $y''_{(\alpha)} = y''\ (x = x_{(\alpha)})$ is calculated. $x_{(\alpha)}$ is

$$\left.\begin{array}{l}\text{a maximum}\\ \text{a minimum}\\ \text{an inflexion}\end{array}\right\}, \text{ if } y''_{(\alpha)} \left.\begin{array}{c}<\\ >\\ =\end{array}\right\} 0 \qquad \text{(B-16)}$$

B.5 Roots of the Cubic Spline

The roots of the cubic spline are sought in each interval. Again x_{n+1} is regarded as an external point (except for $n = N$). In general, up to three roots are possible. The extreme values divide the interval to up to three sub-intervals. Each sub-interval, however, cannot contain more than one root. Thus it is expedient to first identify the sub-intervals and then to determine whether or not a root exists within each sub-interval.

If the spline cubic vanishes at the starting point of the sub-interval, there can be no other root in the same sub-interval, and root-testing can immediately proceed to the next sub-interval. In the following, it will be presumed that the starting point is not a root. The existence of a root within the sub-interval is indicated by a sign reversal of the end points. When this is ascertained, the polynomial is calculated at the midpoint of the sub-interval and its magnitude is checked against $|\Delta y_n|$ for convergence. If necessary, further narrowing down of the sub-interval is done by excluding the half which is bounded by polynomial values of the same sign. The process is continued until a convergence test is passed.

APPENDIX C

LISTING OF SOURCE PROGRAM

```
C     ROTOR SUBSTRUCTURE VIBRATION PROGRAM
C     *RSVP* BASIC VERSION          DECEMBER 31, 1974 CHTP
C     SHAKER RESEARCH CORPORATION
C
C     SUPPLEMENTARY OUTPUT IS WRITTEN INTO I/O DEVICE (10)
C
C     VERSION 76.02          JUNE 7, 1976
C     MODIFIED FOR WPAFB
C
C     VERSION 78.05          MAY 18, 1978
C     FULL CAPABILITY WITH STREAMLINED I/O
C
C     VERSION 79.05          MAY 01, 1979
C     UPDATED FOR UNION COLLEGE COURSE
C
C     VERSION 79.06          MAY 22, 1979
C     PARAMETER 'LIST' REMOVED FROM INPUT
C
C     VERSION 79.07          SEPTEMBER 25, 1979
C     DOUBLE PRECISION REAL ALGEBRA VERSION
C
C     VERSION 79.08          DECEMBER  6, 1979
C     TESTED FOR SAMPLES INCLUDED IN AIR FORCE REPORT
C
C     VERSION 79.09          DECEMBER 13, 1979
C     DIMENSION ALLOWANCE INCREASED TO 75 SEGMENTS
C
C     VERWION 79.10          DECEMBER 20, 1979
C     DOUBLE PRECISION SPECIFICATIONS REMOVED
C
C*********************************************************************
C
C
C FEATURES TO BE INCLUDED IN THE FUTURE:
C     (1) ASSIGN THE LAST STATION WITH DEFAULT K8=8
C     (2) AUTOMATIC RUNNING OF TORSIONAL VIBRATIONS
C         E.G. IRUN=6 FOR TORSIONAL RESONANCE
      INTEGER TITLE(16),IDENT(4)
      COMMON/AWH/F,FF,XS(4,2),F1,FF1,SS
      COMMON/AVIB/AA5(80,8,4)
      COMMON/ATEST1/B1(75),B2(75),B3(75),B4(75),B5(75),B6(75)
      COMMON/ATEST2/U0(80,4)
      COMMON/ATEST3/V0(80,8),A5(80,8,4)
      COMMON/BSTEP/D5(4,2)
      COMMON/CSTAT/X(75),Z(76)
      COMMON/ACOU/S4(4),A6(4,6)
      COMMON/ABRG/S5(4,2),T5(4,2)
      COMMON/AHIN/C4(4),W4(4),P4(4),T4(4),Y4(4)
      COMMON/AEND/U(80,2),ZM(76),ZK(20),FFRT
      COMMON/BNODE/W(76),P(76),T(76),Y(76)
      COMMON/VNODE/PS(76)
      COMMON/VHIN/P4S(4)
      COMMON/AFREQ/FREQ(5,11),PCC,JFQ(5),IFRE
      COMMON/TORL/TORD(5,11)
```

```
      COMMON/IHIN/N1(5),N2(5)
      COMMON/ANODE/N3(76)
      COMMON/ABEND/L1P,L4P1,I8,I5
      COMMON/AINT/N8(76),K8(76),J8
      COMMON/CHEAD1/TITLE,L1,L2,L3,L4,L5,LIST/WR/KW,KR
      COMMON/ASTEP/J4,LB,M5,N5(4)
      COMMON/AINER/TRA(75)
      COMMON/PRMS/IRUN,ITYPE,IBRG,IDIAG
      DIMENSION N4(4),XSS(4,2,11),S00(11)
      DIMENSION X5(4,2),Y5(4,2),W5(4,2),Q5(4,2),ZBRG(4)
      DIMENSION C5(2,5),KTORS(20),KBEND(20),JBRG(4)
 9998 FORMAT (1H )
      KR = 5
      KW = 6
      READ(KR,8000) NCASE,KRUN
      IF(NCASE.EQ 0) GO TO 4321
      READ (KR,1000) IDENT,TITLE
 1000 FORMAT (20A4)
      READ(KR,8000) LSEG,LMAT,LMAS,LBEA,LTYP,LEXI,LPRI
      I5=LTYP
      J8=LEXI
      L1=LSEG
      L2=LMAT
      L3=LMAS
      L4=0
      L5=LBEA
      LIST = 1
      IF (I5.EQ 1) LPRI =-1
 8000 FORMAT (16I5)
      CALL HEAD1
      GO TO 8
C
C INPUT ERROR MESSAGES
C
 2005 WRITE (KW,1005)
 1005 FORMAT (43H FIRST SEGMENT MUST BE ASSIGNED A MATERIAL /)
      GOTO 9999
 1854 WRITE (KW,1007) N
 1007 FORMAT (21H THERE IS NO NODE NO.,I3/)
      GOTO 9999
 12   WRITE (KW,1012)
 1012 FORMAT (31H THERE MUST BE MORE BEARING(S) /)
      GOTO 9999
 14   WRITE (KW,1014)
 1014 FORMAT (35H COUPLING MUST BE AN INTERNAL NODE /)
      GOTO 9999
 8    Z(1) = 0
      WRITE (KW,8006)
 8006 FORMAT (//17H SHAFT DIMENSIONS//8H SEGMENT,T12,6HLENGTH,T24,
     +4HI.D.,T35,4HO.D.,T45,6HM.I.D.,T56,6HM.O.D./6H   NO.
     +5(11H      (IN) )/1H )
C
C     FOR L=1,L1
C
```

```
      DO 9 L=1,L1
      READ(KR,8001) LO,NULL,X(L),D1,D2,D3,D4
8001  FORMAT (2I5,7F10.0)
      Z(L+1) = Z(L)+X(L)
C     CALCULATE SEGMENT FUNCTIONS (B1(L),B2(L),B3(L),B4(L),B5(L))
      WRITE (KW,8002) L,X(L),D1,D2,D3,D4
8002  FORMAT (I5,T10,5(F9.4,2X),F9.4)
      P1 = D2*D2
      P2 = D1*D1
      P3 = P1+P2
      P7 = P1-P2
      P5 = D4*D4
      P6 = D3*D3
      BB=P5+P6
      P5 = P5-P6
      P6=BB*P5
      B1(L) = .7853981633975*P7
      B2(L) = B1(L)*P3/16
      B1(L) = .7853981633975*P5
      A1 = P3*P7
      TRA(L)=B1(L)/16.0*BB*X(L)
      B3(L) = P5/A1
      B4(L)=P6/A1
      B5(L)=0.0
9     B6(L)=BB*B4(L)
      L1P = L1+1
      WRITE (KW,8007)
8007  FORMAT (16H1SHAFT MATERIALS//9H STARTING,T12,7HDENSITY,T21,
     +21HYOUNG'S MOD SHEAR MOD/7H    NODE,T10,11H(LBS/CU-IN),
     +T24,5H(PSI),T35,5H(PSI)/1H )
      DO 21 L=1,2
      READ (KR,8111) NMAT,N,DO,E,G
8111  FORMAT(2I5,3E10.4)
      WRITE (KW,8020) N,DO,E,G
8020  FORMAT (I5,5X,F7.4,3X,1P2E11.4)
      IF (L.GT.1) GO TO 21
      B9 = DO/386.4
      C = B9/G
      BB9=B9
      B9 = B9/E
      D = DO
      E1 = E
      G1=G
      IF (N.NE.1) GO TO 2005
      IF (L2.GE.2) GO TO 21
      GOTO 22
21    CONTINUE
22    L = 2
      DO 25 K=1,L1
      IF (K.EQ.1.OR.K.NE.N) GO TO 24
      B9 = DO/386.4
      C = B9/G
      BB9=B9
      B9 = B9/E
```

```
      D = DO
      E1 = E
      G1=G
      IF (L.EQ.L2) GO TO 24
      L = L+1
      READ (KR,8111) NMAT,N,DO,E,G
      WRITE (KW,8020) N,DO,E,G
 24   B1(K) = D*B1(K)
      B5(K)=2*G1*B2(K)
      B2(K) = E1*B2(K)
      TRA(K)=BB9*TRA(K)
      B6(K)=0.0625*B9*B6(K)
      B3(K) = 2.0*(B9*B3(K))**0.25
 25   B4(K) = SQRT(C*B4(K))
      IF (L3.GE.1) GO TO 30
      M3 = 0
      GOTO 31
 30   WRITE (KW,8008)
 8008 FORMAT (//16H LUMPED INERTIAS//7H  NODAL,T11,6HWEIGHT,T23,
     +5HPOLAR,T34,5HTRANS,T46,2HCG/8H STATION,
     +T22,7HINERTIA,T33,7HINERTIA,T44,7HOFF-SET,
     +/T12,4H(LB),T21,10H(LB-SQ IN),T32,10H(LB-SQ IN),T45,4H(IN))
      DO 26 L=1,L3
      READ(KR,8001) LO,N3(L),W(L),P(L),T(L),Y(L)
      N = N3(L)
      WRITE (KW,8002) N3(L),W(L),P(L),T(L),Y(L)
      IF (N.GT.L1+1) GO TO 1854
 26   CONTINUE
      M3 = "3(1)
 31   J3 = .
      WRITE (KW, 9998)
      IF (L4.EQ.0) GOTO 29
      DO 28 L=1,L4
      READ(KR,8001) LO,N4(L),S4(L),C4(L),W4(L),P4(L),T4(L),Y4(L)
      WRITE (KW,8002) N4(L),W4(L),P4(L),T4(L),Y4(L),C4(L),S4(L)
      N = N4(L)
      IF (N.EQ.1.OR.N.GT.L1) GOTO 14
 28   CONTINUE
 29   L4P1=L4+1
C
C     INTEGRATE STATIC WEIGHT EFFECTS
C
      DO 40 J=1,L4P1
      CALL STACK (J,L1,L4,N4,J4,N1,N2)
      M1 = N1(J)
      DO 36 I=1,4
 36   UO(M1,I)=0
      M2 = N2(J)+1
      DO 39 K=M1,M2
      K3 = K-J4
      IF (K.NE.M1.OR K EQ.1) GOTO 37
      UO(K,1) = UO(K,1)-W4(J4)
      UO(K,2) = UO(K,2)+W4(J4)*Y4(J4)
      P4(J4) = P4(J4)/386.4
```

```
      T4(J4) = T4(J4)/386.4
37    IF (M3.NE.K3) GO TO 39
      U0(K,1) = U0(K,1)-W(J3)
      U0(K,2) = U0(K,2)+W(J3)*Y(J3)
      P(J3) = P(J3)/386.4
      T(J3) = T(J3)/386.4
      J3 = J3+1
      IF (L3 GE J3) M3 = N3(J3)
39    IF(K.LT.M2) CALL STAT(J4,K)
40    CONTINUE
      IF (L5.EQ.0) GOTO 100
      IF (L5.LT.2) GO TO 12
C
C     CALCULATE STATIC BEARING LOADS
C
      DO 43 L=1,L5
      READ (KR,8001) L0,N5(L),D5(L,1),D5(L,2),S5(L,1),S5(L,2)
     +,T5(L,1),T5(L,2)
43    CONTINUE
      DO 75 K=1,2
      DO 75 K1=1,4
      KK3 = KFCN(4,K,K1)
      DO 75 K2=1,2
      KK4 = KFCN(2,K,K2)
      K3 = K1-2
      IF (K2 EQ K3) GO TO 72
      P9 = 0
      GO TO 75
72    P9 = 1
75    A5(1,KK3,KK4) = P9
      M3 = 0
      IF (L3 EQ 0) GO TO 61
      M3 = N3(1)
      J3 = 1
61    L8 = 1
      M5 = N5(L8)
      DO 90 J=1,L4P1
      CALL JFCN (J,M1)
      DO 51 K1=1,8
51    V0(M1,K1) = 0
      IF (J.EQ.1) GO TO 70
      V0(M1,1) = -W4(J4)
      V0(M1,2) = W4(J4)*Y4(J4)
      ISTEP = 2
      CALL STEP (M2,ISTEP,IER)
      IF (IER.EQ.1) GO TO 9999
70    CALL MFCN(J,M21,M2)
      WRITE (KW, 9998)
      DO 90 K0=M1,M2
      K4 = K0-J4
      IF (M3.NE.K4) GO TO 76
      V0(K0,1) = V0(K0,1)-W(J3)
      V0(K0,2) = V0(K0,2)+W(J3)*Y(J3)
      J3 = J3+1
```

```
      IF (L3.GE.J3) M3 = N3(J3)
76    IF (M5.NE.K4) GO TO 82
      WRITE (KW, 9998)
      DO 32 K=1,2
      IF (S5(LB,K).GT.0) GO TO 32
      WRITE(KW,5000)
5000  FORMAT (T10,40HBRG STIFFNESS IS DEFAULTED TO 0.1 LB/IN )
      S5(LB,K) = 0.1
32    CONTINUE
      ISTEP = 3
      CALL STEP (KO,ISTEP,IER)
82    IF (KO.GT.M21) GO TO 90
      ISTEP = 1
      CALL STEP (KO,ISTEP,IER)
90    CONTINUE
      DO 99 K=1,2
      K11 = KFCN(4,K,1)
      K12 = KFCN(4,K,2)
      K21 = KFCN(2,K,1)
      K22 = KFCN(2,K,2)
      K13 = KFCN(4,K,3)
      K14 = KFCN(4,K,4)
      P9 = A5(M2,K11,K21)*A5(M2,K12,K22)
      P9 = P9-A5(M2,K11,K22)*A5(M2,K12,K21)
      C5(2,1) = A5(M2,K12,K21)*VO(M2,K11)-A5(M2,K11,K21)*VO(M2,K12)
      C5(2,1) = C5(2,1)/P9
      P9 = (A5(M2,K11,K22)*VO(M2,K12)-A5(M2,K12,K22)*VO(M2,K11))/P9
      DO 95 L=1,L4P1
      L6 = L4+2-L
      IF (L.GT.1) GO TO 94
93    C5(1,L6) = P9
      GOTO 95
94    J1 = L6-1
      K32 = KFCN(3,K,2)
      K33 = KFCN(3,K,3)
      P9 = A6(J1,K32)*C5(2,1)+A6(J1,K33)*P9
      GO TO 93
95    CONTINUE
      L = 0
      DO 99 J=1,L4P1
      CALL JFCN (J,M1)
      M2 = N2(J)+1
      IF (M1.GT.1) C5(2,J) = VO(M1,K14)
      DO 99 KO=M1,M2
      K4 = KO-J4
      IF (KO.EQ.1) GO TO 71
97    IF (M5.NE.K4) GO TO 99
      X5(L,K) = D5(L,K)-VO(KO,K14)
      Y5(L,K) = VO(KO,K13)
      DO 98 K2=1,2
      KK4 = KFCN(2,K,K2)
      X5(L,K) = X5(L,K)-A5(KO,K14,KK4)*C5(K2,J)
98    Y5(L,K) = Y5(L,K)+A5(KO,K13,KK4)*C5(K2,J)
      W5(L,K) = S5(L,K)*X5(L,K)
```

```
      Q5(L,K) = T5(L,K)*Y5(L,K)
71    IF (L.EQ.L5) GO TO 99
      L = L+1
      M5 = N5(L)
      GOTO 97
99    CONTINUE
      WRITE (KW,5001)
5001  FORMAT(/93H   BRG  ST  LOCATION  **********VERTICAL PLANE*********
     +* **********HORIZONTAL PLANE**********/1X,2(5H  NO.),3X,4H(IN),3X,
     +2(35H MISALGN(IN)   LOAD(LB)  MOM(IN-LB)))
      DO 41 L=1,L5
      N = N5(L)
41    WRITE(KW,9002) L,N,Z(N), (D5(L,K),W5(L,K),Q5(L,K),K=1,2)
9002  FORMAT(2I5,F10.4,2(F11.6,2F11.4,2X))
      P9 = 0
      Q = 0
      DO 42 L=1,L4P1
      M2 = N2(L)+1
      P9 = P9-U0(M2,1)
42    Q = Q-U0(M2,2)
      Z0 = Z(L1+1)-Q/P9
      WRITE (KW,5002) P9,Z0
5002  FORMAT(/T15,12HTOTAL WEIGHT,T33,1H=,1PE11.4,5H (LB)/
     +T15,19HLOCATION OF C.G.  =,E11.4,5H (IN))
      WRITE (KW,5003) NCASE
5003  FORMAT(28H1NUMBER OF SPEED GROUPS    =,I5)
C
C     SETUP OUTPUT LOGIC
C
100   NTORS=0
      NBEND=0
      S = 0
      IF (J8.EQ.0) GO TO 120
      READ (KR,8000) (N8(I),I=1,J8)
      READ (KR,8000) (K8(I),I=1,J8)
      DO 112 I=1,J8
      K8I = K8(I)
      GO TO (112,113,115,115,113,113,114,113), K8I
113   NTORS = NTORS+1
      KTORS(NTORS) = N8(I)
      GO TO (112,112,112,112,115,115,112,114), K8I
114   NBEND = NBEND+1
      KBEND(NBEND) = N8(I)
115   NBEND = NBEND+1
      KBEND(NBEND) = N8(I)
112   CONTINUE
120   CONTINUE
      IF (I5.EQ.2) NTORS = 0
      WRITE(10) NCASE,NTORS,NBEND
      IF (I5-2) 101,102,103
101   WRITE (KW,5007)
5007  FORMAT (39H DYNAMIC RESPONSE OF THE TORSIONAL MODE)
      GO TO 104
102   WRITE (KW,5008)
```

```
5008  FORMAT(37H DYNAMIC RESPONSE OF THE BENDING MODE)
      GO TO 104
103   WRITE(KW,5009)
5009  FORMAT(54H DYNAMIC RESPONSES OF BOTH TORSIONAL AND BENDING MODES)
104   CONTINUE
      DO 126 ICASE=1,NCASE
      WRITE (KW,6030) ICASE
6030  FORMAT (////19H SPEED GROUP NUMBER,I5/)
      READ (KR,8000) LRUN
      PC = .1047197551197
      PCC = 60*PC
      IF (LTYP.EQ.1) LRUN = 0
      READ(KR,8001) I7,IFRE,SPE1,SPE2
      IF(NBEND.GT.0) GO TO 1203
      I7=1
      SPE1=0.0
1203  CONTINUE
      WRITE(10) IDENT,TITLE,I7,L5,L1P,KRUN,LRUN
      IF(I7.GT.1) GO TO 1201
      SRAT=1.0
      GO TO 1202
1201  SRAT=(SPE2/SPE1)**(1./(I7-1))
1202  SO=SPE1/SRAT
      I6=1
      DO 125 JSPD=1,I7
      SO=SO*SRAT
      SOO(JSPD)=SO
      CONTINUE
      IF (NTORS.GT.0) WRITE (10) (KTORS(I),I=1,NTORS)
      IF (NBEND.GT.0) WRITE (10) (KBEND(I),I=1,NBEND)
      IF (I5.EQ.1) GO TO 111
      IF (L5.EQ. 0) GO TO 106
      DO 105 L=1,4
      IF (ICASE.GT.1.OR.JSPD.GT.1) GO TO 1050
      JBRG(L) = L
      IF (L.GT.L5) GO TO 105
      N = N5(L)
      ZBRG(L) = Z(N)
1050  IF (I6.EQ.0.OR.L.GT.L5) GO TO 105
      READ (KR,8001) LO,I6,XS(L,1),XS(L,2)
      IF(I6.EQ.1 .OR. L.EQ.L5) GO TO 105
      WRITE(KW,8004)
8004  FORMAT(///46H I6 MUST BE UNITY EXCEPT FOR THE LAST BEARING.)
      GO TO 9999
105   CONTINUE
      WRITE(10) (N5(L),(XS(L,I),I=1,2),L=1,L5)
      IF (L5.EQ.0) GO TO 106
      DO 106 L=1,L5
      XSS(L,1,JSPD)=XS(L,1)
      XSS(L,2,JSPD)=XS(L,2)
106   CONTINUE
      SSO = PC*SO
      S = SO
      SS = SSO
```

```
      IF (L3.EQ.0) GO TO 109
      DO 108 L=1,L3
108   PS(L) = P(L)*SS
109   IF(L4.EQ.0) GO TO 111
      DO 110 L=1,L4
110   P4S(L)=P4(L)*SS
111   CONTINUE
      IF(LRUN.EQ.0) GO TO 1205
      IFRE=1
      JFQ(1)=1
      FREQ(1,1)=S0/60.0
      GO TO 1208
1205  DO 1103 L=1,IFRE
      READ (KR,8001) L0,I8,FREQ1,FREQ2
      JFQ(L)=I8
      IF(I8.EQ.1) GO TO 1206
      FRAT=(FREQ2/FREQ1)**(1./(I8-1))
      GO TO 1207
1206  FRAT=1.0
1207  FRQY=FREQ1/FRAT
      DO 1101 I=1,I8
      FRQY=FRQY*FRAT
1101  FREQ(L,I)=FRQY
1103  CONTINUE
1208  WRITE(10) IFRE
      DO 1209 L=1,IFRE
      JFK=JFQ(L)
      WRITE(10) JFK
      IF (LRUN.EQ.0) WRITE(KW,5011) L,(FREQ(L,I),I=1,JFK)
5011  FORMAT(/27H EXCITATION FREQUENCY GROUP,I5/5H HZ   ,11F10.4)
      IF(LIST.EQ.2)  GO TO 1209
      IF (LTYP.EQ.1) GO TO 1104
      IF(JSPD.EQ.1.OR.(L.EQ.1.AND.IFRE.GT.1))WRITE(KW,5012)
5012  FORMAT (/23H    SPEED      FREQUENCY,8X,16HEND DETERMINANTS/4X,
     +5H(RPM),7X,4H(HZ),6X,28HCO-ROTATIONAL CTR-ROTATIONAL)
1104  CALL ENDST(S,NTORS,NBEND,JFK,L)
      IF (LRUN.EQ.1) GO TO 1209
      IF (LTYP-2) 1111,1209,1113
1111  WRITE (KW,5021)
5021  FORMAT (1H1)
      GO TO 1115
1113  WRITE (KW,5023)
5023  FORMAT (/1H )
1115  WRITE (KW,5013)
5013  FORMAT (11H  FREQUENCY,5X,9HTORSIONAL/4X,4H(HZ) 8X,9HEND STIFF)
      DO 117 I=1,JFK
      WRITE (KW,5014) FREQ(L,I),TORD(L,I)
117   CONTINUE
5014  FORMAT (1PD12.4,D15.6)
1209  CONTINUE
125   CONTINUE
      IF (LPRI.EQ.-1) GO TO 1106
      WRITE (KW,5004) (JBRG(L),L=1,4),(N5(L),L=1,L5)
5004  FORMAT (//7H BRG NO,I18,3I24/7H STN NO,I18,3I24)
```

```
      WRITE (KW,5005) (ZBRG(L),L=1,L5)
5005  FORMAT (9H LOC (IN),1PE15.4,3(12X,E12.4))
      WRITE (KW,5006)
5006  FORMAT (9H    SPEED,3X,4(4X,3HRAD,9X,3HANG,5X)/
     +4X,8H(RPM)    ,4(3X,21H(LB/IN)    (IN-LB/RAD)))
      DO 1051 II=1,I7
1051  WRITE (KW,5010) SOO(II),((XSS(L,I,II),I=1,2),L=1,L5)
5010  FORMAT (1P11E12.4)
1106  CONTINUE
126   CONTINUE
      IF(KRUN.EQ.0.OR.LIST.EQ.2) GO TO 9999
4321  REWIND 10
      CALL SYDSYN
9999  CALL EXIT
      END
```

```
      SUBROUTINE ENDST(S,NTORS,NBEND,JFK,I9)
      INTEGER TITLE(16)
      COMMON/AWH/F,FF,XS(4,2),F1,FF1,SS
      COMMON/AVIB/AA5(80,8,4)
      COMMON/ATEST1/B1(75),B2(75),B3(75),B4(75),B5(75),B6(75)
      COMMON/ATEST2/UO(80,4)
      COMMON/BSTEP/D5(4,2)
      COMMON/CSTAT/X(75),Z(76)
      COMMON/ACOU/S4(4),A6(4,6)
      COMMON/AHIN/C4(4),W4(4),P4(4),T4(4),Y4(4)
      COMMON/AEND/U(80,2),ZM(76),ZK(20),FFRT
      COMMON/BNODE/W(76),P(76),T(76),Y(76)
      COMMON/VNODE/PS(76)
      COMMON/VHIN/P4S(4)
      COMMON/AFREQ/FREQ(5,11),PCC,JFQ(5),IFRE
      COMMON/TORL/TORD(5,11)
      COMMON/MO/AMAX(4),QC5(80,8),QU(80,8,2)
      COMMON/IHIN/N1(5),N2(5)
      COMMON/ANODE/N3(76)
      COMMON/ABEND/L1P,L4P1,I8,I5
      COMMON/AINT/N8(76),K8(76),J8
      COMMON/CHEAD1/TITLE,L1,L2,L3,L4,L5,LIST/WR/KW,KR
      COMMON/ASTEP/J4,LB,M5,N5(4)
      COMMON/AINER/TRA(75)
      DIMENSION AM(4,4),BM(4,4),AMI(4,4),UU(80,8,2),XD5(80,8),XV(4,2)
      DIMENSION DET(2),TRATE(76),K9(20)
      DIMENSION B(10),AI(4,2,2),E(4),P4F(4),PF(80),VV(80,2)
      EQUIVALENCE (UU(1),VV(1))
      IONE = 1
      ITWO = 2
      IFOUR = 4
      IF (NBEND.EQ.0) GO TO 245
      JMAX = 4
      CSPD=S/60
      SS=CSPD*PCC
      NEND = 2*NBEND
245   CONTINUE
      DO 1000 I=1,JFK
      F=FREQ(I9,I)
      FF = F*PCC
      F2 = FF*FF
C
C     CALCULATE TORSIONAL RESPONSE
C
      M8 = 0
      LL8 = 0
      L8 = 1
      IF (NTORS.GT.0) WRITE(10) F
      IF (J8.EQ.0) GO TO 31
      ITORS = 0
      M8 = N8(1)
      KK8 = K8(1)
31    L81 = L8
      IF (I5.EQ.2) GO TO 400
```

```
      IF( ITORS.GT.0 ) GO TO 300
C *** INITIALIZE HOMOGENEOUS SOLUTION
      U(1,1) = 0
      U(1,2) = 1
300   M3 = 0
      IF (NTORS.EQ.0) GO TO 320
      GO TO (310,320,310,310,320,320,310,320), KK8
310   IF (J8 LE.L8) GO TO 99
      L8 = L8 +1
      KK8=K8(L8)
      GO TO (310,315,310,310,315,315,310,315), KK8
315   L81 = L8
C *** IDENTIFY TORSIONAL EXCITATION STATIONS
      M8=N8(L8)
320   IF (L3 EQ.0) GO TO 1
      M3 = N3(1)
      J3 = 1
C *** INTEGRATION OF THE TORSIONAL PROBLEM
1     DO 4 L=1,L4P1
      CALL JFCN (L,M1)
      IF (L.EQ.1) GO TO 2
      IF( ITORS.GT.0 ) GO TO 32
      P4F(J4) = F2*P4(J4)
      U(M1,1) = U(M2,1)-P4F(J4)*U(M2,2)
      U(M1,2) = U(M2,2)+C4(J4)*U(M1,1)
32    IF (LL8.EQ.0) GO TO 2
      VV(M1,1) = VV(M2,1)-P4F(J4)*VV(M2,2)
      VV(M1,2) = VV(M2,2)+C4(J4)*VV(M1,1)
2     CALL MFCN(L,M21,M2)
      DO 4 K=M1,M2
      K4 = K-J4
      IF (J8.EQ.0) GO TO 33
      IF (K4.NE.M8) GO TO 33
      GO TO (33,87,33,33,87,87,33,87), KK8
87    L82 = L81
      L81 = L81+1
      IF (L81.LE.J8) M8 = N8(L81)
      IF (L82.NE.L8) GO TO 33
      MM9 = K4
      KK8 = 1
      MM8 = K
      LL8 = 1
C *** INITIALIZE PARTICULAR SOLUTION
      VV(K,1) = -1
      VV(K,2) = 0
33    IF (M3.NE.K4) GO TO 3
      IF( ITORS.GT.0 ) GO TO 34
      PF(J3) = F2*P(J3)
      U(K,1) = U(K,1)-PF(J3)*U(K,2)
34    IF (LL8.EQ.0) GO TO 35
      VV(K,1) = VV(K,1)-PF(J3)*VV(K,2)
35    J3 = J3+1
      IF (L3.GE.J3) M3 = N3(J3)
3     IF (K.GT.M21) GO TO 4
```

```
      KP = K+1
      IF( ITORS.GT.0 ) GO TO 36
      B(1) = U(K,1)
      B(2) = U(K,2)
      CALL TORS (K,B)
      U(KP,1) = B(1)
      U(KP,2) = B(2)
36    IF (LL8.EQ.0) GO TO 4
      B(1) = VV(K,1)
      B(2) = VV(K,2)
      CALL TORS (K,B)
      VV(KP,1) = B(1)
      VV(KP,2) = B(2)
4     CONTINUE
      IF( ITORS.GT.0 ) GO TO 46
C *** END EXCITATION RESPONSE
      ZE = U(M2,1)
      TORD(I9,I) = ZE
      YE=ZE
      IF (U(M2,2).NE.0) GO TO 7
      IF (I5.EQ.1)
     +WRITE (KW,5002) F
5002  FORMAT (26H1END NODE IS STATIONARY AT,1PE12.4,3H HZ/9H TORS STI,
     +52HFF BELOW IS END TORQUE PER UNIT TWIST OF FIRST NODE./1H )
      GOTO 8
7     ZE = ZE/U(M2,2)
      IF (I5.EQ.1)
     +WRITE (KW,5006)
5006  FORMAT (1H1)
8     DO 5 L=1,L4P1
      CALL JFCN (L,M1)
      M2 = N2(L)+1
      DO 5 K=M1,M2
      IF (K.EQ M1.AND.L.NE.1) GO TO 5
      K4 = K-J4
      ZM(K4) = U(K,2)
      IF (K4.LT.L1P) BBB = B5(K4)
      TRATE(K4)=U(K,1)/YE/BBB
5     CONTINUE
      CALL MODE
      IF(NTORS.GT.0) WRITE(10) (ZM(K4),K4=1,L1P)
      IF (I5.NE.1) GO TO 9
      WRITE (KW,5007) F,L1P,Z(L1P),ZE
5007  FORMAT (13H FREQUENCY   =,1PE12.4,3H HZ/13H NODE NO     =,I3/
     +13H LOCATION    =,1PE12.4,3H IN/13H TORS STIFF =,1PE12 4,
     +10H IN-LB/RAD/1H )
      WRITE (KW,5013)
5013  FORMAT (T15,8HLOCATION,T31,8HRELATIVE,T49,4HUNIT/T3,4HNODE,T17,
     +4H(IN),T32,5HTWIST,T46,10HTWIST RATE/1H )
      DO 6 K=1,L1P
6     WRITE (KW,5014) K,Z(K),ZM(K),TRATE(K)
5014  FORMAT (I5,2(1X,F15 4),7X,1PE11.4)
      KPAGE = 0
9     IF(NTORS EQ.0) GO TO 99
```

```
C *** ASSIGNED EXCITATION RESPONSE
46      IF (U(M2,1).EQ.0) GO TO 39
        IF (VV(M2,1).EQ.0) GO TO 38
        VU = VV(M2,1)
        IF (U(MM8,2).EQ.0) GO TO 37
        AA = U(MM8,2)
        VU = VU*AA
        GOTO 40
37      IF (I5.EQ.1) WRITE(KW,5003) M8,F,M8
5003    FORMAT (28H1TORS EXCITATION AT NODE NO.,I3,18H CAUSES FORE-SEGMT,
       +13H RESONANCE AT,1PD12.4,3H HZ/20H TORS STIFF BELOW IS,
       +19H TORQUE AT NODE NO.,I3,24H PER UNIT TWIST OF FIRST,
       +6H NODE./1H )
        AA = 1
        GOTO 41
38      IF(I5.EQ.1) WRITE(KW,5004) M8,F,M8
5004    FORMAT (28H1TORS EXCITATION AT NODE NO.,I3,17H CAUSES AFT-SEGMT,
       +13H RESONANCE AT,1PD12.4,3H HZ/20H TORS STIFF BELOW IS,
       +19H TORQUE AT NODE NO.,I3,23H PER UNIT TWIST OF LAST,
       +6H NODE./1H )
        KU = 2
        ZK(L8) = 1/VV(M2,2)
        GOTO 42
39      IF (I5.EQ.1) WRITE (KW,5005) F
5005    FORMAT (27H1TORS RESONANCE IS FOUND AT,1PD12.4,3H HZ/1H )
        KU = 3
        GO TO 90
40      IF (I5.NE.1.OR.KPAGE.NE.0) GO TO 41
        KPAGE = 1
        WRITE (KW,5006)
41      IF (KPAGE.NE.0.AND.I5.EQ.1) WRITE (KW,5008)
5008    FORMAT (//1H )
        ZK(L8) = -U(M2,1)/VU
        KU = 1
42      AMP = -VV(M2,1)/U(M2,1)
        IF(I5.NE.1) GO TO 90
        WRITE (KW,5007) F,MM9,Z(MM9),ZK(L8)
90      JTORS = 0
        IF(I5.EQ.1) WRITE(KW,5013)
        NN8 = N8(1)
        KK8 = K8(1)
        JJ8 = 1
        DO 43 L=1,L4P1
        CALL JFCN (L,M1)
        M2 = N2(L)+1
        DO 43 K=M1,M2
        K4 = K-J4
        IF (L.GT.1.AND.K.EQ.M1) GO TO 43
        IF (K4.NE.NN8) GO TO 43
        IF (KU.NE.3) GO TO 89
        BB=ZM(K4)
        GO TO 890
89      BB = 0
        TRATE(K4) = 0
```

```
      IF (KU.NE.1) GO TO 891
      BB = BB+U(K,2)/AA
      IF (L.LT.L4P1.OR.K.LT.M2) TRATE(K4) = TRATE(K4)+U(K,1)*AMP
891   IF (MM8.GE.K) GO TO 892
      BB = BB+ZK(L8)*VV(K,2)
      IF (L.LT.L4P1.OR.K.LT.M2) TRATE(K4) = TRATE(K4)+VV(K,1)
892   TRATE(K4) = TRATE(K4)/B5(K4)
890   IF(I5.EQ.1) WRITE(KW,5014) K4,Z(K4),BB,TRATE(K4)
      GO TO (94,93,94,94,93,93,94,93), KK8
93    JTORS = JTORS+1
      IF (KU.NE.3) GO TO 91
      GG = ZM(K4)
      GO TO 92
91    GG = AMP*U(K,2)
      IF (MM8.LT.K) GG = GG+VV(K,2)
92    VV(JTORS,2) = GG
94    IF (JJ8.EQ.J8) GO TO 44
      JJ8 = JJ8+1
      NN8 = N8(JJ8)
      KK8 = K8(JJ8)
43    CONTINUE
44    ITORS = ITORS+1
      IF(NTORS.GT.0) WRITE(10) KU,(VV(L,ITWO),L=1,NTORS)
      IF (ITORS.EQ.NTORS) GO TO 99
C *** NEXT EXCITATION STATION
      L8 = L8+1
      KK8 = K8(L8)
      L81 = L8
      M8 = N8(L8)
      LL8 = 0
      GO TO 300
C
C     CALCULATE BENDING RESPONSE
C
99    IF(I5.EQ.1) GO TO 1000
      L8=1
      LL8 = 0
      IF (I5.EQ.1) GO TO 216
      L81 = L8
400   L83 = 1
      L84 = 1
      IF(J8.EQ.0) GO TO 10
      M8 = N8(L8)
      KK8 = K8(L8)
      IF (NBEND.GT.0) WRITE(10) F,CSPD
      IBEND = 0
10    IF (L8.GT.1) GO TO 45
      FFRT = SQRT(FF)
      CALL BEND
C *** INITIALIZE HOMOGENEOUS SOLUTION
      DO 12 K=1,2
      KB = 4*K
      KB1 = 9-KB
      KB4 = KB1+3
```

```
        DO 18 KBO=KB1,KB4
        DO 18 K1=1,2
        LBO = KFCN(ITWO,K,K1)
18      AA5(1,KBO,LBO) = 0
        DO 12 K1=1,4
        KK3 = KFCN(IFOUR,K,K1)
        DO 12 K2=1,2
        KK4 = KFCN(ITWO,K,K2)
        K3 = K1-2
        IF (K2.EQ.K3) GO TO 11
        AA = 0
        GO TO 12
11      AA = 1
12      AA5(1,KK3,KK4) = AA
        DO 95 K1=1,2
        UU(1,1,K1)=0.0
95      UU(1,2,K1)=1.0
C ***   START OF ASSIGNED EXCITATION RESPONSE
45      JBEND = 0
C ***   START OF SENSE-OF-ROTATION CYCLE
451     M3 = 0
        L85 = L8*L83*L84
        IF (L3.EQ.0) GO TO 13
        M3 = N3(1)
        J3 = 1
13      M5 = 0
        LB = 1
        IF (L5.GT.0) M5 = N5(1)
C ***   INTEGRATION OF THE BENDING PROBLEM
        DO 20 J=1,L4P1
        CALL JFCN (J,M1)
        IF (J.EQ.1) GO TO 15
        IF (L8.GT.1.AND.LL8.EQ.0) GO TO 47
        CALL LUMP1 (W4(J4),Y4(J4),T4(J4),P4S(J4),FF,F2,AA,BB,CC,DD)
47      IF (L85.GT.1) GO TO 48
        DO 16 K1=1,4
        DO 14 K=1,8
14      B(K) = AA5(M2,K,K1)
        CALL LUMP (AA,BB,CC,DD,B)
        DO 16 K=1,8
16      AA5(M1,K,K1) = B(K)
        CALL LIMP (M1,AM,BM)
        CALL COUP (M1,BM)
        DO 88 K1=1,2
88      AI(J4,K,K1) = AM(K,K1)
48      IF (LL8.EQ.0) GO TO 15
        DO 49 K=1,8
49      B(K) = QU(M2,K,L83)
        CALL LUMP (AA,BB,CC,DD,B)
        CALL COUP1 (M1,J4,AI,B)
        DO 50 K=1,8
50      QU(M1,K,L83) = B(K)
15      CALL MFCN(J,M21,M2)
        DO 20 KO=M1,M2
```

```
      K4 = K0-J4
      IF (J8.EQ.0) GO TO 56
      IF (K4.NE.M8.OR.KK8.LE.2) GO TO 56
C *** IDENTIFY BENDING EXCITATION STATIONS
      L82 = L81
      L81 = L81+1
      IF (L81.LE.J8) M8 = N8(L81)
      IF (L82.NE.L8) GO TO 56
      K08 = K0
      K09 = K4
      LL8 = 1
      GO TO (356,356,52,53,52,53,51,51),KK8
51    IF (L84.EQ.2) GO TO 53
52    M88 = 1
      GOTO 54
53    M88 = 2
54    KK8 = 1
C *** INITIALIZE PARTICULAR SOLUTION
      DO 55 K=1,2
      DO 55 K1=1,4
      K2 = KFCN(IFOUR,K,K1)
      K3 = KFCN(ITWO,K,K1)
      IF (K1.NE.M88.OR.L83.NE.K) GO TO 855
      AA = 1
      GO TO 955
855   AA = 0
955   IF (K1.LE.2) XV(K3,L83) = AA
      IF (K1.EQ.2.AND.M88.EQ.2) AA = -AA
55    QU(K0,K2,L83) = AA
356   IF (L8.GT.1.AND.LL8.EQ.0) GO TO 216
56    IF (M3.NE.K4) GO TO 17
      IF (L8.GT.1.AND.LL8.EQ.0) GO TO 57
      CALL LUMP1 (W(J3),Y(J3),T(J3),PS(J3),FF,F2,AA,BB,CC,DD)
57    IF (L85.GT.1) GO TO 601
      DO 59 K1=1,4
      DO 58 K=1,8
58    B(K) = AA5(K0,K,K1)
      CALL LUMP (AA,BB,CC,DD,B)
      DO 59 K=1,8
59    AA5(K0,K,K1) = B(K)
601   IF (LL8.EQ.0) GO TO 62
      DO 60 K=1,8
60    B(K) = QU(K0,K,L83)
      CALL LUMP (AA,BB,CC,DD,B)
      DO 61 K=1,8
61    QU(K0,K,L83) = B(K)
62    J3 = J3+1
      IF (L3.GE.J3) M3 = N3(J3)
17    IF (M5.NE.K4) GO TO 19
      IF(L85 GT.1) GO TO 666
      DO 65 K1=1,4
      DO 64 K=1,8
64    B(K) = AA5(K0,K,K1)
      B(K)=AA5(K0,K,K1)
```

```
      B(1)=B(1)-XS(LB,1)*B(4)
      B(2)=B(2)+XS(LB,2)*B(3)
      B(5)=B(5)-XS(LB,1)*B(8)
      B(6)=B(6)+XS(LB,2)*B(7)
      DO 65 K=1,8
65    AA5(KO,K,K1) = B(K)
666   IF (LL8.EQ.0) GO TO 68
      DO 66 K=1,8
66    B(K) = QU(KO,K,L83)
      B(1)=B(1)-XS(LB,1)*B(4)
      B(2)=B(2)+XS(LB,2)*B(3)
      B(5)=B(5)-XS(LB,1)*B(8)
      B(6)=B(6)+XS(LB,2)*B(7)
      DO 67 K=1,8
67    QU(KO,K,L83) = B(K)
68    IF (LB.EQ.L5) GO TO 19
      LB = LB+1
      M5 = N5(LB)
19    IF (KO GT.M21) GO TO 20
      IF (LB.GT.1 .AND. LL8.EQ.0) GO TO 957
      TRAH=0.5*TRA(K4)
      POLH=SS*TRA(K4)
      ALUMP = 0.0
      BLUMP = 0.0
      CALL LUMP1(ALUMP,BLUMP,TRAH,POLH,FF,F2,AA,BB,CC,DD)
957   IF(L85.GT.1) GO TO 6019
      DO 959 K1=1,4
      DO 958 K=1,8
958   B(K)=AA5(KO,K,K1)
      CALL LUMP(AA,BB,CC,DD,B)
      DO 959 K=1,8
959   AA5(KO,K,K1)=B(K)
6019  IF(LL8.EQ.0) GO TO 962
      DO 960 K=1,8
960   B(K)=QU(KO,K,L83)
      CALL LUMP(AA,BB,CC,DD,B)
      DO 961 K=1,8
961   QU(KO,K,L83)=B(K)
962   CONTINUE
      KP = KO+1
      IF (LB.GT 1.AND.LL8.EQ.0) GO TO 69
      CALL FLEX (K4,FFRT,E)
69    IF (L85.GT.1) GO TO 72
      DO 71 K1=1,4
      DO 70 K=1,8
70    B(K) = AA5(KO,K,K1)
      CALL CFLEX (K4,B,E)
      DO 71 K=1,8
71    AA5(KP,K,K1) = B(K)
72    IF (LL8.EQ.0) GO TO 20
      DO 73 K=1,8
73    B(K) = QU(KO,K,L83)
      CALL CFLEX (K4,B,E)
      DO 74 K=1,8
```

```
74    QU(KP,K,L83) = B(K)
      IF(L8.GT.1 .AND. LL8.EQ.0) GO TO 857
      ALUMP = 0.0
      BLUMP = 0.0
      CALL LUMP1(ALUMP,BLUMP,TRAH,POLH,FF,F2,AA,BB,CC,DD)
857   IF(L85.GT.1) GO TO 6018
      DO 859 K1=1,4
      DO 858 K=1,8
858   B(K)=AA5(KP,K,K1)
      CALL LUMP(AA,BB,CC,DD,B)
      DO 859 K=1,8
859   AA5(KP,K,K1)=B(K)
6018  IF(LL8.EQ.0) GO TO 862
      DO 860 K=1,8
860   B(K)=QU(KP,K,L83)
      CALL LUMP(AA,BB,CC,DD,B)
      DO 861 K=1,8
861   QU(KP,K,L83)=B(K)
862   CONTINUE
      KQ=K4+1
      UU(KQ,1,1)=AA5(KP,4,1)
      UU(KQ,2,1)=AA5(KP,4,2)
      UU(KQ,1,2)=AA5(KP,8,3)
      UU(KQ,2,2)=AA5(KP,8,4)
20    CONTINUE
      IF (L85.GT.1) GO TO 207
      IF(LIST.EQ.0) WRITE(KW,5019) S,F
5019  FORMAT (17H1BENDING RESPONSE//12H SPEED     = ,1PD12.4,4H RPM/
     +12H FREQUENCY = ,1PD12.4,3H HZ/1H )
      JORB = 0
      IF(S.NE.0.0) GO TO 9226
      NGYRO = 1
      GO TO 9227
9226  NGYRO = 2
C *** ISOTROPIC END DETERMINANT CHECK
9227  DO 9222 N=1,2
      NN = 2*(N-1)
      DO 9221 J=1,2
      JJ = 2*NN+J
      DO 9221 K=1,2
      KK = NN+K
9221  AM(J,K) = AA5(M2,JJ,KK)
      IMAXA = 2
      IMAXB = 0
      CALL MAXIV(AM,IMAXA,BM,IMAXB,DETERM,JMAX)
      DET(N)=DETERM
      IF (IMAXA.NE.0) GO TO 21
      WRITE (KW,5021) F
5021  FORMAT(/30H BENDING RESONANCE IS FOUND AT,1PE12.4,4H  HZ//)
C *** CALCULATE ISOTROPIC RESONANT MODE
      JORB = N
28    J=1
      K1 = NN+1
      K2 = NN+2
```

```
 23    J1 = NN+J
       J2 = NN+J1
       IF (AA5(M2,J2,K1).EQ.0.0) GO TO 24
       E(K1) = -AA5(M2,J2,K2)/AA5(M2,J2,K1)
       E(K2) = 1
       GO TO 9222
 24    IF (AA5(M2,J2,K2).EQ.0.0) GO TO 25
       E(K1) = 1
       E(K2) = 0
       GO TO 9222
 25    IF (J.EQ.2) GO TO 26
       J = 2
       GO TO 23
 C *** INVERT ISOTROPIC END MATRIX
 21    DO 27 J=1,2
       JJ = NN+J
       DO 27 K=1,2
       KK = NN+K
 27    AMI(JJ,KK) = AM(J,K)
       IF (N.EQ.1) GO TO 28
       WRITE (KW,5025) S,F,(DET(NWHIRL),NWHIRL=1,2)
 5025  FORMAT (1P2E12.4,2E15.6)
       GO TO 28
 9222  CONTINUE
       DO 76 N=1,L1P
       DO 76 J=1,2
       SHAPE=0.0
       DO 75 K=1,2
       K1=K
       IF(J.EQ.2) K1=K1+2
 75    SHAPE=SHAPE+E(K1)*UU(N,K,J)
 76    UU(N,1,J)=SHAPE
       WRITE(10) (DET(N),N=1,2),(Z(NU),(UU(NU,IONE,J),J=1,2),NU=1,L1P
       GO TO 207
 26    WRITE (KW,5022)
 5022  FORMAT (21H END MATRIX IS EMPTY./1H )
       CALL EXIT
 207   IF(LL8.EQ.0) GO TO 216
       IF(L83.EQ.1.AND.LIST.EQ.0) WRITE(KW,5107) S,F,K09
 5107  FORMAT (17H1BENDING RESPONSE//19H SPEED            = ,1PD12.4,
      +4H RPM//19H FREQUENCY        = ,1PD12.4,3H HZ//
      +19H ACTION NODE NO. = ,I3)
       DO 2081 K=1,4
 2081  AMAX(K) = 0
 C *** CALCULATE ISOTROPIC INITIAL INFLUENCE COEFFICIENTS
       NN=2*(L83-1)
       DO 9225 J=1,2
       JA = NN+J
       JB = 5-JA
       E(JA) = 0.0
       IF (JORB.EQ.L83) GO TO 9225
       DO 29 K=1,2
       KA = NN+K
       KB = NN+KA
```

```
29    E(JA) = E(JA)-AMI(JA,KA)*QU(M2,KB,L83)
9225  E(JB) = 0.0
C *** GENERATE COUPLING STATION INFLUENCE COEFFICIENTS
      DO 211 L=1,L4P1
      L6 = L4P1-L+1
      M2 = N2(L6)+1
      IF (L EQ.1) GO TO 210
      BM(1,1) = QC5(M1,4)-AA5(M2,4,2)*E(2)-AA5(M2,4,4)*E(4)
      BM(2,1) = QC5(M1,8)-AA5(M2,8,2)*E(2)-AA5(M2,8,4)*E(4)
      DO 209 K=1,2
      K4 = KFCN(IFOUR,K,4)
      DO 209 J=1,2
      J1 = 2*J-1
209   AM(K,J) = AA5(M2,K4,J1)
      IMAXA = 2
      IMAXB = 1
      CALL MAXIV(AM,IMAXA,BM,IMAXB,DETERM,JMAX)
      E(1) = BM(1,1)
      E(3) = BM(2,1)
210   M1 = N1(L6)
      DO 211 KO=M1,M2
      DO 211 K=1,8
      QC5(KO,K) = 0
      DO 211 J=1,4
211   QC5(KO,K) = QC5(KO,K)+AA5(KO,K,J)*E(J)
C *** CALCULATE TOTAL ROTATIONAL SOLUTION
      DO 214 L=1,L4P1
      CALL JFCN (L,M1)
      M2 = N2(L)+1
      DO 214 K=M1,M2
      K4 = K-J4
      DO 213 N=1,4
      J = N+4
      AA = QC5(K,N)
      BB = QC5(K,J)
      IF(K.LT.KO8) GO TO 212
      AA = AA+QU(K,N,L83)
      BB = BB+QU(K,J,L83)
212   TRY = ABS(AA)+ABS(BB)
      UU(K,N,L83) = AA
      UU(K,J,L83) = BB
213   IF(AMAX(N) LT TRY) AMAX(N) = TRY
214   CONTINUE
      IF (L83.GT.NGYRO) GO TO 2211
      IF(LIST.EQ.1) GO TO 217
      WRITE (KW,5100)
5100  FORMAT (//T22,10HFORCE (LB),T36,14HMOMENT (IN-LB))
      WRITE (KW,5101) (XV(K,L83),K=1,4)
5101  FORMAT (15H CO -ROTATIONAL,2(4X,1PD12.5)/
     +15H CTR-ROTATIONAL,2(4X,1PD12.5)/1H )
      WRITE (KW,5093) (AMAX(K),K=1,4)
5093  FORMAT (T4,4HMODE,T19,5HSHEAR,T31,6HMOMENT,T43,5HSLOPE,T54,
     +10HDEFLECTION/10H VARIABLES,T19,5H(LBS),T31,7H(IN-LB),T57,
     +4H(IN)/6H SCALE,8X,1P4D12.4/1H )
```

```
      WRITE (KW,5023)
 5023 FORMAT (T9,8HLOCATION,21H  CO-ROTATIONAL WHIRL,T72,20HCTR-ROTATION
     +AL WHIRL/14H NODE     (IN),2(53H    SHEAR     MOMENT    SLOPE    D
     +EFLECTION   M/(EI) )/1H )
 217  NN8=N8(1)
      JJ8 = 1
      DO 221 J=1,L4P1
      CALL JFCN (J,M1)
      M2 = N2(J)+1
      DO 221 K=M1,M2
      K4 = K-J4
      IF(K4.NE.NN8) GO TO 221
      DO 220 N=1,4
      L = N+4
      B(N) = UU(K,N,L83)/AMAX(N)
 220  B(L+1) = UU(K,L,L83)/AMAX(N)
      IF (J.LT.L4P1.OR.K.LT.M2) GO TO 250
      B(5) = 0
      B(10) = 0
      GO TO 251
 250  B(5) = UU(K,2,L83)/B2(K4)
      B(10) = UU(K,6,L83)/B2(K4)
 251  IF(LIST.EQ.0) WRITE(KW,5028) K4,Z(K4),(B(N),N=1,10)
 5028 FORMAT (I4,F11.4,2(1X,0P4F10.6,1PD12.4))
      IF(JJ8.EQ.J8) GO TO 2211
      JJ8 = JJ8+1
      NN8 = N8(JJ8)
 221  CONTINUE
 2211 M8 = N8(L8)
      KK8 = K8(L8)
      IF (L83.EQ.2) GO TO 215
C *** SWITCH TO CTR-ROTATIONAL CASE
      L83 = 2
      LL8 = 0
      L81 = L8
      GO TO 451
C *** CARTESIAN REPRESENTATION
 215  IF(LIST.EQ.0) WRITE(KW,5102) S,F
 5102 FORMAT(/25H CARTESIAN REPRESENTATION//19H SPEED
     +1PE12.4,5H RPM //19H FREQUENCY        = ,E12.4,3H HZ)
      KKK8 = KK8
      WRITE(10) JORB,M8,KK8
      M=1
      DO 222 N=1,4
 222  AMAX(N) = 0
      DO 224 J=1,L4P1
      CALL JFCN (J,M1)
      M2 = N2(J)+1
      DO 224 K=M1,M2
      K4 = K-J4
      DO 223 N=1,4
      L = N+4
      AA = 0.0
      IF (JORB.NE.2) AA = AA+UU(K,N,1)
```

```
      BB = 0.0
      IF(JORB.NE.1) BB=-UU(K,L,2)
      XD5(K,N) = (AA-BB)/2
      IF (M.EQ.2) XD5(K,N) = -XD5(K,N)
      TRY = (ABS(AA)+ABS(BB))/2
      IF (AMAX(N).LT.TRY) AMAX(N) = TRY
223   XD5(K,L)= -(AA+BB)/2
224   CONTINUE
      IF(LIST.EQ.1)  GO TO 7002
      WRITE(KW,5103)
5103  FORMAT(/37H STATION  FORCE (LB)   MOMENT (IN-LB))
      WRITE(KW,5106) K09,(XV(J,M),XV(J,M),J=1,2)
5106  FORMAT(I5,2(3X,1PE12.5))
      IF (JORB.NE.0) GO TO 7000
      WRITE (KW,5093) (AMAX(N),N=1,4)
      GO TO 7001
7000  WRITE (KW,9999)
9999  FORMAT (53H THE FOLLOWING ARE NORMALIZED RESONANT DISTRIBUTIONS./)
7001  WRITE(KW,5104)
5104  FORMAT (T9,29HLOCATION  VERTICAL COMPONENTS,T72,21HHORIZONTAL COMP
     +ONENTS/14H NODE     (IN),2(53H    SHEAR     MOMENT     SLOPE     DEF
     +LECTION   M/(EI) ))
7002  NN8=N8(1)
      KK8 = K8(1)
      JJ8 = 1
      DO 226 J=1,L4P1
      CALL JFCN (J,M1)
      M2 = N2(J)+1
      DO 226 K=M1,M2
      K4 = K-J4
      IF (J.GT.1.AND.K.EQ.M1) GO TO 226
      IF (K4.NE.NN8) GO TO 226
      DO 225 N=1,4
      L = N+4
      B(N) = XD5(K,N)/AMAX(N)
225   B(L+1) = XD5(K,L)/AMAX(N)
      IF (J.LT.L4P1.OR.K LT.M2) GO TO 252
      B(5) = 0
      B(10) = 0
      GO TO 253
252   B(5) = XD5(K,2)/B2(K4)
      B(10) = XD5(K,6)/B2(K4)
253   IF(LIST NE.1) WRITE(KW,5105) K4,Z(K4),(B(N),N=1,10)
5105  FORMAT (I3,3H RE,F10.4,4F10 6,1PD12 4/6H    IM,63X,0P4F10 6,1PD12
     +4)
      GO TO (260,260,231,232,231,232,231,231), KK8
231   JBEND = JBEND+1
      K9(JBEND)=1
      LBEND = NBEND+JBEND
      ZM(JBEND) = XD5(K,4)
      ZM(LBEND) = XD5(K,8)
      IF (KK8.LT.7) GO TO 260
232   JBEND = JBEND+1
      K9(JBEND)=2
```

```
      LBEND = NBEND+JBEND
      ZM(JBEND) = XD5(K,3)
      ZM(LBEND) = XD5(K,7)
260   IF (JJ8.EQ.J8) GO TO 227
      JJ8 = JJ8+1
      NN8 = N8(JJ8)
      KK8 = K8(JJ8)
226   CONTINUE
227   CONTINUE
      WRITE(10) (ZM(K),K=1,NEND)
      IBEND = IBEND+1
      IF (KKK8.LT.7.OR.L84.EQ.2) GO TO 244
C *** SWITCH TO MOMENT EXCITATION
      L84 = 2
      L81 = L8
      M8 = N8(L8)
      KK8 = K8(L8)
      L83 = 1
      LL8 = 0
      GOTO 45
244   IF (IBEND.EQ.NBEND) GO TO 100
C *** IDENTIFY NEXT BENDING EXCITATION STATION
216   LL8 = 0
240   IF (J8.LE.L8) GO TO 100
      L8 = L8+1
      KK8 = K8(L8)
      IF (KK8.LT.3) GO TO 240
      L81 = L8
      L83 = 1
      L84 = 1
      M8 = N8(L8)
      GO TO 45
100   CONTINUE
1000  CONTINUE
      WRITE(10) (K9(I),I=1,NBEND)
      RETURN
      END
```

```
      SUBROUTINE SYDSYN
      DIMENSION BRGLX(2,2),TLX(2,4),DXX(2,11),X(40,40)
      DIMENSION M8(20),K8(20)
      DIMENSION DETER(2,11)
      DIMENSION IWARN(2,11),NFORCE(20),KFORCE(20),BIG(20),SMALL(20)
      DIMENSION SLANT(20),PHASE(20),SPEED(11)
      DIMENSION FLX(4,20),PLX(80),WLX(80)
      DIMENSION IDENT(4),NTITLE(16),KEX(40)
      DIMENSION IRLAB(40),ICLAB(40),FREQ(11)
      DIMENSION BRGST(4,2),ENDET(2),SHAPE(76,3),STIF(2,2,20),
     +DAMP(2,2,20),KBG(4),POSIT(76)
      DIMENSION RET(11),XINT(11)
      DIMENSION TABLX(60,122)
      COMMON/BMAT/XLX(60,120),YLX(60,120),ZLX(60,120),
     +XHALF(20,40),YHALF(20,40),ZHALF(20,40),QHALF(20,40)
      COMMON/BINV/XIN(60,122),XOU(60,122),DLX(2),IROW(61),ICOL(61)
      COMMON/WR/LW,LR
      COMMON/CBEG/IBEG
      COMMON/WUADD/FSQ
      COMMON/WUS/M8,K8,X,POSIT
      COMMON/CODA/K9(20)
      COMMON/WUSA/RET,XINT
      COMMON/PRMS/IRUN,ITYPE,IBRG,IDIAG
      COMMON/XGCON/SLOW,RLAX,IGEN
      DATA STIF,DAMP/160*0.0/
      M60 = 60
      JR = 1
      JC = 1
      KEEP=8
      KR=LR
      KW=LW
      KSAVE=9
      ISLO = 0
      READ (10) KASE,NTORS,NBEND
      IRUN = 0
      IF (NBEND.EQ.0.AND.NTORS.EQ.0) GO TO 5
      READ(KR,110) IRUN,ITYPE,IBRG,IPRI,IDIAG,IGEN,ISLO,IBEG
      IF (ISLO) 1,2,3
 1    SLOW = 2.0**ISLO
      RLAX = 0.1**ISLO
      GO TO 5
 2    RLAX = 1.0
 3    SLOW = 1.0
      IF (ISLO.EQ.0) GO TO 5
      RLAX = 10.0**ISLO
 5    LINE=0
      NT1 = NTORS+1
      NT2 = 2*NTORS
      NTB = NTORS+NBEND
      N2 = 2*NBEND
      N3 = N2+1
      NTBB = NTORS+N2
      N22 = 2*N2
      N21 = N22-1
```

```
      N23 = N22+1
      N24 = 2*NTBB
      DO 400 JASE=1,KASE
      DO 6 I=1,N2
      PLX(I) = 0.0
      PLX(N2+I) = 0.0
      DO 6 J=1,4
      FLX(J,I) = 0.0
6     CONTINUE
      REWIND KSAVE
      REWIND KEEP
      READ (10) IDENT,NTITLE,NSPD,LBRG,LNODE,KRUN,LRUN
      IF(NBEND.GT.0) GO TO 2005
      IRUN=0
      GO TO 2017
2005  IF(IRUN.GT.2) GO TO 2016
      IF(IRUN.EQ.0.OR.LRUN.EQ.1) GO TO 2017
2010  WRITE(KW,2015) LRUN,IRUN
2015  FORMAT (//19H PARAMETERS (LRUN =,I5,7H IRUN =,I5,19H) ARE INCOMPAT
     +IBLE.)
      CALL EXIT
2016  IF(LRUN.EQ.1) GO TO 2010
2017  CONTINUE
      IF (IRUN.NE.4) ITYPE = 0
      CALL RUNCON(KW,IRUN,ITYPE,IDENT,NTITLE)
      DO 399 KSPD=1,NSPD
      IF (NTORS.EQ.0) GO TO 120
      READ (10) (KEX(I),I=1,NTORS)
      IF (KSPD.GT.1) GO TO 120
      WRITE (KW,100)
100   FORMAT (/30H TORSIONAL EXCITATION STATIONS)
      WRITE (KW,110) (KEX(I),I=1,NTORS)
110   FORMAT(16I5)
120   IF (NBEND.EQ.0) GO TO 140
      READ (10) (KEX(I),I=NT1,NTB)
      IF(LBRG.EQ.0) GO TO 125
      READ (10) (KBG(IBR),(BRGST(IBR,JBRG),JBRG=1,2),IBR=1,LBRG)
125   CONTINUE
      IF (KSPD.GT.1) GO TO 140
      WRITE(KW,130)
130   FORMAT (/28H BENDING EXCITATION STATIONS)
      WRITE(KW,110) (KEX(I),I=NT1,NTB)
140   READ (10) IFRE
      DO 390 JFRE=1,IFRE
      IF(LRUN.EQ.1) GO TO 3901
      REWIND KSAVE
      REWIND KEEP
3901  READ (10) NFREQ
      DO 380 KJ=1,NFREQ
      IF (NTORS.EQ.0) GO TO 265
      READ (10) FREQ(KJ)
      OMG=6.283185*FREQ(KJ)
      IONE = 1
      READ (10) (SHAPE(IPRINT,IONE),IPRINT=1,LNODE)
```

```
      DO 150 I=1,NTORS
      READ (10) KU,(X(J,I),J=1,NTORS)
150   CONTINUE
      IF(IDIAG.NE.2) GO TO 175
      WRITE (KW,170) IDENT,NTITLE
170   FORMAT (1H1,20A4)
      WRITE (KW,160)
160   FORMAT(26H TORSIONAL MOBILITY MATRIX/1H )
175   DO 180 I=1,NTORS
      IRLAB(I) = I
      ICLAB(I) = I
      DO 180 J=1,NTORS
      J2 = 2*J
      J1 = J2-1
      XIN(I,J1) = X(I,J)
      XIN(I,J2) = 0.0
      XLX(I,J1) = X(I,J)
180   XLX(I,J2) = 0.0
      IF(IDIAG.NE.2) GO TO 185
      DO 181 I=1,NTORS
      DO 181 J=1,NT2
      TABLX(I,J) = XLX(I,J)
181   CONTINUE
      CALL MOUTC (TABLX,NTORS,NTORS,M60,JR,JC,IRLAB,ICLAB,LINE,KW)
185   IF (KU.NE.2) GO TO 210
      WRITE (KW,190) FREQ(KJ)
190   FORMAT (/1PE12.4,38H HZ IS A TORSIONAL RESONANT FREQUENCY /
     +37H EXECUTION HALTED, SEE USER'S MANUAL./1H1)
      CALL EXIT
210   CALL INVC (NTORS,IERR)
      IF (IERR.NE.0) GO TO 4010
      DO 215 I=1,NTORS
      DO 215 J=1,NT2
      ZLX(I,J) = XOU(I,J)
215   CONTINUE
      IF(IDIAG.NE.2) GO TO 240
      WRITE (KW,230)
230   FORMAT (////27H TORSIONAL IMPEDANCE MATRIX/1H )
      DO 231 I=1,NTORS
      DO 231 J=1,NT2
      TABLX(I,J) = ZLX(I,J)
231   CONTINUE
      CALL MOUTC (TABLX,NTORS,NTORS,M60,JR,JC,IRLAB,ICLAB,LINE,KW)
240   IF (NBEND.EQ.0) GO TO 360
      DO 260 I=1,NTORS
      DO 260 J=1,NT2
      YLX(I,J) = ZLX(I,J)
260   CONTINUE
265   IF (NBEND.EQ.0) GO TO 360
      READ (10) FREQ(KJ),SPD
      OMG=6 283185*FREQ(KJ)
      SRPM = 60.0*SPD
      READ (10) (ENDET(IDET),IDET=1,2), (POSIT(IPRINT),(SHAPE(IPRINT
     + ISHAPE),ISHAPE=2,3),IPRINT=1,LNODE)
```

```
      DO 280 I=1,NBEND
      READ (10) JORB,M8(I),K8(I)
      READ (10) (X(J,I),J=1,N2)
280   CONTINUE
      IF(IDIAG.NE.2) GO TO 295
      WRITE (KW,170) IDENT,NTITLE
      WRITE (KW,270) FREQ(KJ),SPD
270   FORMAT(/10H FREQ    = ,1PE12.4,3H HZ/10H RPS     = ,E12.4,3H HZ)
      WRITE (KW,290)
290   FORMAT(24H0BENDING MOBILITY MATRIX)
295   NM1=NBEND-1
      DO 300 I=1,NM1
      IRLAB(I)=I
      ICLAB(I)=I
      L = NBEND+I
      IRLAB(L)=L
      ICLAB(L)=L
      I2 = 2*I
      I1 = I2-1
      XLX(I,I1) = X(I,I)
      XLX(I,I2) = 0.0
      XLX(L,I1) = 0.0
      XLX(L,I2) = X(L,I)
      L2 = 2*L
      L1 = L2-1
      XLX(I,L1) = -XLX(L,I1)
      XLX(I,L2) = -XLX(L,I2)
      XLX(L,L1) = XLX(I,I1)
      XLX(L,L2) = XLX(I,I2)
      IPLUS=I+1
      DO 300 J=IPLUS,NBEND
      J2 = 2*J
      J1 = J2-1
      X(J,I)=0.5*(X(J,I)+X(I,J))
      XLX(J,I1) = X(J,I)
      XLX(J,I2) = 0.0
      XLX(I,J1) = XLX(J,I1)
      XLX(I,J2) = XLX(J,I2)
      M = NBEND+J
      X(M,I)=0.5*(X(M,I)+X(L,J))
      XLX(M,I1) = 0.0
      XLX(M,I2) = X(M,I)
      XLX(L,J1)=XLX(M,I1)
      XLX(L,J2)=XLX(M,I2)
      XLX(J,L1)=-XLX(M,I1)
      XLX(J,L2)=-XLX(M,I2)
      M2 = 2*M
      M1 = M2-1
      XLX(I,M1)=XLX(J,L1)
      XLX(I,M2)=XLX(J,L2)
      XLX(L,M1) = XLX(I,J1)
      XLX(L,M2) = XLX(I,J2)
      XLX(M,L1) = XLX(J,I1)
300   XLX(M,L2) = XLX(J,I2)
```

```
      IRLAB(NBEND) = NBEND
      ICLAB(NBEND) = NBEND
      IRLAB(N2)=N2
      ICLAB(N2)=N2
      XLX(NBEND,N2-1) = X(NBEND,NBEND)
      XLX(NBEND,N2) = 0.0
      XLX(N2,N2-1) = 0.0
      XLX(N2,N2) = X(N2,NBEND)
      XLX(NBEND,N21) =-XLX(N2,N2-1)
      XLX(NBEND,N22) =-XLX(N2,N2)
      XLX(N2,N21) = XLX(NBEND,N2-1)
      XLX(N2,N22) = XLX(NBEND,N2)
      DO 302 I=1,N2
      DO 302 J=1,N22
  302 XIN(I,J) = XLX(I,J)
      IF(IDIAG.NE.2) GO TO 305
      DO 301 I=1,N2
      DO 301 J=1,N22
  301 TABLX(I,J) = XLX(I,J)
      CALL MOUTC (TABLX,N2,N2,M60,JR,JC,IRLAB,ICLAB,LINE,KW)
  305 IF (JORB.EQ.0) GO TO 320
      WRITE (KW,310) FREQ(KJ)
  310 FORMAT (1H1,1PE12.4,36H HZ IS A BENDING RESONANT FREQUENCY /
     +37H EXECUTION HALTED, SEE USER'S MANUAL./1H1)
      CALL EXIT
  320 CALL INVC (N2,IERR)
      IF (IERR.NE.0) GO TO 4010
      DO 321 I=1,N2
      DO 321 J=1,N22
  321 ZLX(I,J) = XOU(I,J)
      IF(NTORS.EQ.0) GO TO 360
      DO 331 I=1,N2
      DO 331 J=N23,N24
      ZLX(I,J) = 0.0
  331 CONTINUE
      DO 333 I=N3,NTBB
      IRLAB(I) = I
      ICLAB(I) = I
      DO 332 J=1,N22
      ZLX(I,J) = 0.0
  332 CONTINUE
      L = I-N22
      DO 333 J=N23,N24
      K = J-N22
      ZLX(I,J) = YLX(L,K)
  333 CONTINUE
      IF(IDIAG.NE.2) GO TO 360
      WRITE (KW,340)
  340 FORMAT (25H1BENDING IMPEDANCE MATRIX/1H )
      DO 334 I=1,N2
      DO 334 J=1,N22
      TABLX(I,J) = ZLX(I,J)
  334 CONTINUE
      CALL MOUTC (TABLX,N2,N2,M60,JR,JC,IRLAB,ICLAB,LINE,KW)
```

```
360    IF (IDIAG.NE.2) GO TO 3702
       WRITE (KW,370) IDENT,NTITLE,SRPM,FREQ(KJ),NTBB
370    FORMAT (43H1UNMODIFIED IMPEDANCE MATRIX RECORD SUMMARY/
      +10H0IDENT  = ,4A4/10H NTITLE = ,16A4/10H RPM     = ,1PE12 4,
      +3H HZ/10H FREQ    = ,E12.4,3H HZ/10H RANK    = ,I3/1H )
       DO 3701 I=1,NTBB
       DO 3701 J=1,N24
       TABLX(I,J) = ZLX(I,J)
3701   CONTINUE
       CALL MOUTC (TABLX,NTBB,NTBB,N60,JR,JC,IRLAB,ICLAB,LINE,KW)
3702   IF (IRUN EQ.0) GO TO 780
       FSQ=FREQ(KJ)**2
       IF (IRUN GT.2) GO TO 42
C SYNCHRONISM CHECK
       CHECK=FREQ(KJ)-SPD
       CHECK=CHECK*CHECK/(FSQ+SPD*SPD)
       IF (CHECK.LE.1.0E-08) GO TO 41
       WRITE (KW,2000)
2000   FORMAT (/60H SPEED-FREQUENCY DATA ARE FOUND INCONSISTENT IN SYNCHR
      +25H CHECK. EXECUTION HALTED )
       CALL EXIT
41     INDEX = KSPD
       SPEED(INDEX) = SPD
       IF (IRUN.EQ.2) CENT = 0 0063856*FSQ
       GO TO 43
42     INDEX = KJ
C IMPEDANCE REVISION
43     IF(IBRG.GT.0.OR IRUN.LT.2.OR IRUN EQ 3) GO TO 441
       WRITE(KW,4224)
4224   FORMAT(73H COMPLEX EIGENVALUE LOGIC BEING USED UNNECESSARILY FOR A
      +N UNDAMPED ROTOR //)
441    IF(IBRG GT 0 OR LBRG GT 0) GO TO 44
       DO 600 I=1,N2
       DO 600 J=1,N22
600    XLX(I,J)=ZLX(I,J)
       GO TO 500
44     I=1
       KBGZ=0
       NTEMP=1
       NBRG = 0
       IREV=0
       KOUNT = 1
       KBOTH = 1
       ITIME = 1
       KBRG = 0
       IF (IDIAG.NE.3) GO TO 4490
       WRITE (KW,170)
       WRITE (KW,4410) LBRG
4410   FORMAT (40H NUMBER OF BRGS ASSIGNED IN LEVEL 1     =,I5)
       IF (LBRG.EQ.0) GO TO 4420
       WRITE (KW,4411) (KBG(IBR),IBR=1,LBRG)
4411   FORMAT (/24H BEARING STATION NUMBERS,4I15)
       IONE = 1
       WRITE (KW,4412) (BRGST(IBR,IONE),IBR=1,LBRG)
```

```
4412  FORMAT (25H RADIAL STIFFNESS (LB/IN),5X,1P4D15.6)
      ITWO = 2
      WRITE (KW,4413) (BRGST(IBR,ITWO),IBR=1,LBRG)
4413  FORMAT (30H ANGULAR STIFFNESS (IN-LB/RAD),1P4D15.6)
4420  IF (IBRG.EQ.0) IPRI =-1
4490  GO TO (46,47,45 47,45), IRUN
45    IF (KJ.EQ.1) GO TO 47
      IF (IPRI) 379,48,48
46    IF (KSPD.EQ.1) GO TO 47
      IF (IPRI) 379,48,48
47    IF (IPRI.EQ.-1) GO TO 379
      WRITE (KW,170)
48    WRITE (KW,610) SRPM, FREQ(KJ)
610   FORMAT (/21H LEVEL 2 BEARING DATA/19H ROTATIONAL SPEED =,1PE11.4,4
     +H RPM,6X,11HFREQUENCY =,E11.4,3H HZ//8H BEARING,2X,4(1H*),6H TYPE
     +,4(1H*),2X,8(1H*),3H XX,2(1X,8(1H*),1X),2HXY,2(1X,8(1H*),1X),2HYX,
     +2(1X,8(1H*),1X),3HYY ,8(1H*)/25H NO. STN  RADIAL ANGULAR ,4(5X,1HK
     +,10X,1HB,5X))
379   IF (IBRG.EQ.0) GO TO 378
C READ BEARING DATA CONTROLS
377   READ(KR,110) LO,NBRG,KBRG,IREV
378   NBO=NTEMP
      DO 376 J=NBO,NBEND
      J02 = 2*J
      J01 = J02-1
C MATCH STATION NUMBER AND ACTION TYPE
      DO 401 IZ=1,2
      DO 401 JZ=1,2
401   BRGLX(IZ,JZ) = 0.0
      NBRG = NB(J)
      LBG = KB(J)
      J1 = J+1
      JTYPE=LBG-2
      IF (KOUNT.GT.LBRG) GO TO 410
      KBGZ=KBG(KOUNT)
      IF (NBRG.NE.KBGZ) GO TO 410
      IF (JTYPE.LE.2) GO TO 406
      IF (JTYPE-5) 404,403,402
402   JTYPE=JTYPE-1
403   JTYPE=JTYPE-3
      IF (ITIME.EQ.2) GO TO 405
      ITIME=2
      JTYPE=JTYPE-1
      GO TO 406
404   JTYPE=JTYPE-2
      GO TO 406
405   ITIME=1
406   DO 407 IZ=1,2
      BRGLX(IZ,IZ) = BRGST(KOUNT,JTYPE)
      DO 407 JZ=1,2
      JZ2 = 2*JZ
      JZ1 = JZ2-1
      TLX(IZ,JZ1) = 0.0
407   TLX(IZ,JZ2) = 0.0
```

```
      IF (JTYPE.EQ.2.OR.LBG.LT.6) KOUNT=KOUNT+1
410   IF (MBRG.NE.NBRG) GO TO 375
      IF (IREV.EQ.0) GO TO 411
      IF (KBRG-1) 371,372,374
C ERROR MESSAGE FOR (NBRG,KBRG)
371   WRITE (KW,10) I
10    FORMAT (1H1,7(1H*),32H INPUT DATA IS FAULTY FOR BRG NO,I3,7H. CHEC
     +34HK NBRG AND KBRG AGAINST NB AND KB.,7(1H*))
      GO TO 400
372   IF (LBG.LT.3.OR.LBG.EQ.4.OR.LBG.EQ.6) GO TO 371
C READ BEARING DATA
373   READ(KR,2020)  (((STIF(IONE,ITWO,I),DAMP(IONE,ITWO,I)),ITWO
     + =1,2),IONE=1,2)
411   NTEMP = J1
2020  FORMAT(8D10.4)
      IF (IDIAG.NE.3) GO TO 375
      IF (KBRG.EQ.2) GO TO 4802
      WRITE (KW,4801) LO,NBRG,(((STIF(IONE,ITWO,I),DAMP(IONE,ITWO,I)),
     +ITWO=1,2),IONE=1,2)
4801  FORMAT (I3,I4,5X,1H*,12X,1P8D11.4)
      GO TO 4804
4802  WRITE (KW,4803) LO,NBRG,(((STIF(IONE,ITWO,I),DAMP(IONE,ITWO,I)),
     +ITWO=1,2),IONE=1,2)
4803  FORMAT (I3,I4,13X,1H*,4X,1P8D11.4)
4804  IPRI =-1
      GO TO 375
374   IF (LBG.LT.4.OR LBG.EQ.5) GO TO 371
      GO TO 373
375   DO 369 K1=1,2
      L = NBEND*(K1-1)+J
      DO 369 K=1,NBEND
      DO 369 K2=1,2
      M = NBEND*(K2-1)+K
      M2 = 2*M
      M1 = M2-1
      XLX(L,M1) = ZLX(L,M1)
      XLX(L,M2) = ZLX(L,M2)
      IF(NTEMP.NE.J1 OR K.NE J) GO TO 369
      K22 = 2*K2
      K21 = K22-1
      TLX(K1,K21) = STIF(K1,K2,I)-BRGLX(K1,K2)
      TLX(K1,K22) = OMG*DAMP(K1,K2,I)
      XLX(L,M1)=XLX(L,M1)+TLX(K1,K21)
      XLX(L,M2)=XLX(L,M2)+TLX(K1,K22)
369   CONTINUE
      IF (NTEMP.NE J1.AND.MBRG.NE KBGZ) GO TO 376
      IF (IPRI NE.-1)
     +CALL BRGTAB (KW,I,KBOTH,NBRG,KBRG,JTYPE,KBGZ,TLX,BRGLX,OMG)
      KBOTH=KBOTH+1
      IF(NTEMP.NE.J1 OR I EQ IBRG) GO TO 376
      I = I+1
      GO TO 377
376   CONTINUE
500   CONTINUE
```

```
      CALL ISTRP(ISO,IBRG,STIF,DAMP,IRUN)
      IF(IBRG.EQ.0.OR.IDIAG.NE.2) GO TO 1000
      WRITE(KW,501) SPD,FREQ(KJ)
501   FORMAT(50H1CARTESIAN IMPEDANCE MATRIX AFTER BEARING REVISION/20H
     +ROTATIONAL SPEED =,1PE11.4,3H HZ,7X,11HFREQUENCY =,E11.4,3H HZ//)
      DO 502 I=1,NTBB
      DO 502 J=1,N24
      TABLX(I,J) = XLX(I,J)
502   CONTINUE
      CALL MOUTC (TABLX,NTBB,NTBB,M60,JR,JC,IRLAB,ICLAB,LINE,KW)
C  IMPEDANCE MATRIX TRANSFORMED TO ROTATING COORDINATES
C  UNDER BOTH ITYPE=1 AND 2
1000  CALL ROTATE(NBEND)
      WRITE (KSAVE) SPD,FREQ(KJ),((YLX(I,J),J=1,N22),I=1,N2)
      IF(IDIAG.NE.2) GO TO 1001
      WRITE (KW,1510)
1510  FORMAT(41H1IMPEDANCE MATRIX IN ROTATING COORDINATES//)
      DO 503 I=1,NTBB
      DO 503 J=1,N24
      TABLX(I,J) = YLX(I,J)
503   CONTINUE
      CALL MOUTC (TABLX,NTBB,NTBB,M60,JR,JC,IRLAB,ICLAB,LINE,KW)
1001  IF(IRUN.EQ.5) GO TO 380
      IF (IRUN.EQ.2.OR.IRUN.EQ.4) GO TO 71
      CALL IGNRL(NBEND,INDEX,ISO,DETER,IWARN)
      GO TO 380
C RESPONSE ORBITS
71    DO 702 I=1,N2
      DO 702 J=1,N22
      XIN(I,J) = YLX(I,J)
702    CONTINUE
      CALL INVC (N2,IERR)
C     NEED IERR CHECK HERE!          CHTP OCT 31,1979
      DO 703 I=1,N2
      DO 703 J=1,N22
      XLX(I,J) = XOU(I,J)
703   CONTINUE
C READ EXCITATION CONTROLS
      IF (IRUN.EQ.0) GO TO 380
      READ (KR,110) IFORCE,IFDATA
      IF (IFDATA.EQ.0) GO TO 39
      READ (KR,110) (NFORCE(I),I=1,IFORCE)
      kEAD (KR,110) (KFORCE(I),I=1,IFORCE)
39    NTFO=1
      NF = 0
      I = 1
      IF (IDIAG.NE.3) GO TO 72
      WRITE(KW,3907)
3907  FORMAT(1X,27HEXCITATION INPUT DATA CHECK,/,1X,15HIFORCE, IFDATA /:
      WRITE (KW,110) IFORCE,IFDATA
      WRITE(KW,3902)
3902  FORMAT(1X,6HNFORCE/)
      WRITE (KW,110) (NFORCE(IPRINT),IPRINT=1,IFORCE)
      WRITE(KW,3903)
```

```
3903  FORMAT(1X,6HKFORCE/)
      WRITE (KW,110) (KFORCE(IPRINT),IPRINT=1,IFORCE)
72    NF = NFORCE(I)
      KF = KFORCE(I)
C  SEQUENCE THROUGH HALF-RANK
      NFO=NTFO
      DO 80 J=NFO,NBEND
      J02 = 2*J
      J01 = J02-1
C  IDENTIFY STATION NUMBER AND ACTION TYPE
      MF = M8(J)
      LF = K8(J)
      J1 = J+1
C  MATCH STATION NUMBERS
      IF (MF.NE.NF) GO TO 77
C  CHECK COMPATIBILITY
      IF (KF-1) 73,74,76
C  ERROR MESSAGE FOR (NFORCE,KFORCE)
73    WRITE (KW,11) I
11    FORMAT (1H1,3(1H*),34H INPUT DATA IS FAULTY FOR ENTRY NO,I3,
     +27H O EITHER NFORCE OR KFORCE.,3(1H*))
      GO TO 400
74    IF (LF.LT.3.OR.LF.EQ.4.OR.LF.EQ.6) GO TO 73
75    NTFO=J1
      IF (IFDATA.EQ.0) GO TO 78
C  READ EXCITATION DATA
      READ (KR,2020) (FLX(IONE,I),IONE = 1,4)
      IF (IRUN.NE.2) GO TO 78
      FLX(3,I) = 0.0
      FLX(4,I) = 0.0
78    DO 79 K1=1,2
      L = NBEND*(K1-1)+J
      K12 = 2*K1
      K11 = K12-1
      L2 = 2*L
      L1 = L2-1
      PLX(L1) = FLX(K11,I)
      PLX(L2) = FLX(K12,I)
      IF (IRUN.NE.2) GO TO 79
      PLX(L1) = CENT*PLX(L1)
      PLX(L2) = CENT*PLX(L2)
79    CONTINUE
      IF (I.EQ.IFORCE) GO TO 83
      I = I+1
      GO TO 72
76    IF (LF.LT.4.OR.LF.EQ.5) GO TO 73
      GO TO 75
77    PLX(J01) = 0.0
      PLX(J02) = 0.0
      PLX(N2+J01) = 0.0
      PLX(N2+J02) = 0 0
80    CONTINUE
83    JF1=J02+1
      DO 82 J=JF1,N2
```

```
          L=N2+J
          PLX(J)=0.0
82        PLX(L)=0.0
        IF(IPRI.NE.-1) CALL XCITAB(KW,ITYPE,NBEND,M8,K8,PLX)
        WRITE (KSAVE) (PLX(I),I=1,N22)
C  COMPUTE RESPONSE VECTOR
        IF(ITYPE.EQ.0) GO TO 832
C  TRANSFORM FORCES FROM CARTESIAN TO ROTATING COORDINATES
        DO 831 IK=1,NBEND
        IK2 = 2*IK
        IK1 = IK2-1
        LK1 = N2+IK1
        LK2 = N2+IK2
        WLX(IK1) = 0.5*(PLX(IK1)-PLX(LK2))
        WLX(IK2) = 0.5*(PLX(IK2)+PLX(LK1))
        WLX(LK1) = 0.5*(PLX(IK1)+PLX(LK2))
        WLX(LK2) = 0.5*(PLX(IK2)-PLX(LK1))
        PLX(IK1) = WLX(IK1)
        PLX(IK2) = WLX(IK2)
        PLX(LK1) = WLX(LK1)
        PLX(LK2) = WLX(LK2)
831     CONTINUE
832     DO 84 I=1,N2
        I2 = 2*I
        I1 = I2-1
        WLX(I1) = 0.0
        WLX(I2) = 0.0
        DO 84 J=1,N2
        J2 = 2*J
        J1 = J2-1
        WLX(I1) = XLX(I,J1)*PLX(J1)-XLX(I,J2)*PLX(J2)+WLX(I1)
        WLX(I2) = XLX(I,J1)*PLX(J2)+XLX(I,J2)*PLX(J1)+WLX(I2)
84      CONTINUE
        IF (IDIAG.NE.3) GO TO 1010
        WRITE(KW,3904)
3904    FORMAT(1X,20HRESPONSE DIAGNOSTICS/1X,14HEXCITING FORCE/)
        WRITE (KW,1002) (PLX(IPRINT),IPRINT=1,N22)
        WRITE(KW,3905)
3905    FORMAT(1X,30HRESPONSE DUE TO EXCITING FORCE/)
        WRITE (KW,1002) (WLX(IPRINT),IPRINT=1,N22)
1002    FORMAT(10E12.3/10E12.3)
        DO 81 I=1,NTBB
        DO 81 J=1,N24
        TABLX(I,J) = XLX(I,J)
81      CONTINUE
        CALL MOUTC (TABLX,NTBB,NTBB,M60,JR,JC,IRLAB,ICLAB,LINE,KW)
C  COMPUTE ORBIT PARAMETERS
1010    DO 85 I=1,NBEND
        I2 = 2*I
        I1 = I2-1
        L = NBEND+I
        L2 = 2*L
        L1 = L2-1
85      CALL ORBIT(ITYPE,BIG(I),SMALL(I),SLANT(I),PHASE(I),
```

```
     +              WLX(I1),WLX(I2),WLX(L1),WLX(L2))
C WRITE PART II OF SYDSYN OUTPUT
      IORB = 1
      CALL ORBTAB(KW,NBEND,M8,K8,SRPM,FREQ(KJ),BIG,SMALL,SLANT,
     +PHASE,POSIT,IORB)
380   CONTINUE
      IF (NBEND.EQ.0) GO TO 400
      READ (10) (K9(I),I=1,NBEND)
      IF(IRUN-3) 390,386,385
386   CONTINUE
      WRITE(KW,382) SRPM
382   FORMAT (45H1UNDAMPED ASYNCHRONOUS STIFFNESS DETERMINANTS/
     +19H ROTATIONAL SPEED =,1PE11.4,4H RPM/)
      IF (ISO.EQ.1) GO TO 383
      WRITE (KW,387)
387   FORMAT (/10H FREQUENCY,7X,6(1H*),14H DETERMINANTS ,7(1H*)/
     +7H   (HZ),12X,7HCO -ROT,9X,7HCTR-ROT)
      GO TO 384
383   WRITE (KW,388)
388   FORMAT (/10H FREQUENCY,6X,11HDETERMINANT/7H   (HZ),11X,7HCOUPLED)
384   CALL ASNTAB(KW,NFREQ,IWARN,FREQ,DETER,ISO)
C INTERPOLATION FOR ASYNCHRONOUS RESONANCE
      CALL ENTER(KSAVE,KEEP,KW,NBEND,NFREQ,ISO,DETER,FREQ,SPEED,
     + RET,XINT,IRUN,IDIAG)
      GO TO 390
C STABILITY-MARGIN/DAMPED RESONANCE
385   IF (IGEN.EQ.0) GO TO 390
      CALL KRAUTE(NFREQ,NBEND,KSAVE,KEEP,KW,ITYPE,ISO,FREQ,SPEED,IRUN,
     +TABLX,PLX,WLX,BIG,SMALL,SLANT,PHASE,IDIAG)
390   CONTINUE
399   CONTINUE
      IF (IRUN.LE.0) GO TO 400
      IF (IRUN-2) 391,394,400
391   WRITE(KW,392)
392   FORMAT(28H1CRITICAL SPEED DETERMINANTS//16H SPEED (RPM)
     +11HDETERMINANT)
      CALL SYNTAB(KW,NSPD,IWARN,SPEED,DETER,ISO)
C CRITICAL SPEED AND UNBALANCE RESPONSE INTERPOLATION
      CALL ENTER(KSAVE,KEEP,KW,NBEND,NSPD,ISO,DETER,SPEED,FREQ,
     + RET,XINT,IRUN,IDIAG)
      GO TO 400
C UNBALANCE RESPONSE AT RESONANCE
394   IF (IGEN.EQ.0) GO TO 400
      KTYPE = 0
      CALL KRAUTE(NSPD,NBEND,KSAVE,KEEP,KW,KTYPE,ISO,SPEED,FREQ,IRUN,
     +TABLX,PLX,WLX,BIG,SMALL,SLANT,PHASE,IDIAG)
400   CONTINUE
4005  RETURN
4010  WRITE (KW,4011)
4011  FORMAT (50H1INVERSION OF A SINGULAR COMPLEX MATRIX ATTEMPTED!)
      GO TO 4005
      END
```

```
      SUBROUTINE SPLINE(NI,NP,RET,XINT.X,Y,Z,KW,IER)
C     NI     RUN CONTROL -  <0 OBTAIN SPLINE COEFFICIENTS
C                             0 LOOK FOR ROOTS
C                           100 LOCATE WELL
C                        OTHERS NUMBER OF INTERPOLATION POINTS
C            ON RETURN - NUMBER OF ROOTS FOUND
C     NP     NUMBER OF GIVEN DATA POINTS, MUST NOT EXCEED 11
C     XINT   X VALUES TO BE INTERPOLATED  OR
C            SLOPE AT ROOTS
C     RET    Y VALUES FROM INTERPOLATION OF GIVEN X'S
C            X VALUES OF COMPUTED ROOTS
C            ARRAY SIZE MUST BE AT LEAST 11 IN THE CALLING PROGRAM
C     X      X DATA VALUES
C     Y      Y DATA VALUES
C     Z      SPLINE COEFFICIENTS
C     KW     OUTPUT DEVICE NUMBER
C     IER    DIAGNOSTIC CONTROL
C              0 - NO PRINTOUT
C              1 - PRINT ONLY ANSWERS
C              2 - FULL DIAGNOSTIC PRINTOUT
C            ON RETURN - ERROR STATUS
C              0 - NO ERRORS
C              1 - VALUE FOR INTERPOLATION OUT OF BOUNDS
C              2 - ITERATION ERROR
C     DCB'S MODIFICATIONS ADDED MAY 23, 1979 BY CHTP
C
      INTEGER O
      DIMENSION XINT(1),RET(1),X(1),Y(1),Z(3,1)
      DIMENSION D(2,2),D1(2,2),D9(2,2),E(2),E1(2),E9(2),
     +R1(20,2),R2(20)
      NPM = NP-1
      IF (NI.NE.100) GO TO 2000
 2000 IPR=IER
      IER=0
      I=0
      D(1,1)=1.
      D(1,2)=0.
      D(2,1)=0.
      D(2,2)=1.
      E(1)=0.
      E(2)=0.
      D1(1,1)=-2.
      D1(2,2)=-2.
      E1(2)=1.
  111 I=I+1
      IFLAG=1
      GO TO 720
  120 DO 124 I1=1,2
      DO 124 I2=1,2
      D9(I1,I2)=0.
      DO 124 I3=1,2
  124 D9(I1,I2)=D9(I1,I2) + D1(I1,I3)*D(I3,I2)
      D(1,1)=D9(1,1)
      D(1,2)=D9(1,2)
```

```
      D(2,1)=D9(2,1)
      D(2,2)=D9(2,2)
      GO TO 710
 131  IF(I.LT.NP-1) GO TO 111
      Y1= -(1./D(2,1))*E(2)
      IF(NI-0) 300,300,140
 140  I2=0
      NWELL=0
      IF(NI.NE.100) GO TO 141
      I=1
      TANK=1.0E+5
      X1=0.0
      IFLAG=5
      E(1)=Y1
      E(2)=0.
      GO TO 730
 143  I2=0
      IF (I.EQ.NPM) GO TO 747
      GO TO 720
 145  I1=1
      B1=E(2)/Y3
      A1=2.*E(1)/Y3
      C1=B1*B1-A1
      IF(C1.LT.0.) GO TO 143
      C2=SQRT(C1)
      XQ=-A1+C2
      IF(XQ.LT.0.0) GO TO 143
 146  RET(1)=X(I)+XQ
      IF(RET(1).GE.X(I+1)) GO TO 147
      T9=E(2)+Y3*XQ
      IF(T9.LT.0.) GO TO 147
      NWELL=1
      NI=NWELL
      Y1=Y(I)+XQ*(E(1)+XQ*(0.5*E(2)+XQ*Y3/6.))
      IF(Y1-0.0) 147,147,50
 50   IF(Y1.GT.TANK.OR.RET(1).LT.X(1).OR.RET(1).GT.X(NP)) GO TO 147
      TANK=Y1
      X1=RET(1)
      GO TO 147
 747  IF(X1.LT.X(1).OR.X1.GT.X(NP)) GO TO 700
      RET(1)=X1
      Y1=TANK
      GO TO 700
 147  IF(I2.EQ.2) GO TO 143
      I2=2
      XQ=-B1-C2
      GO TO 146
 141  DO 270 INDEX=1,NI
      XS=XINT(INDEX)
      I=0
      IF(XS.GE.X(1).AND.XS.LE.X(NP))        GO TO 210
      IF (IPR.GT.0) WRITE (KW,200) X(1),X(NP),INDEX,XS
 200  FORMAT (/25H DATA PASSING ERROR. LOW= ,E16.8,12H      HIGH=
     +E16.9,8H     AT ,I2,9HX VALUE= ,E16.8)
```

```
      IER=1
      GO TO 700
210   I=I+1
      IF (I.EQ.NP) GO TO 220
      IF (XS.GE.X(I)) GO TO 210
220   K=I-1
      I=0
      E(1)=Y1
      E(2)=0.
230   I=I+1
      IFLAG=2
      IF (I.EQ.K) GO TO 260
      GO TO 720
260   GO TO 730
261   YS=Y(I)+(XS-X(I))*E(1)+0.5*(XS-X(I))**2*E(2)+
     +(1./6.)*(XS-X(I))**3*Y3
      RET(INDEX)=YS
      Z(1,I)=E(1)
      Z(2,I)= 0.5*E(2)
      Z(3,I)= Y3/6.
      IF (IPR.GT.0) WRITE (KW,264) INDEX,XS,YS
264   FORMAT (5X,I3,3X,29HTHE INTERPOLATED VALUE AT X = ,E16.8,
     +4H IS ,E16.8)
270   CONTINUE
      GO TO 700
C     ROOT FINDING
300   I=0
      DO 3003 IC=1,11
3003  RET(IC)=0.0
      RET(1)=0.0
      RET(2)=0 0
      RET(3)=0 0
      RET(4)=0 0
      R1(1,1)=X(1)
      R1(1,2)=Y(1)
      E(1)=Y1
      E(2)=0.
      J=1
310   I=I+1
      IFLAG=3
      IF (I LT.NP) GO TO 730
      IF (NI LT 0) GO TO 700
      GO TO 470
321   Z(1,I)=E(1)
      Z(2,I)=0 5*E(2)
      Z(3,I)=Y3/6
      IF (NI LT.0) GO TO 720
      A=Y3/2
      B=E(2)-X(I)*Y3
      C=E(1)-X(I)*E(2)+0 5*X(I)**2*Y3
      IF (SQRT((A*X(I+1)**2)**2+(B*X(I+1))**2)/SQRT((A*X(I+1)**2)**2
     ++(B*X(I+1))**2+C**2).LT 0 001) GO TO 720
      IF (ABS(A*X(I+1)**2)/SQRT((A*X(I+1)**2)**2
     ++(B*X(I+1))**2+C**2) GT 0 001) GO TO 360
```

```
      Z2=-C/B
      GO TO 420
360   ZZ=B**2-4.*A*C
      IF(ZZ.LT.0.) GO TO 720
      Z1=-B-SQRT(ZZ)
      Z2=-B+SQRT(ZZ)
      Z1=Z1/(2.*A)
      Z2=Z2/(2.*A)
      IF (Z1.LT.X(I).OR.Z1.GT.X(I+1)) GO TO 420
      XS=Z1
      YS=Y(I)+(XS-X(I))*E(1)+0.5*(XS-X(I))**2*E(2)+
     +(1./6.)*(XS-X(I))**3*Y3
      IF (R1(J,2)*YS.LE.0.0) GO TO 410
      IF (J.GT.1) GO TO 420
      J=0
410   J=J+1
      R1(J,1)=Z1
      R1(J,2)=YS
420   IF (Z2.LE.X(I).OR.Z2.GT.X(I+1)) GO TO 720
      XS=Z2
      YS=Y(I)+(XS-X(I))*E(1)+0.5*(XS-X(I))**2*E(2)+
     +(1./6.)*(XS-X(I))**3*Y3
      IF (R1(J,2)*YS.LE.0.) GO TO 450
      IF (J.GT.1) GO TO 720
      J=0
450   J=J+1
      R1(J,1)=Z2
      R1(J,2)=YS
      GO TO 720
470   IF (R1(J,2)*Y(NP).GT.0.) GO TO 480
      J=J+1
      R1(J,1)=X(NP)
      R1(J,2)=Y(NP)
480   IF (IPR.LT.2) GO TO 490
      DO 481 I1=1,J
481   WRITE (KW,482) R1(I1,1),R1(I1,2)
482   FORMAT (5X,E16.8,5X,E16.8)
490   O=0
      IF (J.EQ.1) GO TO 532
      IUP=J-1
      DO 520 I=1,IUP
      IF (R1(I+1,1).NE.R1(I,1)) GO TO 510
      O=O+1
      GO TO 520
510   R2(I-O)=R1(I+1,1)-R1(I+1,2)*(R1(I+1,1)-R1(I,1))/(R1(I+1,2)
     +-R1(I,2))
520   CONTINUE
      IF (IPR.LE.1) GO TO 532
      IUP=J-1-O
      DO 531 I1=1,IUP
531   WRITE (KW,482) R2(I1)
532   NI=J-O-1
      IF (NI.LE.0) GO TO 700
      DO 690 I7=1,NI
```

```
      XS=R2(I7)
      OLDDX=1.0E-25
540   I=0
      IF (XS.GE.X(1).AND.XS.LE.X(NP)) GO TO 570
      IF (IPR.GT.0) WRITE (KW,561) X(1),XS,X(NP)
561   FORMAT (24H ITERATION ERROR X(1)= ,E16 8,4H XS ,E16.8,7H X(NP)
     +E16.8)
      IER=2
      GO TO 700
570   I=I+1
      IF (I.EQ.NP) GO TO 580
      IF (XS.GE.X(I)) GO TO 570
580   K=I-1
      I=0
      E(1)=Y1
      E(2)=0
590   I=I+1
      IFLAG=4
      IF (I.NE.K) GO TO 720
      GO TO 730
621   YS=Y(I)+(XS-X(I))*E(1)+0.5*(XS-X(I))**2*E(2)+
     +(1./6.)*(XS-X(I))**3*Y3
      S1=E(1)+(XS-X(I))*E(2)+0 5*(XS-X(I))**2*Y3
      S2=E(2)+(XS-X(I))*Y3
      IF (ABS(S1).LE.1 0E-15) GO TO 660
      X9=XS-YS/S1
      IF (IPR.GT.1) WRITE (KW,632) XS,X9,YS,S1
632   FORMAT (4(5X,E16.8))
      CHECKX=0 001*ABS(OLDDX)
      IF (ABS(XS-X9).LE.CHECKX) GO TO 660
C   ADDITION (DCB)
      IF(X9.GE.R1(I7,1)) GOTO 633
      XS=R1(I7,1)
      GOTO 540
633   IF(X9.LE.R1(I7+1,1)) GOTO 634
      XS=R1(I7+1,1)
      GOTO 540
C   END ADDITION (DCB)
634   OLDDX=XS-X9
      Y9=YS+S1*(X9-XS)+(S2/2.)*(X9-XS)**2+(Y3/6 )*(X9-XS)**3
      XS=XS-YS*(X9-XS)/(Y9-YS)
      GO TO 540
660   RET(I7)=XS
      XINT(I7)=S1
      IF (IPR.GT.0) WRITE (KW,680) I7,XS,S1
680   FORMAT (5X,13H ROOT NUMBER ,I2,4H AT ,E16 8,9H SLOPE = ,E16.8)
690   CONTINUE
700   CONTINUE
      RETURN
C     MAT E9=D1*E   AND    MAT E=E9+E1
710   DO 714 I1=1,2
      E9(I1)=0.0
      DO 714 I2=1,2
714   E9(I1)=E9(I1)+D1(I1,I2)*E(I2)
```

```
      E(1)=E9(1)+E1(1)
      E(2)=E9(2)+E1(2)
      GO TO (131,230,310,590,728),IFLAG
C     SUBROUTINE *2
720   VAL=X(I+1)-X(I)
      D1(1,2)=-.5*VAL
      D1(2,1)=-6./VAL
      E1(1)=-D1(1,2)
      E1(2)=1.0
      DO 726 I1=1,2
726   E1(I1)=E1(I1)*(6.*(Y(I+1)-Y(I))/VAL**2)
      GO TO (120,710,710,710,710),IFLAG
C     SUBROUTINE  *3
728   I=I+1
730   VAL=X(I+1)-X(I)
      Y3=(Y(I+1)-Y(I))-VAL*E(1)-0.5*VAL**2*E(2)
      Y3=6.*Y3/VAL**3
      GO TO (131,261,321,621,145),IFLAG
      END
```

```
      SUBROUTINE PINVC(ID,KW,N,NA,NB,IS,IROT,TABLX,JDIAG)
C     TABLX      INPUT COMPLEX MATRIX
C                REPLACED BY ITS INVERSE UPON RETURN
C     ID         0    CALCULATE DETERMINANT DLX ONLY
C                1    PERFORM INVERSION AND CALCULATE DETERMINANT
C
C               A0   B0                          A1   B1
C     TABLX =                     INV(TABLX) =
C               C0   D0                          C1   D1
C
      COMMON/BMAT/XLX(60,120),YLX(60,120),ZLX(60,120),
     +XHALF(20,40),YHALF(20,40),ZHALF(20,40),QHALF(20,40)
      COMMON/BINV/XIN(60,122),XOU(60,122),DLX(2),IROW(61),ICOL(61)
      DIMENSION TABLX(60,1)
      DIMENSION SUMX(2)
      IW=0
      N2 = 2*N
      M60=60
      M20=20
      JR=1
      LINE=0
      IF (JDIAG.NE.5) GO TO 105
      DO 100 I=1,N2
      IROW(I) = I
      ICOL(I) = I
100   CONTINUE
      WRITE(KW,9876)
9876  FORMAT(//17H SUBROUTINE PINVC//)
      CALL MOUTC(TABLX,N2,N2,M60,JR,JR,IROW,ICOL,LINE,KW)
105   DO 160 L= 1,2
      IF (ID - 1) 110,130,190
110   IF (IS .EQ. 1) GO TO 130
      IF (IROT .NE. L) GO TO 160
130   MB= (L-1)*N
      MB2 = 2*MB
      IB= (2-L)*N
      IB2 = 2*IB
C     FOR        L = 1          2
C     SET UP         A0         D0
      DO 140 I= 1,N
      IO= I+MB
      DO 140 J= 1,N2
      JO= J+MB2
      XIN(I,J)= TABLX(IO,JO)
140   CONTINUE
C     FOR        L = 1          2
C     INVERT         A0         D0
      IF(JDIAG.EQ.5) CALL MOUTC(XIN,N,N,M60,JR,JR,IROW,ICOL,LINE,KW)
      CALL INVC(N,IERR)
      DRE = DLX(1)
      DIM = DLX(2)
      IF (JDIAG.NE.5) GO TO 1405
      DO 1402 I=1,N2
      IROW(I) = I
```

```
      ICOL(I) = I
 1402 CONTINUE
      CALL MOUTC(XOU,N,N,M60,JR,JR,IROW,ICOL,LINE,KW)
      WRITE(KW,1100)DLX(1),DLX(2)
 1100 FORMAT(13H DETERMINANT=,1PE13.6,1H,,1PE13 6)
C     FOR          L = 1           2
C     STORE            INV(AO)     INV(DO)
 1405 DO 1410 I=1,N
      DO 1410 J=1,N2
      YHALF(I,J) = XOU(I,J)
 1410 CONTINUE
      IF (IS .EQ.1) GO TO 145
      IF (IERR.NE.0) WRITE(KW,1000)
C     FOR          L = 1           2
C     STORE            A1          D1          FOR ISOTROPIC CASE
      DO 142 I=1,N
      IO=I+MB
      DO 142 J=1,N2
      JO=J+MB2
 142  TABLX(IO,JO)=YHALF(I,J)
      IF(JDIAG.EQ 5) CALL MOUTC(YHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
      GO TO 160
C     FOR          L = 1           2
C     NULL TEST OF  DET(AO)     DET(DO)
 145  IF (IERR .EQ. 0) GO TO 150
      IF (L .EQ. 1) IW=1
      IF (L.EQ.1.OR.IW.EQ.O) GO TO 160
      WRITE (KW,1000)
 1000 FORMAT (1H1,45H ILL-CONDITIONED MATRIX ENCOUNTERED IN PINVC )
      CALL EXIT
C     FOR          L = 1           2
C    SET UP            CO,BO       BO,CO
 150  DO 170 I= 1,N
      IO= I+IB
      I1=I+MB
      DO 170 J=1,N2
      JO= J+IB2
      J1= J+MB2
      XHALF(I,J)=TABLX(IO,J1)
 170  ZHALF(I,J)=TABLX(I1,JO)
      IF (JDIAG.NE.5) GO TO 1710
      CALL MOUTC(XHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
      CALL MOUTC(ZHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
C     FOR          L = 1                 2
C     CALC             CO*INV(AO)        BO*INV(DO)
 1710 DO 171 I=1,N
      DO 171 J=1,N
      J2 = 2*J
      J1 = J2-1
      SUMX(1) = 0 0
      SUMX(2) = 0.0
      DO 169 K=1,N
      K2 = 2*K
      K1 = K2-1
```

```
      SUMX(1) = SUMX(1)+XHALF(I,K1)*YHALF(K,J1)-XHALF(I,K2)*YHALF(K,J2)
      SUMX(2) = SUMX(2)+XHALF(I,K1)*YHALF(K,J2)+XHALF(I,K2)*YHALF(K,J1)
169   CONTINUE
      QHALF(I,J1) = SUMX(1)
      QHALF(I,J2) = SUMX(2)
171   CONTINUE
      IF(JDIAG.EQ.5) CALL MOUTC(QHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
C     FOR        L = 1          2
C     CALC            D1,C1      A1,B1
      DO 172 I=1,N
      IO=I+IB
      DO 172 J=1,N
      J2 = 2*J
      J1 = J2-1
      SUMX(1) = 0.0
      SUMX(2) = 0.0
      J02 = J2+IB2
      J01 = J02-1
      DO 168 K=1,N
      K2 = 2*K
      K1 = K2-1
      SUMX(1) = SUMX(1)+QHALF(I,K1)*ZHALF(K,J1)-QHALF(I,K2)*ZHALF(K,J2)
      SUMX(2) = SUMX(2)+QHALF(I,K1)*ZHALF(K,J2)+QHALF(I,K2)*ZHALF(K,J1)
168   CONTINUE
      XIN(I,J1) = TABLX(IO,J01)-SUMX(1)
      XIN(I,J2) = TABLX(IO,J02)-SUMX(2)
172   CONTINUE
      IF(JDIAG.EQ.5) CALL MOUTC(XIN,N,N,M60,JR,JR,IROW,ICOL,LINE,KW)
      CALL INVC(N,IERR)
      D1=DRE*DLX(1)-DIM*DLX(2)
      DIM=DRE*DLX(2)+DIM*DLX(1)
      DRE=D1
      IF (JDIAG.NE.5) GO TO 1720
      DO 1725 I=1,N2
      IROW(I) = I
      ICOL(I) = I
1725  CONTINUE
      CALL MOUTC(XOU,N,N,M60,JR,JR,IROW,ICOL,LINE,KW)
      WRITE(KW,1100)DLX(1),DLX(2)
1720  IF (ID.EQ.0) GO TO 190
      DO 173 I=1,N
      DO 173 J=1,N2
      ZHALF(I,J) = XOU(I,J)
173   CONTINUE
      IF(JDIAG.EQ.5) CALL MOUTC(ZHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
      DO 180 I= 1,N
      IO= I+IB
      I1= I+MB
      DO 180 J= 1,N
      J2 = 2*J
      J1 = J2-1
      J02 = J2+IB2
      J01 = J02-1
      J12 = J2+MB2
```

```
      J11 = J12-1
      TABLX(I0,J01) = ZHALF(I,J1)
      TABLX(I0,J02) = ZHALF(I,J2)
      XHALF(I,J1) = 0.0
      XHALF(I,J2) = 0.0
      DO 175 K= 1,N
      K2 = 2*K
      K1 = K2-1
      XHALF(I,J1) = XHALF(I,J1)-ZHALF(I,K1)*QHALF(K,J1)+ZHALF(I,K2)
     +                                              *QHALF(K,J2)
      XHALF(I,J2) = XHALF(I,J2)-ZHALF(I,K1)*QHALF(K,J2)-ZHALF(I,K2)
     +                                              *QHALF(K,J1)
175   CONTINUE
      TABLX(I0,J11) = XHALF(I,J1)
      TABLX(I0,J12) = XHALF(I,J2)
180   CONTINUE
C     FOR        L = 1          2
C     STORE           A1        D1
      IF(JDIAG.EQ.5) CALL MOUTC(XHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
      DO 181 I=1,N
      I1=I+MB
      DO 181 J=1,N2
      J0=J+IB2
181   QHALF(I,J)=TABLX(I1,J0)
      IF(JDIAG.EQ.5) CALL MOUTC(QHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
      DO 182 I=1,N
      DO 182 J=1,N
      J2 = 2*J
      J1 = J2-1
      SUMX(1) = 0.0
      SUMX(2) = 0.0
      DO 183 K=1,N
      K2 = 2*K
      K1 = K2-1
      SUMX(1) = SUMX(1)+QHALF(I,K1)*XHALF(K,J1)-QHALF(I,K2)*XHALF(K,J2)
      SUMX(2) = SUMX(2)+QHALF(I,K1)*XHALF(K,J2)+QHALF(I,K2)*XHALF(K,J1)
183   CONTINUE
      ZHALF(I,J1) = -SUMX(1)
      ZHALF(I,J2) = -SUMX(2)
      IF (I.NE.J) GO TO 182
      ZHALF(I,J1) = 1.0-SUMX(1)
182   CONTINUE
      IF(JDIAG.EQ.5) CALL MOUTC(ZHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
C     FOR        L = 1          2
C     CALC            INV(A0)   INV(D0)
      DO 184 I=1,N
      I0=I+MB
      DO 184 J=1,N
      J2 = 2*J
      J1 = J2-1
      J02 = J2+MB2
      J01 = J02-1
      SUMX(1) = 0.0
      SUMX(2) = 0.0
```

```
      DO 185 K=1,N
      K2 = 2*K
      K1 = K2-1
      SUMX(1) = SUMX(1)+YHALF(I,K1)*ZHALF(K,J1)-YHALF(I,K2)*ZHALF(K,J2)
      SUMX(2) = SUMX(2)+YHALF(I,K1)*ZHALF(K,J2)+YHALF(I,K2)*ZHALF(K,J1)
185   CONTINUE
      TABLX(IO,J01) = SUMX(1)
      TABLX(IO,J02) = SUMX(2)
184   CONTINUE
C     FOR        L = 1          2
C     STORE           B1         C1
      DO 186 I=1,N
      I1 = I+MB
      DO 186 J=1,N
      J2 = 2*J
      J1 = J2-1
      XHALF(I,J1) = 0.0
      XHALF(I,J2) = 0.0
      JB2 = J2+IB2
      JB1 = JB2-1
      DO 186 K=1,N
      KI = K+IB
      K2 = 2*KI
      K1 = K2-1
      XHALF(I,J1) = XHALF(I,J1)+TABLX(I1,K1)*TABLX(KI,JB1)-TABLX(I1,K2)
     +                                                    *TABLX(KI,JB2)
      XHALF(I,J2) = XHALF(I,J2)+TABLX(I1,K1)*TABLX(KI,JB2)+TABLX(I1,K2)
     +                                                    *TABLX(KI,JB1)
186   CONTINUE
      IF(JDIAG.EQ.5) CALL MOUTC(XHALF,N,N,M20,JR,JR,IROW,ICOL,LINE,KW)
      DO 188 I=1,N
      DO 188 J=1,N
      J2 = 2*J
      J1 = J2-1
      SUMX(1) = 0.0
      SUMX(2) = 0.0
      DO 189 K=1,N
      K2 = 2*K
      K1 = K2-1
      SUMX(1) = SUMX(1)-YHALF(I,K1)*XHALF(K,J1)+YHALF(I,K2)*XHALF(K,J2)
      SUMX(2) = SUMX(2)-YHALF(I,K1)*XHALF(K,J2)-YHALF(I,K2)*XHALF(K,J1)
189   CONTINUE
      TABLX(I+MB,J1+IB2) = SUMX(1)
      TABLX(I+MB,J2+IB2) = SUMX(2)
188   CONTINUE
      IF (ID.EQ.1) GO TO 190
160   CONTINUE
190   DLX(1) = DRE
      DLX(2) = DIM
      IF (JDIAG.NE.5) GO TO 200
      CALL MOUTC(TABLX,N2,N2,M60,JR,JR,IROW,ICOL,LINE,KW)
      WRITE(KW,1100)DLX(1),DLX(2)
200   RETURN
      END
```

```
      SUBROUTINE HEAD1
      INTEGER TITLE(16)
      COMMON/CHEAD1/TITLE,L1,L2,L3,L4,L5,LIST/WR/KW,KR
      WRITE(KW,1) TITLE,L1,L2,L3,L5
    1 FORMAT (1H1,T4,16A4//T10,14HSHAFT SEGMENTS,T38,1H=,T40,I5/T10,
     +15HSHAFT MATERIALS,T38,1H=,T40,I5/T10,15HLUMPED INERTIA
     +8HSTATIONS,T38,1H=,T40,I5/
     +T10,8HBEARINGS,T38,1H=,T40,I5)
      RETURN
      END
```

```
      SUBROUTINE STAT(J,K)
      COMMON/ATEST1/B1(75),B2(75),B3(75),B4(75),B5(75),B6(75)
      COMMON/ATEST2/UO(80,4)
      COMMON/CSTAT/X(75),Z(76)
      DIMENSION D(4)
      KMJ=K-J
      KP1=K+1
      DO 1 L=1,4
      D(L)=UO(K,L)
      IF(L.GE.3) D(L)=D(L)*B2(KMJ)
    1 UO(KP1,L)=D(L)
      C = 1
      DO 4 L=1,4
      C=C*X(KMJ)/L
      IF (L.EQ.4) GO TO 4
      LL1 = 4-L
      DO 3 I=1,LL1
      IP1=I+L
    3 UO(KP1,IP1)=UO(KP1,IP1)+D(I)*C
    4 UO(KP1,L)=UO(KP1,L)-B1(KMJ)*C
      DO 5 L=3,4
    5 UO(KP1,L)=UO(KP1,L)/B2(KMJ)
      RETURN
      END
```

```
      SUBROUTINE STEP (K,IS,IER)
      INTEGER TITLE(16)
      COMMON/ATEST1/B1(75),B2(75),B3(75),B4(75),B5(75),B6(75)
      COMMON/ATEST3/V0(80,8),A5(80,8,4)
      COMMON/BSTEP/D5(4,2)
      COMMON/CSTAT/X(75),Z(76)
      COMMON/ATOU/S4(4),A6(4,6)
      COMMON/ABRG/S5(4,2),T5(4,2)
      COMMON/CHEAD1/TITLE,L1,L2,L3,L4,L5,LIST/WR/KW,KR
      COMMON/ASTEP/J4,LB,M5,H5(4)
      DIMENSION D(4)
      K1 = K
      KK = K-J4
      KP = K+1
      DO 50 I=1,2
      I3 = I-1
      I2 = 2*I3
      I4 = 2*I2
      I3 = 3*I3
      DO 40 M=1,3
      MM = M+I2-1
      DO 3 IL=1,4
      LL = IL+I4
      IF (M-2) 1,2,2
1     D(IL) = V0(K1,LL)
      GO TO 3
2     D(IL) = A5(K,LL,MM)
3     CONTINUE
      IF (IS-2) 10,20,30
10    DO 11 IL=3,4
11    D(IL) = D(IL)*B2(KK)
      CALL TRANS (D,KK)
      IF (M.EQ.1.AND.I.EQ.1) CALL WT (D,KK)
      DO 14 IL=1,4
      IF (IL.GE.3) D(IL) = D(IL)/B2(KK)
      LL = IL+I4
      IF (M-2) 12,13,13
12    V0(KP,LL) = D(IL)
      GO TO 14
13    A5(KP,LL,MM) = D(IL)
14    CONTINUE
      GO TO 40
20    JJ = M+I3
      CALL HINGE (D,JJ)
      GO TO 40
30    CALL BEAR (D,I)
      IF (M.EQ.1) D(1) = D(1)+S5(LB,I)*D5(LB,I)
      DO 33 IL=1,4
      LL = IL+I4
      IF (M-2) 31,32,32
31    V0(K,LL) = D(IL)
      GO TO 33
32    A5(K,LL,MM) = D(IL)
33    CONTINUE
```

```
40    CONTINUE
      IF (IS-2) 50,41,50
41    I31 = I3+1
      I32 = I3+2
      I33 = I3+3
      AAA = A6(J4,I32)
      IF (AAA.EQ.0) GO TO 70
      A6(J4,I31) = -A6(J4,I31)/AAA
      A6(J4,I32) = -A6(J4,I33)/AAA
      A6(J4,I33) = S4(J4)/AAA
      I21 = I2+1
      I22 = I2+2
      DO 42 IL=1,4
      LL = IL+I4
      IF (IL.EQ.3) GO TO 42
      A5(KP,LL,I22) = A5(K,LL,I21)*A6(J4,I32)+A5(K,LL,I22)
      A5(KP,LL,I21) = A5(K,LL,I21)*A6(J4,I33)
      VO(KP,LL) = VO(K1,LL)+A5(K,LL,I21)*A6(J4,I31)
42    CONTINUE
      I43 = I4+3
      A5(KP,I43,I21) = 0
      A5(KP,I43,I22) = 1
50    CONTINUE
      IF (IS.NE.3.OR.LB.EQ.L5) GO TO 60
      LB = LB+1
      M5 = N5(LB)
60    RETURN
70    WRITE (KW,71) J4
71    FORMAT (/22H FREE COUPLING AT NODE,I3,19H SHOULD BE PRECEDED,
     +14H BY A BEARING.)
      IER = 1
      GOTO 60
      END
```

```
      SUBROUTINE BEAR (A,K)
      COMMON/BSTEP/D5(4,2)
      COMMON/ABRG/S5(4,2),T5(4,2)
      COMMON/ASTEP/J4,LB,M5,N5(4)
      DIMENSION A(4),BB(2)
      BB(1) = -S5(LB,K)
      BB(2) = T5(LB,K)
      DO 1 I=1,2
      J = 5-I
    1 A(I) = A(I)+BB(I)*A(J)
      RETURN
      END
```

```
      FUNCTION KFCN(N,K,I)
      KFCN = N*(K-1)+I
      RETURN
      END
```

```
SUBROUTINE JFCH (J,N)
COMMON/IMIN/N1(5),M2(5)
COMMON/ASTEP/J4,LB,M5,N5(4)
J4 = J-1
N = N1(J)
RETURN
END
```

```
      SUBROUTINE MFCN (J,N,M)
      COMMON/IHIN/N1(5),N2(5)
      N = N2(J)
      M = N+1
      RETURN
      END
PROGRAM SIZE:   PROCEDURE - 000020   LINKAGE - 000022   STACK - 000024
0000 ERRORS [<MFCN  >FTN-REV15.3]
```

```
      SUBROUTINE STACK (I,J,K,N,I1,K1,K2)
      DIMENSION N(4),K1(5),K2(5)
      I1=I-1
      IF (I1.GT.0) GO TO 3
      K1(I) = 1
      IF (K.GT.0) GO TO 2
1     K2(I) = J+K
      GO TO 4
2     K2(I) = N(I)+I1-1
      GO TO 4
3     K1(I) = N(I1)+I1
      IF (K.GT.I1) GO TO 2
      GO TO 1
4     RETURN
      END
```

```
      SUBROUTINE TRANS (A,KK)
      COMMON/CSTAT/X(75),Z(76)
      DIMENSION A(4),G(3)
      X1 = X(KK)
      XX = 1
      DO 1 M=1,3
      XX = XX*X1/M
    1 G(M) = XX
      DO 3 I=1,3
      M = 5-I
      MM = M-1
      DO 2 J=1,MM
      K = M-J
      B = G(J)*A(K)
    2 A(M) = A(M)+B
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE WT (D,KK)
      COMMON/ATEST1/B1(75),B2(75),B3(75),B4(75),B5(75),B6(75)
      COMMON/CSTAT/X(75),Z(76)
      DIMENSION D(4)
      BB = B1(KK)
      X1 = X(KK)
      DO 1 I=1,4
      BB = BB*X1/I
    1 D(I) = D(I)-BB
      RETURN
      END
```

```
      SUBROUTINE HINGE (A,JJ)
      COMMON/ACOU/S4(4),A6(4,6)
      COMMON/ASTEP/J4,LB,M5,N5(4)
      DIMENSION A(4)
      A6(J4,JJ) = A(2)+S4(J4)*A(3)
      RETURN
      END
```

```
      SUBROUTINE TORS (K,D)
      INTEGER TITLE(16)
      COMMON/AWH/F,FF,XS(4,2),F1,FF1,SS
      COMMON/ATEST1/B1(75),B2(75),B3(75),B4(75),B5(75),B6(75)
      COMMON/CSTAT/X(75),Z(76)
      COMMON/ASTEP/J4,LB,M5,N5(4)
      COMMON/CHEAD1/TITLE,L1,L2,L3,L4,L5,LIST/WR/KW,KR
      DIMENSION D(2)
      KK=K-J4
   10 CC = FF*B4(KK)
      AA = B5(KK)*CC
      D(1)=D(1)/AA
      CALL TWIST(D,KK,CC)
      D(1) = D(1)*AA
   30 RETURN
      END
```

```
      SUBROUTINE TWIST(D,K,C)
      INTEGER TITLE(16)
      COMMON/CSTAT/X(75),Z(76)
      COMMON/CHEAD1/TITLE,L1,L2,L3,L4,L5,LIST/WR/KW,KR
      DIMENSION D(2)
      C1=C*X(K)
      A = COS(C1)
      B = SIN(C1)
      E=A*D(1)-B*D(2)
      D(2)=B*D(1)+A*D(2)
      D(1)=E
      RETURN
      END
```

```
      SUBROUTINE BEND
      INTEGER TITLE(16)
      COMMON/AWH/F,FF,XS(4,2),F1,FF1,SS
      COMMON/ATEST1/B1(75),B2(75),B3(75),B4(75),B5(75),B6(75)
      COMMON/ATEST2/UO(80,4)
      COMMON/CSTAT/X(75),Z(76)
      COMMON/AEND/U(80,2),ZM(76),ZK(20),FFRT
      COMMON/CHEAD1/TITLE,L1,L2,L3,L4,L5,LIST/WR/KW,KR
      DO 1 L=1,L1
      BB = FFRT*B3(L)*X(L)
      CS = COS(BB)
      SN = SIN(BB)
      XH = EXP(BB)
      XO = 1.0/XH
      CH = 0.5*(XH+XO)
      SH = CH-XO
      UO(L,1) = 0.5*(CS+CH)
      UO(L,2) = 0.5*(SH-SN)
      UO(L,3) = UO(L,1)-CS
    1 UO(L,4) = UO(L,2)+SN
      RETURN
      END
```

```
      SUBROUTINE MODE
      COMMON/AEND/U(80,2),ZM(76),ZK(20),FFRT
      COMMON/ABEND/L1P,L4P1,I8,I5
      MO=1
      A=ZM(1)*ZM(1)
      DO 1 I=2,L1P
      B=ZM(I)*ZM(I)
      IF(A.GE.B) GO TO 1
      MO=I
      A=B
    1 CONTINUE
      A = A/ZM(MO)
      DO 2 I=1,L1P
    2 ZM(I)=ZM(I)/A
      RETURN
      END
```

```
      SUBROUTINE COUP (M1,B)
      COMMON/AVIB/AA5(80,8,4)
      COMMON/ACOU/S4(4),A6(4,6)
      COMMON/ASTEP/J4,LB,M5,N5(4)
      DIMENSION D(8,4),B(4,4)
      DO 1 K=1,8
      DO 1 K1=1,4
    1 D(K,K1) = AA5(M1,K,K1)
      DO 5 K=1,8
      DO 5 K1=1,4
      IF (K.NE 3.AND K.NE.7) GO TO 3
      KK = 2*K1+1
      IF (K.EQ.KK) GO TO 2
      AA5(M1,K,K1) = 0.0
      GOTO 5
    2 AA5(M1,K,K1) = 1.0
      GO TO 5
    3 DO 4 K2=1,2
      KK = 2*K2-1
    4 AA5(M1,K,K1) = AA5(M1,K,K1)+D(K,KK)*B(K2,K1)
    5 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COUP1 (M,L,A,B)
      COMMON/AVIB/AA5(80,8,4)
      COMMON/ACOU/S4(4),A6(4,6)
      DIMENSION A(4,2,2),B(8),C(2),AA(2)
      C(1) = -B(2)-S4(L)*B(3)
      C(2) = -B(6)-S4(L)*B(7)
      DO 1 I=1,2
      AA(I) = 0
      DO 1 J=1,2
    1 AA(I) = AA(I)+A(L,I,J)*C(J)
      DO 2 I=1,8
      DO 2 J=1,2
      K = 2*J-1
    2 B(I) = B(I)+AA5(M,I,K)*AA(J)
      RETURN
      END
```

```
      SUBROUTINE LUMP (A,B,C,D,BB)
      DIMENSION BB(8)
      DO 1 K=1,2
      K1 = KFCN(4,K,1)
      K2 = KFCN(4,K,2)
      K3 = KFCN(4,K,3)
      K4 = KFCN(4,K,4)
      BB(K1) = BB(K1)+A*BB(K4)+B*BB(K3)
      AA = D
      IF (K.EQ.2) AA = -AA
    1 BB(K2) = BB(K2)-B*BB(K4)+(AA-C)*BB(K3)
      RETURN
      END
```

```
      SUBROUTINE LUMP1 (W,Y,T,P,F,G,A,B,C,D)
      A = G*W/386.4
      B = A*Y
      C = B*Y+G*T
      D = F*P
      RETURN
      END
```

```
      SUBROUTINE LIMP (M,A,B)
      COMMON/AVIB/AA5(80,8,4)
      COMMON/ACOU/S4(4),A6(4,6)
      COMMON/ASTEP/J4,LB,M5,N5(4)
      DIMENSION A(4,4),B(4,4)
      DO 2 I=1,2
      K = KFCN(4,I,2)
      L = K+1
      DO 2 J=1,2
      J2 = 2*J
      J1 = J2-1
      B(I,J2) = -AA5(M,L,J2)-S4(J4)*AA5(M,L,J2)
      IF (I.EQ.J) GO TO 1
      B(I,J1) = 0
      GOTO 2
    1 B(I,J1) = S4(J4)
    2 A(I,J) = AA5(M,K,J1)+S4(J4)*AA5(M,L,J1)
      KA = 2
      KB = 4
      IMAX = 4
      CALL MAXIV(A,KA,B,KB,DETERM,IMAX)
      RETURN
      END
```

```
      SUBROUTINE FLEX (K,F,E)
      COMMON/ATEST1/B1(75),B2(75),B3(75),B4(75),B5(75),B6(75)
      DIMENSION E(4)
      B = F*B3(K)
      E(1) = 1
      DO 2 I=2,4
      J = I-1
      A = E(J)
      IF (J-2) 2,1,2
    1 A = A*B2(K)
    2 E(I) = A*B
      RETURN
      END
```

```
      SUBROUTINE CFLEX (K,B,E)
      COMMON/ATEST2/UO(80,4)
      DIMENSION D(4),E(4),B(8),F(8)
      DO 1 I=1,4
    1 D(I) = UO(K,I)
      DO 5 J=1,4
      DO 3 L=1,2
      I = KFCN(4,L,J)
      A = 0
      DO 2 N=1,4
      M = KFCN(4,L,N)
    2 A = A+D(N)*E(N)*B(M)
    3 F(I) = A/E(J)
      IF (J.EQ.4) GO TO 5
      A = D(4)
      DO 4 N=1,3
      L = 5-N
      I = L-1
    4 D(L) = D(I)
      D(1) = A
    5 CONTINUE
      DO 6 I=1,8
    6 B(I) = F(I)
      RETURN
      END
```

```
      SUBROUTINE MAXIV(AS,N,BS,M,DS,NJ)
C
C     MATRIX INVERSION AND SOLUTION OF SIMULTANEOUS LINEAR EQUATIONS
C     OF THE FORM    AS*X=BS
C     INV(AS) AND X ARE RETURNED AS (AS,BS), THE ORIGINAL (AS,BS) ARE
C     LOST
C     N(.LT.61)  RANK OF AS AND BS, RETURNED AS 0 IF AS IS SINGULAR
C     M          COLUMNS OF BS, M MUST NOT EXCEED N
C     DS         DETERMINANT OF AS
C     AS AND BS MUST BE DIMENSIONED (NJ,NJ) IN THE CALLING PROGRAM
C     NJ MUST BE NO LESS THAN EITHER M OR N
C
      DIMENSION A1(60,60),PIVOT(60)
      DIMENSION AS(NJ,1),BS(NJ,1)
      DIMENSION IPIVOT(20),INDEX(20,2)
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX,T,SWAP)
C
C     COMPUTATION OF NORM
C
1     C = 1.0E-20
2     ANO = 0.0
3     DO 6 I=1,N
4     DO 6 J=1,N
5     ANO = ANO+AS(I,J)*AS(I,J)
6     A1(I,J) = AS(I,J)
7     ANO = ANO/N
C
C     INITIALIZATION
C
10    DETERM = 1.0
15    DO 20 J=1,N
20    IPIVOT(J) = 0
30    DO 550 I=1,N
C
C     SEARCH FOR PIVOT ELEMENT
C
40    AMAX = 0 0
45    DO 100 J=1,N
50    IF (IPIVOT(J) EQ 1) GO TO 100
60    DO 100 K=1,N
70    IF (IPIVOT(K)-1) 80,100,840
80    IF (ABS(AMAX)-ABS(AS(J,K))) 85,100,100
85    IROW = J
90    ICOLUM = K
95    AMAX = AS(J,K)
100   CONTINUE
110   IPIVOT(ICOLUM) = IPIVOT(ICOLUM)+1
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
130   IF (IROW.EQ.ICOLUM) GO TO 260
140   DETERM = -DETERM
150   DO 200 L=1,N
160   SWAP = AS(IROW,L)
```

```
 170   AS(IROW,L) = AS(ICOLUM,L)
 200   AS(ICOLUM,L) = SWAP
 205   IF (M.LE.0) GO TO 260
 210   DO 250 L=1,M
 220   SWAP = BS(IROW,L)
 230   BS(IROW,L) = BS(ICOLUM,L)
 250   BS(ICOLUM,L) = SWAP
 260   INDEX(I,1) = IROW
 270   INDEX(I,2) = ICOLUM
 310   PIVOT(I) = AS(ICOLUM,ICOLUM)
 320   DETERM = DETERM*PIVOT(I)
C
C      ROW NORMALIZATION
C
 330   AS(ICOLUM,ICOLUM)=1.0
 340   DO 350 L=1,N
 350   AS(ICOLUM,L) = AS(ICOLUM,L)/PIVOT(I)
 355   IF (M.LE.0) GO TO 380
 360   DO 370 L=1,M
 370   BS(ICOLUM,L) = BS(ICOLUM,L)/PIVOT(I)
C
C      REDUCE NON-PIVOT ROWS
C
 380   DO 550 L1=1,N
 390   IF (L1.EQ.ICOLUM) GO TO 550
 400   T = AS(L1,ICOLUM)
 420   AS(L1,ICOLUM) = 0.0
 430   DO 450 L=1,N
 450   AS(L1,L) = AS(L1,L)-AS(ICOLUM,L)*T
 455   IF (M.LE.0) GO TO 550
 460   DO 500 L=1,M
 500   BS(L1,L) = BS(L1,L)-BS(ICOLUM,L)*T
 550   CONTINUE
C
C      INTERCHANGE COLUMNS
C
 600   DO 720 I=1,N
 610   L = N+1-I
 620   IF (INDEX(L,1) EQ.INDEX(L,2)) GO TO 720
 630   JROW = INDEX(L,1)
 640   JCOLUM = INDEX(L,2)
 650   DO 710 K=1,N
 660   SWAP = AS(K,JROW)
 670   AS(K,JROW) = AS(K,JCOLUM)
 700   AS(K,JCOLUM) = SWAP
 710   CONTINUE
 720   CONTINUE
C
C      TEST SINGULARITY
C
 730   DO 780 I=1,N
 740   D = 0.0
 750   DO 760 J=1,N
 760   D = D+A1(I,J)*AS(J,I)
```


', u)

```
770   DETERM = D*DETERM
780   CONTINUE
790   DETERN = DETERM*DETERM
800   DO 810 I=1,N
810   DETERN = DETERN/ANO
820   IF (DETERN.LT.C) GO TO 840
830   DS = DETERM
      RETURN
C
C     SET ERROR INDEX FOR SINGULAR INPUT MATRIX
C
840   N = 0
850   GO TO 830
      END
```

```
      SUBROUTINE ISTRP(ISO,IBRG,STIF,DAMP,IRUN)
      DIMENSION STIF(2,2,20),DAMP(2,2,20)
      ISO=0
      IF (IBRG.EQ.0) GO TO 105
      DO 101 I=1,IBRG
      ANORM=STIF(1,1,I)**2+STIF(2,2,I)**2+STIF(1,2,I)**2+STIF(2,1,I)**2
      BNORM=DAMP(1,1,I)**2+DAMP(2,2,I)**2+DAMP(1,2,I)**2+DAMP(2,1,I)**2
      ASUM=(STIF(1,2,I)+STIF(2,1,I))**2+(STIF(1,1,I)-STIF(2,2,I))**2
  101 BSUM=(DAMP(1,2,I)+DAMP(2,1,I))**2+(DAMP(1,1,I)-DAMP(2,2,I))**2
      THORM=ANORM+BNORM
      IF (TNORM .EQ. 0.0) GO TO 105
      RTOR=(ASUM+BSUM)/TNORM
      IF (RTOR.GT.1.0D-8) ISO=1
  105 RETURN
      END
```

```
      SUBROUTINE IGNRL(NBEND,INDEX,ISO,DETER,IWARN)
      COMMON/BMAT/XLX(60,120),YLX(60,120),ZLX(60,120),
     +XHALF(20,40),YHALF(20,40),ZHALF(20,40),QHALF(20,40)
      COMMON/BINV/XIN(60,122),XOU(60,122),DLX(2),IROW(61),ICOL(61)
      COMMON/WUADD/FSQ
      DIMENSION DETER(2,1),IWARN(2,2)
      IONE = 1
      ITWO = 2
      L=ISO*NBEND
      I20 = 20
      GSQ = 39.478417604*FSQ
      DO 91 I=1,NBEND
      DO 91 K=1,NBEND
      J = 2*K-1
      IF (ISO.EQ.0) GO TO 91
      J1 = J+2*NBEND
      YHALF(I,K) = -YLX(I,J1)/GSQ
 91   XHALF(I,K) = YLX(I,J)/GSQ
      NIN = NBEND
      CALL MAXIV(XHALF,NIN,YHALF,L,DETER(IONE,INDEX),I20)
      IF (NIN.GT.0) GO TO 92
      IW=1
      GO TO 93
 92   IW=0
      L=0
 93   IWARN(1,INDEX)=IW
      DO 95 I=1,NBEND
      I1=I+NBEND
      DO 95 K=1,NBEND
      J = 2*K-1
      J1 = J+2*NBEND
      XHALF(I,K) = YLX(I1,J1)/GSQ
      IF (ISO EQ 0) GO TO 95
      IF (IW.EQ 0) GO TO 101
      YHALF(I,K) = -YLX(I1,J)/GSQ
      GO TO 95
 101  DO 94 M=1,NBEND
      M1 = 2*M-1
      XHALF(I,K) = XHALF(I,K)+YLX(I1,M1)*YHALF(M,K)/GSQ
 94   CONTINUE
 95   CONTINUE
      NIN = NBEND
      CALL MAXIV(XHALF,NIN,YHALF,L,DETER(ITWO,INDEX),I20)
      IF (NIN GT 0) GO TO 102
      IWARN(2,INDEX)=1
      GO TO 200
 102  IWARN(2,INDEX)=0
      IF (ISO.EQ.0) GO TO 200
      IF (IW.EQ.0) GO TO 97
      DO 96   I=1,NBEND
      DO 96   M=1,NBEND
      J = 2*M-1
      XHALF(I,M) = YLX(I,J)/GSQ
      DO 96 K=1,NBEND
```

```
      K1 = 2*(K+NBEND)-1
96    XHALF(I,M) = XHALF(I,M)+YLX(I,K1)*YLX(K,J)/GSQ
      CALL MAXIV(XHALF,NBEND,YHALF,L,DETER(IONE,INDEX),I20)
97    DETER(2,INDEX)=DETER(1,INDEX)*DETER(2,INDEX)
200   RETURN
      END
```

```
      SUBROUTINE ENTER(KS,KE,KW,NB,NF,IS,DT,F,SP,RT,XI,IR,IDG)
      INTEGER STEP,FINAL,DIRECT,POINT6,POINT7
      COMMON/CODA/K9(20)
      DIMENSION TABLX(60,122),CN(3,10),CO(4,10,40),DT(2,11),V(60),F(11),
     + SP(11),RT(11),XI(11),F1(4),SP1(4)
C    SET UP FILE OF INTERPOLATION COEFFICIENTS
      DIRECT = 0
      REWIND KS
      REWIND KE
      MF=NF-1
      N10=NB+1
      N20=NB+NB
      N202.= 2*N20
      N1=1
      N2=NB
      NR = -1
 10   DO 70 I=N1,N2
        IO=I
        IF (I.LE.NB) GO TO 11
        ISTART = I-NB
        GO TO 12
 11     ISTART = I
 12     IF(IS.EQ 1.AND.N1.GE.N10) GO TO 60
 20     I1=I
        M2=N2
 30     CONTINUE
        DO 50 J=I1,M2
        J1 = 2*J-1
        REWIND KS
        DO 40 N=1,NF
          READ (KS) SO,F(N),((TABLX(L,M),M=1,N202),L=1,N20)
 40     SP(N) = TABLX(IO,J1)
        IER=0
        NPTS = NF
        CALL SPLINE(NR,NF,RT,XI,F,SP,CN,KW,IER)
        IF (IER.NE.0) GO TO 170
        DO 50 N=1,MF
          CO(1,N,J)=SP(N)
          DO 50 L=1,3
            M=L+1
            CO(M,N,J)=CN(L,N)
 50     CONTINUE
        IF(M2.EQ.N20) GO TO 56
        M2=N20
        I1=I
        IF(I1.LT.N10) I1=I1+NB
        IF(IS.EQ.0) IO=I1
        GO TO 30
 56     WRITE(KE) IS,((((CO(L,M,J),CO(L,M,J+NB)),L=1,4),J=ISTART,NB),
     +  M=1,MF)
        GO TO 70
 60     I1=I-NB
        M2=NB
        GO TO 30
```

```
 70   CONTINUE
      IF(IS.EQ.0.OR.N2.EQ.N20) GO TO 80
      N1=N10
      N2=N20
      GO TO 10
 80   REWIND KE
      IF(IS.EQ.1) GO TO 160
      IROT=1
 100  DO 110 I=1,NF
 110  SP(I)=DT(IROT,I)
      NR=0
      IF(NF.LE.4) GOTO 300
      POINT6=0
      POINT7=0
      IF(NF.NE.5) GOTO 6
 1    NF1=3
      STEP=2
 3    NRO = 0
      DO 2 I=1,NF1,STEP
        NR1 = NRO+1
        J=I+2+POINT7
        IF(I.NE.3) GOTO 4
        J=J+POINT6
        JJ=1
        DO 5 II=I,J
          F1(JJ)=F(II)
          SP1(JJ)=SP(II)
 5      JJ=JJ+1
        NR=0
        IER = 0
        NPTS = NF1
        CALL SPLINE(NR,NF1,RT,XI,F1,SP1,CN,KW,IER)
        IF (IER.NE.0) GO TO 170
        IF (NR.LE.0) GO TO 2
        CALL ROOTAB(IR,IS,IROT,NR,RT,NB,NF,KE,KW,F1,CO,V,DIRECT,IDG)
        NRO = NRO+NR
 2    CONTINUE
      NR = NRO
      GOTO 155
 6    IF(NF.NE.6) GOTO 7
      POINT6=1
      GOTO 1
 7    IF(NF.NE.7) GOTO 8
      POINT7=1
      STEP=3
      NF1=4
      GOTO 3
 8    NF2=NF-1
      NRO = 0
      DO 9 I=2,NF2
        NR1 = NRO+1
        ABSF=ABS(SP(I))
        ABSF1=ABS(SP(I-1))
        ABSF2=ABS(SP(I+1))
```

```
           IF(ABSF GT ABSF1.OR.ABSF.GT.ABSF2) GOTO 9
           INC=I-1
           IF(ABSF1.LT ABSF2) INC=I-2
           IF(INC.LT.1) INC=1
           JJ=1
 17        FINAL=INC+3
           IF(FINAL.LE.NF) GOTO 16
           INC=INC-1
           GOTO 17
 16        DO 18 II=INC,FINAL
             F1(JJ)=F(II)
             SP1(JJ)=SP(II)
 18        JJ=JJ+1
           NF1=4
           IER=0
           NR=0
           NPTS = NF1
           CALL SPLINE(NR,NF1,RT,XI,F1,SP1,CN,KW,IER)
           IF (IER NE 0) GO TO 170
           IF (NR.LE.0) GO TO 9
           CALL ROOTAB(IR,IS,IROT,NR,RT,NB,NF,KE,KW,F1,CO,V,DIRECT,IDG)
           NRO = NRO+NR
 9       CONTINUE
         NR = NRO
         GOTO 155
 300     IER = 0
         INC = 1
         NPTS = NF
         CALL SPLINE(NR,NF,RT,XI,F,SP,CN,KW,IER)
         IF (IER.NE.0) GO TO 170
         IF (NR.LE.0) GO TO 155
         CALL ROOTAB(IR,IS,IROT,NR,RT,NB,NF,KE,KW,F,CO,V,DIRECT,IDG)
 155     IF (IS.EQ.1) GO TO 130
         IF (IROT.EQ.2) GO TO 120
         WRITE (KW,2000) NR
 2000    FORMAT(/29H NO OF  CO-ROTATIONAL ROOTS =,I5/)
         GO TO 156
 120     WRITE(KW,2001) NR
 2001    FORMAT(/29H NO OF CTR-ROTATIONAL ROOTS =,I5/)
         GO TO 200
 130     WRITE (KW,2002) NR
 2002    FORMAT(/29H NO OF        COUPLED ROOTS =,I5/)
         GO TO 200
 156     IF(DIRECT.EQ.20) GOTO 200
         IF((IROT.EQ. 2).OR.(IR.EQ.1.AND.IS.EQ.0)) GO TO 200
 160     IROT=2
         GO TO 100
 170     WRITE (KW,1000)
 200     RETURN
 1000    FORMAT (/49H ERROR ENCOUNTERED DURING EXECUTION OF SUBROUTINE,
        +8H SPLINE.)
         END
```

```
      SUBROUTINE ROOTAB(IR,IS,IROT,NR,RT,NB,NF,KE,KW,F,CO,V,DIRECT,IDG)
      INTEGER DIRECT
      DIMENSION CO(4,10,1),RT(1),F(1),V(1)
      DIRECT=0
      IF (NR.EQ.0) GO TO 110
      IF (IS.EQ.1) GO TO 143
      IF (IROT-1) 140,140,142
  110 IF (IS.EQ.1.OR.IROT.EQ.2) GO TO 115
      DIRECT = 15
      GO TO 100
  115 DIRECT = 20
  140 WRITE(KW,1000)
 1000 FORMAT(/19H CO-ROTATIONAL MODE)
  141 IF (IR.EQ.1) GO TO 1141
      WRITE(KW,1001)
 1001 FORMAT(/20X,10H FRFQ (HZ),6X,42HLOCATION (IN)  DEFLECTION      SLO
     +PE        )
      GO TO 145
 1141 WRITE (KW,1011)
 1011 FORMAT(/20X,10H    RPM    ,6X,42HLOCATION (IN)  DEFLECTION      SLO
     +PE        )
      GO TO 145
  142 WRITE(KW,1002)
 1002 FORMAT(/20H CTR-ROTATIONAL MODE)
      GO TO 141
  143 WRITE(KW,1003)
 1003 FORMAT(/13H COUPLED MODE,38X,23HCO-ROTATIONAL COMPONENT,6X,
     +24HCTR-ROTATIONAL COMPONENT)
      IF (IR.EQ.1) GO TO 1143
      WRITE (KW,1004)
 1004 FORMAT(21X,10H FREQ (HZ),6X,13HLOCATION (IN),2(27H DEFLECTION
     + SLOPE       ,3X))
      GO TO 145
 1143 WRITE (KW,1014)
 1014 FORMAT(21X,10H    RPM    ,6X,13HLOCATION (IN),2(27H DEFLECTION
     + SLOPE       ,3X))
  145 DO 150 NP=1,NR
      RES=RT(NP)
  150 CALL BMODE (IR,IS,IROT,NB,NF,KW,KE,F,CO,RES,V,IDG)
  100 RETURN
      END
```

```
      SUBROUTINE BMODE(IRUN,IS,IR,NB,NF,KW,KE,F,C,R,V,IDG)
      DIMENSION F(1),C(4,10,1),V(1),X(40,40),POSIT(76),M8(20),K8(20)
     +W(60)
      COMMON/WUS/M8,K8,X,POSIT
      COMMON/CODA/K9(20)
      N40 = 40
      RSQ = 39.47841760436*R*R
C     GENERATE INTERPOLATED IMPEDANCE MATRIX
      MF=NF-1
      N20=NB+NB
      N2=NB+IS*NB
 10   DO 100 I=1,N2
      IF (I.LE.NB) GO TO 11
      ISTART = I-NB
      GO TO 12
 11   ISTART = I
 12   IF (IS .EQ.1 .AND. I.GT.NB) GO TO 90
      I1=I
      M2=NB
 25   READ (KE) IS,((((C(L,M,J),C(L,M,J+NB)),L=1,4),J=ISTART,NB),M=1,
     +MF)
 30   DO 80 J=I1,M2
      K=J
 35   IF (IS.EQ.1) GO TO 35
      IF (IR EQ.2) K=K+NB
      DO 40 L=1,MF
      U=F(L)
      IF (R.GT.U) GO TO 40
      LQ=L
      GO TO 50
 40   CONTINUE
      L=MF
      GO TO 60
 50   IF (R.EQ.U) GO TO 70
      L=LQ-1
 60   D=R-F(L)
      X(I,J)=(C(1,L,K)+D*(C(2,L,K)+D*(C(3,L,K)+D*C(4,L,K))))/RSQ
 65   IF (J.EQ.I1) GO TO 80
      IF (I1-I) 71,72,73
 71   JO=J+NB
      IO=I-NB
      GO TO 75
 72   JO=J
      IO=I
      GO TO 75
 73   JO=J-NB
      IO=I+NB
 75   X(JO,IO)=X(I,J)
      GO TO 80
 70   X(I,J)=C(1,L,K)/RSQ
      GO TO 65
 80   CONTINUE
      IF (IS.EQ.0) GO TO 100
      IF (M2.EQ.N20) GO TO 100
```

```
      N2=N20
      I1=I1+NB
      GO TO 30
90    I1=I-NB
      N2=NB
      GO TO 25
100   CONTINUE
      REWIND KE
C   CALCULATE CHARACTERISTIC VECTOR
      IF (IDG.NE.4) GO TO 5101
      WRITE (KW,2000)
2000  FORMAT (/30H INTERPOLATED STIFFNESS MATRIX//)
      CALL MATPRT(N2,N2,N40,N40,X)
C   INVERT INTERPOLATED STIFFNESS MATRIX
5101  IMAX = 0
      CALL MAXIV(X,N2,X,IMAX,DET,N40)
      IF (N2.GT.0) GO TO 5102
      WRITE (KW,51) R
51    FORMAT (34H1EXACT RESONANCE IS ENCOUNTERED AT,1PD12.5,4H HZ,/
     +31H MODE SHAPE CANNOT BE COMPUTED.)
      GO TO 510
5102  IF (IDG.NE.4) GO TO 81
      WRITE (KW,2001)
2001  FORMAT (/18H MODE SHAPE MATPIX//)
      CALL MATPRT(N2,N2,N40,N40,X)
C   SEARCH FOR LARGEST DIAGONAL ELEMENT OF THE MOBILITY MATRIY
81    KMAX = 0
      DO 103 I=1,N2
      IF (K9(I).EQ.2) GO TO 103
      TRY = ABS(X(I,I))
      IF (KMAX.EQ.1) GO TO 101
      KMAX = 1
      GO TO 102
101   IF (TRY.LE.AMAX) GO TO 103
102   IX = I
      AMAX = TRY
103   CONTINUE
C   CALCULATE NORMALIZED MODE SHAPE
      AMAX = X(IX,IX)
      DO 104 I=1,N2
      V(I) = X(I,IX)/AMAX
104   CONTINUE
C   PRINT MODE SHAPE
      IF (IRUN.EQ.1) R = 60.0*R
      ITIME=1
      DO 500 I=1,NB
      N=N8(I)
      JT=K8(I)-2
      IF (JT.LE.2) GO TO 406
      IF (JT-5) 404,403,402
402   JT=JT-1
403   JT=JT-3
      IF (ITIME.EQ.2) GO TO 405
      ITIME=2
```

```
      W1=V(I)
      IF (IS.EQ.0) GO TO 500
      I1=I+NB
      W3=V(I1)
      GO TO 500
404   JT=JT-2
      GO TO 406
405   ITIME=1
      W2=V(I)
      IF (IS.EQ.0) GO TO 410
      I1=I+NB
      W4=V(I1)
      IF (I.GT.1) GO TO 425
      WRITE (KW,1011) R,POSIT(N),W1,W2,W3,W4
1011  FORMAT (20X,1PE11.4,5E15.4)
      GO TO 500
425   WRITE(KW,1001) POSIT(N),W1,W2,W3,W4
1001  FORMAT(31X,1P5E15.4)
      GO TO 500
406   IF (JT.EQ.2) GO TO 408
      IF (IS.EQ.1) GO TO 407
      IF (I.GT.1) GO TO 426
      WRITE (KW,1011) R,POSIT(N),V(I)
      GO TO 500
426   WRITE(KW,1001) POSIT(N),V(I)
      GO TO 500
407   W1=V(I)
      W3=V(I+NB)
      IF (I.GT.1) GO TO 427
      WRITE (KW,1012) R,POSIT(N),W1,W3
1012  FORMAT (20X,1PE11.4,2E15.4,15X,E15.4)
      GO TO 500
427   WRITE(KW,1002) POSIT(N),W1,W3
1002  FORMAT(31X,1P2E15.4,15X,E15.4)
      GO TO 500
408   IF (IS.EQ.1) GO TO 409
      IF (I.GT.1) GO TO 428
      WRITE (KW,1013) R,POSIT(N),V(I)
1013  FORMAT (20X,1PE11.4,2(E15.4,15X),E15 4)
      GO TO 500
428   WRITE(KW,1003) POSIT(N),V(I)
      GO TO 500
409   IF (I.GT.1) GO TO 429
      WRITE (KW,1011) R,POSIT(N),V(I),V(I+NB)
      GO TO 500
429   WRITE(KW,1003) POSIT(N),V(I),V(I+NB)
1003  FORMAT(16X,3(15X,1PE15.4))
      GO TO 500
410   IF (I.GT.1) GO TO 430
      WRITE (KW,1011) R,POSIT(N),W1,W2
      GO TO 500
430   WRITE(KW,1001) POSIT(N),W1,W2
500   CONTINUE
510   RETURN
      END
```

```
      SUBROUTINE MATPRT(M1,N1,M2,N2,DMAT)
      COMMON/WR/KW,KR
      DIMENSION DMAT(M2,N2)
      WRITE(KW,50)
      DO 10 I=1,M1
10    WRITE(KW,100) (DMAT(I,J),J=1,N1)
      WRITE(KW,150)
50    FORMAT(25H ***** START MATPRT *****//)
100   FORMAT(1X,1P10E13.5)
150   FORMAT(/25H *****  END  MATPRT *****)
      RETURN
      END
```

```
      SUBROUTINE RUNCON(KW,IR,IT,ID,NT)
      DIMENSION ID(4),NT(16)
      WRITE (KW,10) ID,NT
 10   FORMAT (1H1,30(1H*),19HCALCULATION SUMMARY,30(1H*)//1H ,20A4)
      IF (IR.EQ.0) GO TO 50
      GO TO (100,200,300,400,500,600), IR
 50   WRITE (KW,20)
 20   FORMAT(/54H RSVP SUPPLEMENTARY DATA AND ORIGINAL IMPEDANCE MATRIC,
     +2HES)
      GO TO 700
 100  WRITE (KW,110)
 110  FORMAT (/27H CRITICAL SPEED CALCULATION)
      GO TO 700
 200  WRITE (KW,210)
 210  FORMAT (/19H UNBALANCE RESPONSE)
      GO TO 700
 300  WRITE (KW,310)
 310  FORMAT (/35H ASYNCHRONOUS RESONANCE CALCULATION)
      GO TO 700
 400  WRITE (KW,410)
 410  FORMAT (/22H ASYNCHRONOUS RESPONSE)
      IF (IT.EQ.1) GO TO 420
      WRITE (KW,411)
 411  FORMAT (36H EXCITATION IS OF THE ROTATING TYPE.)
      GO TO 700
 420  WRITE (KW,421)
 421  FORMAT (26H EXCITATION IS STATIONARY.)
      GO TO 700
 500  WRITE (KW,510)
 510  FORMAT (/25H WHIRL STABILITY ANALYSIS)
      GO TO 700
 600  WRITE (KW,610)
 610  FORMAT (/32H TORSIONAL RESONANCE CALCULATION)
 700  RETURN
      END
```

```
      SUBROUTINE BRGTAB(KW,I,KBO,NB,KB,IT,KZ,T,B,O)
      DIMENSION  T(2,1),B(2,1),C(2,4)
      IF (NB-KZ) 1,3,2
    1 MB = KZ
      GO TO 4
    2 LB = KB
      MB = NB
      GO TO 6
    3 MB = NB
    4 LB = IT
      DO 5 IZ=1,2
      DO 5 JZ=1,2
      JZ2 = 2*JZ
      JZ1 = JZ2-1
      C(IZ,JZ1) = B(IZ,JZ)
      C(IZ,JZ2) = 0.0
      IF (KB NE.IT.OR.NB.NE.KZ) GO TO 5
      C(IZ,JZ1) = C(IZ,JZ1)+T(IZ,JZ1)
      C(IZ,JZ2) = C(IZ,JZ2)+T(IZ,JZ2)/O
    5 CONTINUE
      GO TO 8
    6 DO 7 IZ=1,2
      DO 7 JZ=1,4
      TO = T(IZ,JZ)
      JZ1 = 2*(JZ/2)
      IF (JZ1.EQ.JZ) TO = TO/O
      C(IZ,JZ) = TO
    7 CONTINUE
      GO TO (10,20), LB
      WRITE (KW,9) KB,I
    9 FORMAT (/35H ERROR MESSAGE **** VALUE OF KBRG (,I3,12H) AT BRG NO.
     +,I3,35H IS ILLEGAL. COMPUTATION IS HALTED.)
      CALL EXIT
    8 IF (LB.EQ.2) GO TO 20
   10 WRITE(KW,11) KBO,MB,((C(M,N),N=1,4),M=1,2)
   11 FORMAT (I3,I4,5X,1H*,12X,1P8E11.4)
      GO TO 100
   20 WRITE(KW,21) KBO,MB,((C(M,N),N=1,4),M=1,2)
   21 FORMAT (I3,I4,13X,1H*,4X,1P8E11.4)
  100 RETURN
      END
```

```
      SUBROUTINE ROTATE(N1)
      COMMON/BMAT/XLX(60,120),YLX(60,120),ZLX(60,120),
     +XHALF(20,40),YHALF(20,40),ZHALF(20,40),QHALF(20,40)
      COMMON/BINV/XIN(60,122),XOU(60,122),DLX(2),IROW(61),ICOL(61)
      DO 10 I=1,N1
      L = N1+I
      DO 10 J=1,N1
      M = N1+J
      J2 = 2*J
      J1 = J2-1
      M2 = 2*M
      M1 = M2-1
      ALX1 = 0.5*(XLX(I,J1)+XLX(L,M1))
      ALX2 = 0.5*(XLX(I,J2)+XLX(L,M2))
      BLX1 = -0.5*(XLX(L,J2)-XLX(I,M2))
      BLX2 = 0.5*(XLX(L,J1)-XLX(I,M1))
      CLX1 = ALX1-XLX(L,M1)
      CLX2 = ALX2-XLX(L,M2)
      DLX1 = BLX1-XLX(I,M2)
      DLX2 = BLX2+XLX(I,M1)
      YLX(I,J1) = ALX1+BLX1
      YLX(I,J2) = ALX2+BLX2
      YLX(I,M1) = CLX1+DLX1
      YLX(I,M2) = CLX2+DLX2
      YLX(L,J1) = CLX1-DLX1
      YLX(L,J2) = CLX2-DLX2
      YLX(L,M1) = ALX1-BLX1
   10 YLX(L,M2) = ALX2-BLX2
      RETURN
      END
```

```
      SUBROUTINE ORBTAB(KW,N,M,K,SP,FR,A,B,S,P,POSIT,IPN)
      DIMENSION M(1),K(1),A(1),B(1),S(1),P(1)
      DIMENSION POSIT(76)
      IF(IPN.EQ.1) WRITE(KW,10) SP,FR
 10   FORMAT(//,1X,14HRESPONSE ORBIT/19H ROTATIONAL SPEED =,1PE11.4,
     +4H RPM,6X,11HFREQUENCY =,1PE11.4,3H HZ//)
      WRITE(KW,101)
 101  FORMAT(33X,42H** PRINCIPAL RADII  ** INCLINATION   PHASE/
     +31H NO. STN  LOCATION  DISPL SLOPE,5X,
     +5HMAJOR,6X,5HMINOR,7X,5H(DEG),4X,9HREFERENCE,4X,11HELLIPTICITY)
      LO = 0
      DO 60 I=1,N
      ELP=(A(I)+B(I))/(A(I)-B(I))
      IF(ELP.GT.1.0) ELP=1.0/ELP
      IG=M(I)
      LF = K(I)-2
      GO TO (20,30,20,30,40,40),LF
 100  WRITE (KW,11) K(I),M(I)
 11   FORMAT (/33H ERROR MESSAGE **** VALUE OF KB (,I3,17H  AT STATION
     +I3,35H IS ILLEGAL. COMPUTATION IS HALTED )
      CALL EXIT
 20   WRITE(KW,21) I,M(I),POSIT(IG),A(I),B(I),S(I),P(I),ELP
 21   FORMAT(I3,I4,F10.4,5X,1H*,9X,2(1X,1P2E11.4),2X,1PE12.4)
      GO TO 60
 30   WRITE(KW,31) I,M(I),POSIT(IG),A(I),B(I),S(I),P(I),ELP
 31   FORMAT(I3,I4,F10.4,5X,1H*,9X,2(1X,1P2E11.4),2X,1PE12.4)
      GO TO 60
 40   LO = LO+1
      GO TO (20,50),LO
      GO TO 100
 50   LO = 0
      GO TO 30
 60   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XCITAB(KW,IT,NO,M,K,PLX)
      DIMENSION PLX(1)
      DIMENSION M(1),K(1)
      WRITE (KW,10)
10    FORMAT (////16H EXCITATION DATA)
      IF (IT.NE.0) GO TO 20
100   WRITE (KW,11)
11    FORMAT(/9X,4(1H*),6H TYPE ,4(1H*),2X,5(1H*),9H FORWARD ,2(1X,5(
     +1H*),1X),8HBACKWARD,1X,5(1H*))
      GO TO 25
20    WRITE (KW,21)
21    FORMAT(/9X,4(1H*),6H TYPE ,4(1H*),2X,5(1H*),10H VERTICAL ,5(1H*
     +),2X,4(1H*),12H HORIZONTAL ,4(1H*))
25    WRITE (KW,26)
26    FORMAT (23H NO. STN  FORCE  MOMENT,2(3X,8HIN-PHASE,6X,5HLAG  ))
      L0 = 0
      DO 70 I=1,NO
      I2 = 2*I
      I1 = I2-1
      L = NO+I
      L2 = 2*L
      L1 = L2-1
      LF = K(I)-2
      GO TO (40,50,40,50,60,60), LF
200   WRITE (KW,31) K(I),M(I)
31    FORMAT (/33H ERROR MESSAGE **** VALUE OF K8 (,I3,13H) AT STATION
     +I3,35H IS ILLEGAL. COMPUTATION IS HALTED )
      CALL EXIT
40    WRITE (KW,41) I,M(I),PLX(I1),PLX(I2),PLX(L1),PLX(L2)
41    FORMAT (I3,I4,5X,1H*,12X,1P4E11.4)
      GO TO 70
50    WRITE (KW,51) I,M(I),PLX(I1),PLX(I2),PLX(L1),PLX(L2)
51    FORMAT (I3,I4,12X,1H*,5X,1P4E11.4)
      GO TO 70
60    L0 = L0+1
      GO TO (40,65), L0
      GO TO 200
65    L0 = 0
      GO TO 50
70    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ORBIT(IT,B,S,T,P,ULX1,ULX2,VLX1,VLX2)
      R = 57.29578
      IF (IT.EQ.0) GO TO 1
      RLX1 = 0.5*ULX1
      RLX2 = 0.5*ULX2
      BLX1 =-0.5*VLX2
      BLX2 = 0.5*VLX1
      ALX1 = RLX1+BLX1
      ALX2 = RLX2+BLX2
      BLX1 = RLX1-BLX1
      BLX2 = RLX2-BLX2
      GO TO 10
1     ALX1 = ULX1
      ALX2 = ULX2
      BLX1 = VLX1
      BLX2 = VLX2
10    F = SQRT(ALX1*ALX1+ALX2*ALX2)
      B2 = BLX1*BLX1+BLX2*BLX2
      G = SQRT(B2)
      B = F+G
      S = F-G
      F = F/B
      G = G/B
      RLX1 = (ALX1*BLX1+ALX2*BLX2)/B2
      RLX2 = (ALX2*BLX1-ALX1*BLX2)/B2
      T = 0.5*R*ATAN2(RLX2,RLX1)
20    X = ALX1+BLX1
      Y = ALX2-BLX2
      P = R*ATAN2(Y,X)
      RETURN
      END
```

```
      SUBROUTINE ASNTAB(K,NF,IW,F,D,IS)
      DIMENSION IW(2,1),F(1),D(2,1)
      DO 50 I = 1,NF
      L = IW(1,I)
      M = L+IW(2,I)
      IF (L.EQ.1) GO TO 10
      IF (M.EQ.1) GO TO 20
      IF (IS.EQ.1) GO TO 5
      WRITE (K,100) F(I),(D(J,I),J=1,2)
      GO TO 50
5     WRITE (K,100) F(I),D(2,I)
      GO TO 50
10    WRITE (K,200) F(I),(D(J,I),J=1,2)
      GO TO 50
20    IF (M.EQ.2) GO TO 30
      WRITE (K,300) F(I),(D(J,I),J=1,2)
      GO TO 50
30    WRITE (K,400) F(I),(D(J,I),J=1,2)
50    CONTINUE
100   FORMAT (1PE11.4,2(5X,E11.4))
200   FORMAT (1PE11.4,2(5X,E11.4),5X,34HFORWARD WHIRL MOTION IS DEGENERA
     +TE)
300   FORMAT (1PE11.4,2(5X,E11.4),5X,35HBACKWARD WHIRL MOTION IS DEGENER
     +ATE)
400   FORMAT (1PE11.4,2(5X,E11 4),5X,33HBOTH WHIRL MOTIONS ARE DEGENERAT
     +E)
      RETURN
      END
```

```
      SUBROUTINE SYNTAB(K,NS,IW,S,D,ISO)
C
C     DOUBLE PRECISION REAL ALGEBRA VERSION         OCT 3, 1979
C
      DIMENSION IW(2,1),S(1),D(2,1)
      JSO=ISO+1
      DO 30 I=1,NS
      RPM = 60.0*S(I)
      L = IW(1,I)
      IF (L.EQ.1) GO TO 10
      WRITE(K,100) RPM,D(JSO,I)
      GO TO 30
10    WRITE(K,200) RPM,D(JSO,I)
30    CONTINUE
      RETURN
100   FORMAT (1PE11.4,5X,E11.4)
200   FORMAT (1PE11.4,5X,E11.4,5X,27HFORWARD WHIRL IS DEGENERATE)
      END
```

```
      SUBROUTINE INVC (N,IERR)
C
C     INVERSION OF COMPLEX MATRIX
C     XOU = INVERSE OF XIN
C     ORIGINAL XIN IS DESTROYED
C     N = RANK OF SYSTEM
C     IERR = ERROR FLAG, RETURNED AS -1 IF XIN IS SINGULAR
C     IROW = INTEGER ARRAY
C     ICOL = INTEGER ARRAY
C     DLX = COMPLEX DETERMINANT OF D
C
C     XOU,XIN,IROW,ICOL,DLX ARE ASSIGNED IN LABELED COMMON/BINV/
C
      COMMON/BINV/XIN(60,122),XOU(60,122),DLX(2),IROW(61),ICOL(61)
      IERR = 0
      DLX(1) = 1.0
      DLX(2) = 0.0
      N2 = 2*N
      DO 100 I=1,N
      DO 100 J=1,N2
100   XOU(I,J) = XIN(I,J)
      M = N+1
      M2 = 2*M
      M1 = M2-1
      DO 110 I=1,N
      IROW(I) = I
110   ICOL(I) = I
      DO 250 K=1,N
      K2 = 2*K
      K1 = K2-1
      AREA = XOU(K,K1)
      AIMA = XOU(K,K2)
      AMAX = AREA*AREA+AIMA*AIMA
      IC=K
      JC=K
      JC2 = K2
      JC1 = K1
      DO 130 I=K,N
      DO 130 J=K,N
      J2 = 2*J
      J1 = J2-1
      BREA = XOU(I,J1)
      BIMA = XOU(I,J2)
      BMAX = BREA*BREA+BIMA*BIMA
      IF(BMAX-AMAX) 130,120,120
120   AMAX = BMAX
      IC = I
      JC = J
      JC2 = J2
      JC1 = J1
130   CONTINUE
      IF (K.EQ.IC) GO TO 131
      KI = ICOL(K)
      ICOL(K) = ICOL(IC)
```

```
      ICOL(IC) = KI
  131 IF (K EQ.JC) GO TO 134
      KI = IROW(K)
      IROW(K) = IROW(JC)
      IROW(JC) = KI
  134 IF (AMAX) 150,140,150
  140 IERR = -1
      GO TO 320
  150 DO 160 J=1,N
      J2 = 2*J
      J1 = J2-1
      E1 = XOU(K,J1)
      E2 = XOU(K,J2)
      XOU(K,J1) = XOU(IC,J1)
      XOU(K,J2) = XOU(IC,J2)
      XOU(IC,J1) = E1
  160 XOU(IC,J2) = E2
      DO 170 I=1,N
      E1 = XOU(I,K1)
      E2 = XOU(I,K2)
      XOU(I,K1) = XOU(I,JC1)
      XOU(I,K2) = XOU(I,JC2)
      XOU(I,JC1) = E1
  170 XOU(I,JC2) = E2
      DO 200 I=1,N
      IF (I-K) 190,180,190
  180 XOU(I,M1) = 1.0
      XOU(I,M2) = 0.0
      GO TO 200
  190 XOU(I,M1) = 0.0
      XOU(I,M2) = 0.0
  200 CONTINUE
      PVT = XOU(K,K1)
      QVT = XOU(K,K2)
      U = PVT
      V = QVT
      IF (K EQ IC.AND.K.EQ.JC.OR K.NE IC.AND K.NE.JC) GO TO 205
      U = -U
      V = -V
  205 W = U*U+V*V
      PVT = PVT/W
      QVT = -QVT/W
      U1 = DLX(1)*U-DLX(2)*V
      U2 = DLX(1)*V+DLX(2)*U
      DLX(1) = U1
      DLX(2) = U2
      DO 210 J=1,M
      J2 = 2*J
      J1 = J2-1
      AREA = XOU(K,J1)
      AIMA = XOU(K,J2)
      XOU(K,J1) = AREA*PVT-AIMA*QVT
  210 XOU(K,J2) = AREA*QVT+AIMA*PVT
      DO 240 I=1,N
```

```
      IF(I-K) 220,240,220
220   BREA = XOU(I,K1)
      BIMA = XOU(I,K2)
      DO 230 J=1,M
      J2 = 2*J
      J1 = J2-1
      XOU(I,J1) = XOU(I,J1)-BREA*XOU(K,J1)+BIMA*XOU(K,J2)
230   XOU(I,J2) = XOU(I,J2)-BREA*XOU(K,J2)-BIMA*XOU(K,J1)
240   CONTINUE
      DO 250 I=1,N
      XOU(I,K1) = XOU(I,M1)
250   XOU(I,K2) = XOU(I,M2)
      DO 280 I=1,N
      DO 260 L=1,N
      IF(IROW(I)-L) 260,270,260
260   CONTINUE
270   DO 280 J=1,N2
280   XIN(L,J) = XOU(I,J)
      DO 310 J=1,N
      J2 = 2*J
      J1 = J2-1
      DO 290 L=1,N
      L2 = 2*L
      L1 = L2-1
      IF(ICOL(J)-L) 290,300,290
290   CONTINUE
300   DO 310 I=1,N
      XOU(I,L1) = XIN(I,J1)
310   XOU(I,L2) = XIN(I,J2)
320   RETURN
      END
```

```
      SUBPOUTINE MOUTC (A,L,M,NRA,IR,IC,IRLAB,ICLAB,LINE,KW)
C                            A IS COMPLEX ARRAY
C                              REAL PARTS ARE IN ODD COLUMNS
C                              IMAGINARY PARTS ARE IN EVEN COLUMNS
C                            IR IS ARRAY OF INDICES FOR ROW (USED WHEN IR=1)
C                            IC IS ARRAY OF INDICES FOR COL (USED WHEN IC=1)
C                            NRA IS DIMENSIONED NO OF ROWS IN A
C                            OUTPUT FORM  RE,IM     X XXXE XX,  X XXXE XX
C                            LINE IS LINE NO ON PAGE OF FIRST OUTPUT LINE
      DIMENSION A(NRA,1)
      DIMENSION IRLAB( 1),ICLAB( 1)
      IC1 = 1
      IA1 = 1
      IC2 = MINO(5,M)
      IA2 = 2*IC2
      LINE=LINE+2
  100 IL1 = 1
      IL2 = MINO (58-LINE,L)
      IF (IL2.NE L)  GO TO 195
  110 IF (IC.EQ 1) GO TO 120
      WRITE (KW,130) (I,I=IC1,IC2)
      GO TO 140
  120 WRITE (KW,130) (ICLAB(I),I=IC1,IC2)
  130 FORMAT (/ I23,4I24/)
  140 DO 170 I=IL1,IL2
      IF (IR.EQ.1) GO TO 150
      WRITE (KW,160) I,(A(I,J),J=IA1,IA2)
      GO TO 170
  150 WRITE (KW,160) IRLAB(I),(A(I,J),J=IA1,IA2)
  160 FORMAT (I6,4X,1P5(E12.3,1H,.E11.3))
  170 CONTINUE
      IF (I.GE.L) GO TO 190
      IL1 = IL2+1
      IL2 = MINO (IL2+58,L)
      WRITE (KW,180)
  180 FORMAT(/)
      GO TO 110
  190 IF (IC2.GE.M ) GO TO 200
      IC1 = IC2+1
      IA1 = IA2+1
      IC2 = MINO (IC2+5,M)
      IA2 = 2*IC2
      LINE=LINE+2
      GO TO 100
  195 WRITE (KW,185)
  185 FORMAT(1H1)
      LINE=0
      GO TO 100
  200 RETURN
      END
```

```
      SUBROUTINE KRAUTE(NF,NB,KS,KE,KW,IT,IS,F,Y,IRUN,TABLX,PLX,WLX,
     +BIG,SMALL,SLANT,PHASE,JDG)
      DIMENSION TABLX(60,1),PLX(1),WLX(1)
      DIMENSION ALX(22),AOLX(22),AA2LX(22),DOLX(20),D1LX(2,22),ULX(22),
     +Y1LX(22),ZZLX(6),DETLX(2,22)
      DIMENSION F(1),Y(1),M8(20),K8(20),BIG(1),SMALL(1)
      DIMENSION X(40,40),POSIT(76),RET(11),XINT(11)
      DIMENSION A1(10,10),B1(10,10),AC5(10),AM8(10),AN9(10),B(10),EO(10)
     +,Q2(11),Z1(3,10),Z2(3,10),FSQ(11),SLANT(1),PHASE(1),AM7(10)
      DIMENSION IRLAB(40),ICLAB(40)
      COMMON/BMAT/XLX(60,120),YLX(60,120),ZLX(60,120),
     +XHALF(20,40),YHALF(20,40),ZHALF(20,40),QHALF(20,40)
      COMMON/BINV/XIN(60,122),XOU(60,122),DLX(2),IROW(61),ICOL(61)
      COMMON/WUS/M8,K8,X,POSIT
      COMMON/CODA/K9(20)
      COMMON/WUSA/RET,XINT
      COMMON/XGCON/SLOW,RLAX,IGEN
      COMMON/WR/LW,LR
      COMMON/CBEG/IBEG
      LINE = 0
      N40 = 40
      M6=60
      JR=1
      IF (IGEN) 1,810,2
1     K5MAX =-IGEN
      GO TO 3
2     K5MAX = IGEN
3     CONTINUE
C     CREATE MATRIX INTERPOLATION FILE
      MF = NF-1
      MF2 = 2*MF
      N1 = 1
      N2 = NB
      N10 = N1+NB
      N20 = N2+NB
      N202 = 2*N20
      ISP = -1
10    DO 60 I=N1,N2
      DO 30 J=I,N2
      J2 = 2*J
      J1 = J2-1
      REWIND KS
      DO 15 K=1,NF
      K2 = 2*K
      K1 = K2-1
      FSQ(K)=6.283185307**2*(F(K)*F(K))
      READ (KS) SPD,FRE,((YLX(IA,JA),JA=1,N202),IA=1,N20)
      IF((FRE-F(K)).LE.1.E-5) GO TO 14
      WRITE(KW,1000) K
1000  FORMAT(44H1FAULTY DATA IN FILE (KS) AT FREQUENCY POINT,I3)
      REWIND KS
      REWIND KE
      CALL EXIT
14    IF(IRUN .EQ. 2 .OR. IRUN .EQ 4) READ (KS) (PLX(IA),IA=1,N202)
```

```
      ULX(K1) = YLX(I,J1)
15    ULX(K2) = YLX(I,J2)
      DO 20 K=1,NF
      K1 = 2*K-1
20    Y(K) = ULX(K1)
      CALL SPLINE(ISP,NF,RET,XINT,F,Y,Z1,KW,IER)
      IF (IER.NE.0) GO TO 806
      DO 25 K=1,NF
      K2 = 2*K
25    Y(K) = ULX(K2)
      IER = 0
      CALL SPLINE(ISP,NF,RET,XINT,F,Y,Z2,KW,IER)
      IF (IER.NE.0) GO TO 806
      WRITE (KE) ((ULX(K),K=1,MF2),((Z1(IA,K),Z2(IA,K)),IA=1,3),K=1,MF)
30    CONTINUE
      IF (IS.EQ.0) GO TO 60
      REWIND KS
      DO 45 K=1,NF
      K2 = 2*K
      K1 = K2-1
      READ (KS) SPD,FRE,((YLX(IA,JA),JA=1,N202),IA=1,N20)
      IF((FRE-F(K)).LE.1.E-3) GO TO 34
      WRITE(KW,1000) K
      REWIND KS
      REWIND KE
      CALL EXIT
34    IF(IRUN.EQ.2 .OR. IRUN.EQ.4) READ (KS) (PLX(IA),IA=1,N202)
      IF (N1.EQ.1) GO TO 40
      J = I-NB
      GO TO 42
40    J = I+NB
42    J2 = 2*J
      J1 = J2-1
      ULX(K1) = YLX(I,J1)
45    ULX(K2) = YLX(I,J2)
      DO 50 K=1,NF
      K1 = 2*K-1
50    Y(K) = ULX(K1)
      IER = 0
      CALL SPLINE(ISP,NF,RET,XINT,F,Y,Z1,KW,IER)
      IF (IER.NE.0) GO TO 806
      DO 55 K=1,NF
      K2 = 2*K
55    Y(K) = ULX(K2)
      IER = 0
      CALL SPLINE(ISP,NF,RET,XINT,F,Y,Z2,KW,IER)
      IF (IER.NE.0) GO TO 806
      WRITE (KE) ((ULX(K),K=1,MF2),((Z1(IA,K),Z2(IA,K)),IA=1,3),K=1,MF)
60    CONTINUE
      IF (N1.EQ.N10) GO TO 65
      N1 = N10
      N2 = N20
      GO TO 10
65    IROT = 1+IS
```

```
C     BEGIN ITERATION FOR EIGENVALUE
C     K05M       NUMBER OF EIGEN ROOTS FOUND
C     K05        ROOT NUMBER BEING SOUGHT
100   K05M = 0
      KBEG = 0
      JBEG = 0
101   IF (IBEG.EQ.0.AND.JBEG.EQ.0) GO TO 105
102   IF (KBEG.GE.K5MAX) GO TO 5001
      KBEG = KBEG+1
      IF (IBEG.EQ.0) GO TO 102
      KP19 = 1
      READ (LR,3000) P10,C5
3000  FORMAT (2D20.4)
105   K05 = K05M+1
      K052 = 2*K05
      K051 = K052-1
      IDOWN = 0
      KDOWN = 0
      RESOLD = 0.0
      P7 = 0.
      KP9 = 0
      IF (IBEG.NE.0) GO TO 120
      C5 = 0
      KP19=0
C     COMPUTE DETERMINANTS AT CONTROL FREQUENCIES
120   CONTINUE
      REWIND KS
      K = 0
121   K=K+1
      K2 = 2*K
      K1 = K2-1
      READ (KS) SPD,FRE,((YLX(I,J),J=1,N202),I=1,N20)
      IF(IRUN.EQ.2.OR.IRUN.EQ.4) READ(KS)(PLX(IQ),IQ=1,N202)
      C5K = C5/F(K)
      DO 160 L=1,2
        IF (IS.EQ.1) GO TO 130
        IF (IROT.NE.L) GO TO 160
130     MB = (L-1)*NB
        LB = (2*L-3)*NB
        DO 140 I=1,NB
          IO = I+MB
          I2 = I+NB
          DO 135 J=1,NB
            J0 = J+MB
            J02 = 2*J0
            J01 = J02-1
            J1 = J0-LB
            J12 = 2*J1
            J11 = J12-1
            TABLX(IO,J01) = YLX(IO,J01)/FSQ(K)
            TABLX(IO,J02) = YLX(IO,J02)/FSQ(K)
            IF (I.EQ.J) TABLX(IO,J02) = TABLX(IO,J02)-C5K
            IF (IS.EQ.0) GO TO 135
            TABLX(IO,J11)=YLX(IO,J11)/FSQ(K)
```

```
                  TABLX(IO,J12)=YLX(IO,J12)/FSQ(K)
 135            CONTINUE
 140          CONTINUE
 160        CONTINUE
            IPID = 0
            CALL PINVC(IPID,KW,NB,N40,N40,IS,IROT,TABLX,JDG)
 C          FACTORIZATION OF KNOWN EIGENVALUES
            IF (K05.EQ.1) GO TO 205
            K17 = 0
            A4LX1 = 0.0
            A4LX2 = 0.0
            A1LX1 = 1.0
            A1LX2 = 0.0
            DO 195 K07=1,K05M
            K072 =2*K07
            K071 = K072-1
            Q = Q2(K07)
            XQ = F(K)*F(K)
            U = AM8(K07)
            SIZEB = E0(K07)/XQ
            AX = SIZEB-U
            AY = AM7(K07)/XQ-C5/F(K)
            SIZEA = AX*AX+AY*AY
            SIZEB = SIZEB*SIZEB*2.5D-08*RLAX
            IF (SIZEA.GE.SIZEB) GO TO 180
            WRITE (KW,1010) K07
 1010       FORMAT (/44H TRUNCATION UNCERTAINTY ENCOUNTERED AT ROOT ,I2,1H )
            K17 = K07
            A4LX1 = 0.0
            A4LX2 = 0.0
            A1LX1 = 1.0
            A1LX2 = 0.0
            DLX(1) =-Q*D1LX(1,K072)
            DLX(2) = Q*D1LX(1,K071)
            GO TO 195
 180        B2LX1 = AX
            B2LX2 = AY
            A1LX0 = A1LX1*B2LX1-A1LX2*B2LX2
            A1LX2 = A1LX1*B2LX2+A1LX2*B2LX1
            A1LX1 = A1LX0
            B2LX0 = B2LX1*B2LX1+B2LX2*B2LX2
            A4LX1 = A4LX1+B2LX2/B2LX0
            A4LX2 = A4LX2+B2LX1/B2LX0
 195        CONTINUE
            A1LX0 =A1LX1*A1LX1+A1LX2*A1LX2
            A0LX(K1) = A1LX1/A1LX0
            A0LX(K2) =-A1LX2/A1LX0
            AA2LX(K1) = A4LX1/F(K)
            AA2LX(K2) = A4LX2/F(K)
            DRE = A0LX(K1)*DLX(1)-A0LX(K2)*DLX(2)
            DIM = A0LX(K1)*DLX(2)+A0LX(K2)*DLX(1)
            DLX(1) = DRE
            DLX(2) = DIM
 205        DETLX(IROT,K1) = DLX(1)
```

```
      DETLX(IROT,K2) = DLX(2)
      IF (K.LT.NF) GO TO 121
C     DETERMINANT INTERPOLATION
      ISP = 1
      IF(KP19.EQ.1) GO TO 228
      IF (KBEG.EQ.0.AND.IBEG.EQ.-1) GO TO 228
      DO 226 K=1,NF
      K2 = 2*K
      K1 = K2-1
226   Y(K) = ALOG10((DETLX(IROT,K1)**2+DETLX(IROT,K2)**2)*10.0+1.0)
      NI=100
      IER = 0
      CALL SPLINE(NI,NF,RET,XINT,F,Y,Z1,KW,IER)
      IF (IER.NE.0) GO TO 806
      IF(NI.EQ.1.AND.RET(1).GE.F(1).AND.RET(1).LE F(NF)) GO TO 227
      P10 = SQRT(F(1)*F(NF))
      GO TO 228
227   P10=RET(1)
228   DO 230 K=1,NF
      K2 = 2*K
      K1 = K2-1
      Y1LX(K1) = DETLX(IROT,K1)
230   Y(K) = Y1LX(K1)
      XINT(1) = P10
      IER = 0
      CALL SPLINE(ISP,NF,RET,XINT,F,Y,Z1,KW,IER)
      IF (IER.NE.0) GO TO 806
      P7 = RET(1)
      DO 235 K=1,NF
      K2 = 2*K
      Y1LX(K2) = DETLX(IROT,K2)
235   Y(K) = Y1LX(K2)
      IER = 0
      CALL SPLINE(ISP,NF,RET,XINT,F,Y,Z2,KW,IER)
      IF (IER.NE.0) GO TO 806
      R4LX1 = P7
      R4LX2 = RET(1)
C  LOCATE FREQUENCY INTERVAL
      K0=1
      DO 1115 IK=1,MF
      IF (P10.GE.F(IK)) GO TO 1115
      K0=IK-1
      GO TO 236
1115  CONTINUE
      K0=MF
236   DF=P10-F(K0)
      DO 240 K=1,3
      K2 = 2*K
      K1 = K2-1
      ZZLX(K1) = Z1(K,K0)
240   ZZLX(K2) = Z2(K,K0)
      R5LX1 = ZZLX(1)
      R5LX2 = ZZLX(2)
      D0LX(K051) = 2.0*ZZLX(3)
```

```
      DOLX(K052) = 2.0*ZZLX(4)
      IF (DF.EQ.0.) GO TO 241
      R5LX1 = R5LX1+DF*(2.0*ZZLX(3)+3.0*DF*ZZLX(5))
      R5LX2 = R5LX2+DF*(2.0*ZZLX(4)+3.0*DF*ZZLX(6))
      DOLX(K051) = DOLX(K051)+6.0*DF*ZZLX(5)
      DOLX(K052) = DOLX(K052)+6.0*DF*ZZLX(6)
241   CONTINUE
      RESNEW = R4LX1**2+R4LX2**2
      IF (KP9.EQ.0) GO TO 243
      RESRAT = RESNEW/RESOLD
      IF (IDOWN.EQ.1) GO TO 242
      IF (RESNEW.LE.RESOLD) GO TO 243
      KDOWN = KDOWN+1
      IF (KDOWN.EQ.4) GO TO 393
      IDOWN = 1
      GO TO 243
242   IF (RESNEW.LE.RESOLD) IDOWN = 0
243   RESOLD = RESNEW
C     COMPUTE C-DERIVATIVES OF DETERMINANTS AT CONTROL FREQUENCIES
      REWIND KS
      K = 0
245   K = K+1
      K2=2*K
      K1=K2-1
      READ (KS) SPD,FRE,((YLX(I,J),J=1,N202),I=1,N20)
      IF(IRUN.EQ.2.OR.IRUN.EQ 4) READ(KS) (PLX(IQ),IQ=1,N202)
      P5RE = 0.0
      P5IM = 0 0
      C5K = C5/F(K)
      X1 = 1./F(K)
      DO 330 N=1,2
      IF (IS.EQ.1) GO TO 252
      IF (IROT.NE.N) GO TO 330
252   MA=(N-1)*NB
      DO 325 M=1,NB
      MX=M+MA
      MX2 = 2*MX
      MX1 = MX2-1
      DO 310 L=1,2
      IF (IS.EQ.1) GO TO 255
      IF (IROT.NE.L) GO TO 310
255   MB = (L-1)*NB
      LB = (3-2*L)*NB
      DO 280 I=1,NB
      I0 = I+MB
      I2 = I+NB
      DO 275 J=1,NB
      J0 = J+MB
      J02 = 2*J0
      J01 = J02-1
      J1 = J0+LB
      J12 = 2*J1
      J11 = J12-1
      IF (I.EQ.J) GO TO 265
```

```
      TABLX(IO,J01) = YLX(IO,J01)/FSQ(K)
      TABLX(IO,J02) = YLX(IO,J02)/FSQ(K)
      IF (IS.EQ.0) GO TO 275
      TABLX(IO,J11) = 0.0
      TABLX(IO,J12) = 0.0
      GO TO 275
265   IF (N.NE L.OR.I.NE.M) GO TO 270
      TABLX(IO,J01) = 1.0
      TABLX(IO,J02) = 0.0
      GO TO 272
270   TABLX(IO,J01) = YLX(IO,J01)/FSQ(K)
      TABLX(IO,J02) = YLX(IO,J02)/FSQ(K)-C5K
272   IF (IS.EQ.0) GO TO 275
      TABLX(IO,J11) = YLX(IO,J11)/FSQ(K)
      TABLX(IO,J12) = YLX(IO,J12)/FSQ(K)
275   CONTINUE
260   CONTINUE
310   CONTINUE
      IPID = 0
      CALL PINVC(IPID,KW,NB,N40,N40,IS,IROT,TABLX,JDG)
      P5RE = P5RE+DLX(1)
      P5IM = P5IM+DLX(2)
      TABLX(MX,MX1) = 0.0
      TABLX(MX,MX2) = 0.0
      IPID = 0
      CALL PINVC(IPID,KW,NB,N40,N40,IS,IROT,TABLX,JDG)
      P5RE = P5RE-DLX(1)
      P5IM = P5IM-DLX(2)
325   CONTINUE
330   CONTINUE
      DLX(1) = X1*P5IM
      DLX(2) =-X1*P5RE
C     FACTORIZATION OF KNOWN EIGENVALUES
      IF (K05.EQ.1) GO TO 340
      IF (K17.EQ.0) GO TO 335
      Q = Q2(K17)
      DLX(1) = (R5LX1-R4LX1/Q)/Q
      DLX(2) = (R5LX2-R4LX2/Q)/Q
      K172 = 2*K17
      K171 = K172-1
      DLX(1) = DLX(1)+D1LX(2,K172)
      DLX(2) = DLX(2)-D1LX(2,K171)
      CONST = 2.0*AM8(K17)/Q
      DRE = DLX(2)/CONST
      DIM =-DLX(1)/CONST
      DLX(1) = A0LX(K1)*DRE-A0LX(K2)*DIM+AA2LX(K1)*Y1LX(K1)
     +                                 -AA2LX(K2)*Y1LX(K2)
335   DLX(2) = A0LX(K1)*DIM+A0LX(K2)*DRE+AA2LX(K1)*Y1LX(K2)
     +                                 +AA2LX(K2)*Y1LX(K1)
340   Y1LX(K1) = DLX(1)
      Y1LX(K2) = DLX(2)
      IF (K.LT.NF) GO TO 245
C     C-DERIVATIVE INTERPOLATION
      DO 345 K=1,NF
```

```
      K1 = 2*K-1
345   Y(K) = Y1LX(K1)
      XINT(1)=P10
      IER = 0
      CALL SPLINE(ISP,NF,RET,XINT,F,Y,Z1,KW,IER)
      IF (IER.NE.0) GO TO 806
      R6 = RET(1)
      DO 350 K=1,NF
      K2 = 2*K
350   Y(K) = Y1LX(K2)
      IER = 0
      CALL SPLINE(ISP,NF,RET,XINT,F,Y,Z2,KW,IER)
      IF (IER.NE.0) GO TO 806
      D1LX(1,K051) = R6
      D1LX(1,K052) = RET(1)
      DO 355 K=1,3
      K2 = 2*K
      K1 = K2-1
      ZZLX(K1) = Z1(K,K0)
355   ZZLX(K2) = Z2(K,K0)
      IF (DF.EQ.0.) GO TO 356
      ZZLX(1) = ZZLX(1)+DF*(2.0*ZZLX(3)+3.0*DF*ZZLX(5))
      ZZLX(2) = ZZLX(2)+DF*(2.0*ZZLX(4)+3.0*DF*ZZLX(6))
356   D1LX(2,K051) = ZZLX(1)
      D1LX(2,K052) = ZZLX(2)
C     ESTIMATE RESIDUE
      D1LX0 = D1LX(1,K051)**2+D1LX(1,K052)**2
      DLX(1) = (R4LX1*D1LX(1,K051)+R4LX2*D1LX(1,K052))/D1LX0
      DLX(2) = (R4LX2*D1LX(1,K051)-R4LX1*D1LX(1,K052))/D1LX0
      WNUV1 = (R5LX1*D1LX(1,K051)+R5LX2*D1LX(1,K052))/D1LX0
      WNUV2 = (R5LX2*D1LX(1,K051)-R5LX1*D1LX(1,K052))/D1LX0
      P8 =-DLX(2)/WNUV2
      C9 =-DLX(1)-WNUV1*P8
      E1 = (P8*P8+C9*C9)/(P10*P10+C5*C5)
      KP9 = KP9+1
      IF(ABS(C9).GE.100.0.AND.E1.GT.1.0E-03) C9=C9*0.01
      E1 = E1/RLAX
      Q3=P10
      KP19=0
      Q2(K05) = P10
      AC5(K05) = C5
      AM8(K05) =-WNUV2/2.0
      E0(K05) = AM8(K05)*P10*P10
      AM7(K05) = (P10*WNUV1+C5)*P10/2.0
      IF (ABS(WNUV2).LT.1.0E-20) GO TO 364
      B(K05) =-C5/WNUV2/P10
      AM9(K05) = (C5/P10-WNUV1)/WNUV2/2.0
364   IF (IGEN.GE.0.OR.KP9.NE 1) GO TO 3641
      WRITE(KW,1091)
1091  FORMAT(1H1)
      WRITE (KW,1060)
1060  FORMAT (/49H I     FREQ        DAMP      MODAL MASS DAMP RATIO
     +41HF-CHANGE   C-CHANGE    RESIDUE     D CHECK)
3641  IF (IGEN.LT.0) WRITE (KW,1061) KP9,P10,C5,AM8(K05),B(K05),
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```
           +P8,C9,RESOLD,AM9(K05)
     1061  FORMAT (I3,1P9D11.3)
           IF(E1.GT.1 0E-12.OR.RESRAT GT 1.0E-02) GO TO 365
           K05M = K05
           GO TO 395
     C     TWO-PARAMETER NEWTON-RAPHSON ITERATION
     365   IF (KP9.GE.20) GO TO 394
           PNEW = P10+P8
           P10=PNEW
           IF(PNEW-F(1)) 370,382,380
     C  LOWER-BOUND RESET
     370   IF(Q3.EQ.F(1)) GO TO 371
           IF(Q3.EQ.F(NF)) GO TO 371
           C9=C9*(F(1)-Q3)/P8
           P10=F(1)
           IF (IGEN.GT.0) GO TO 382
           WRITE(KW,1080)
     1080  FORMAT(/18H LOWER-BOUND RESET)
     382   KP19=1
     371   C5=C5+C9*SLOW
           GO TO 120
     380   IF(PNEW-F(NF)) 382,382,385
     385   IF(Q3.EQ.F(NF)) GO TO 371
           IF(Q3.EQ.F(1)) GO TO 371
           C9=C9*(F(NF)-Q3)/P8
           P10=F(NF)
           IF (IGEN GT.0) GO TO 382
           WRITE(KW,1090)
     1090  FORMAT(/18H UPPER-BOUND RESET)
           GO TO 382
     393   IF (IGEN.LT.0) WRITE (KW,1085) KP9,K05
     1085  FORMAT(/27HOITERATION OSCILLATES AFTER,I3,17H TRIALS FOR ROOT#,I3)
           GO TO 500
     3931  KDOWN = 0
           IDOWN = 0
           GO TO 243
     394   IF (IGEN.LT.0) WRITE (KW,499)
     499   FORMAT (//45H AFTER 20 ITERATIONS, NO EIGENVALUE IS FOUND /)
           GO TO 500
     C     COMPUTE FACTORIZATION FUNCTIONS
     395   IF(K05.GE.K5MAX) GO TO 5001
           DO 400 K=1,NF
           R9 = (F(K)-Q2(K05M))*(F(K)+Q2(K05M))/FSQ(K)*39.478417
           A1(K05M,K) = R9*AM8(K05M)
     400   CONTINUE
           GO TO 101
     C     COMPUTE EIGENVECTORS
     500   JBEG = 1
           GO TO 101
     5001  IF (IS.EQ.1) GO TO 503
           IF (IROT.EQ.2) GO TO 502
           WRITE(KW,1092) K05M
     1092  FORMAT (1H1,32HNUMBER OF CO -ROTATIONAL MODES =,I3)
           GO TO 503
```

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502   WRITE(KW,1093) K05M
1093  FORMAT (1H1,32HNUMBER OF CTR-ROTATIONAL MODES =,I3)
503   CONTINUE
      IF (K05M.EQ.0) GO TO 805
C          REORDER EIGENVALUES
      DO 510 M=1,K05M
      IF (M.EQ.K05M) GO TO 510
      AMIN = Q2(M)
      INDEX = M
      MP1 = M+1
      DO 505 N=MP1,K05M
      IF (AMIN.LE.Q2(N)) GO TO 505
      INDEX = N
      AMIN = Q2(N)
505   CONTINUE
      IF (INDEX.EQ.M) GO TO 510
      SWAP = Q2(M)
      Q2(M) = Q2(INDEX)
      Q2(INDEX) = SWAP
      SWAP = B(M)
      B(M) = B(INDEX)
      B(INDEX) = SWAP
      SWAP = AM8(M)
      AM8(M) = AM8(INDEX)
      AM8(INDEX) = SWAP
      SWAP = AM9(M)
      AM9(M)=AM9(INDEX)
      AM9(INDEX)=SWAP
      SWAP = AM7(M)
      AM7(M)=AM9(INDEX)
      AM7(INDEX)=SWAP
      SWAP=E0(M)
      E0(M)=E0(INDEX)
      E0(INDEX)=SWAP
510   CONTINUE
      DO 800 M=1,K05M
C         LOCATE FREQUENCY INTERVAL
      DO 515 K=2,MF
      IF (Q2(M).LT.F(K)) GO TO 520
515   CONTINUE
      K0 = MF
      GO TO 525
520   K0 = K-1
525   K02 = 2*K0
      K01 = K02-1
      DF = Q2(M)-F(K0)
C         CALCULATE INTERPOLATED MATRIX ELEMENTS
      REWIND KE
      C50=Q2(M)*AC5(M)
      N1 = 1
      N2 = NB
      DO 576 L=1,2
      IF(IS.EQ.0.AND.L.NE.IROT) GO TO 576
      MB = (L-1)*NB
```

```
      LB = (2*L-3)*NB
      DO 575 I=N1,N2
      I0 = I+MB
      I02 = 2*I0
      I01 = I02-1
      I1 = I0-LB
      I12 = 2*I1
      I11 = I12-1
      DO 570 J=I,N2
      J0 = J+MB
      J02 = 2*J0
      J01 = J02-1
      J1 = J0-LB
      J12 = 2*J1
      J11 = J12-1
      READ (KE) ((ULX(K),K=1,MF2),((Z1(IA,K),Z2(IA,K)),IA=1,3),K=1,MF)
      A1LX1 = 0.0
      A1LX2 = 0.0
      IF (DF.EQ.0.) GO TO 555
      DO 550 K=1,3
      K1 = 4-K
      A1LX1 = DF*(A1LX1+Z1(K1,K0))
550   A1LX2 = DF*(A1LX2+Z2(K1,K0))
555   TABLX(I0,J01) = A1LX1+ULX(K01)
      TABLX(I0,J02) = A1LX2+ULX(K02)
      YLX(I0,J01) = TABLX(I0,J01)
      IF (I.NE.J) GO TO 557
      YLX(I0,J02) = TABLX(I0,J02)-C50
      GO TO 570
557   YLX(I0,J02) = TABLX(I0,J02)
      TABLX(J0,I01) = TABLX(I0,J01)
      TABLX(J0,I02) = TABLX(I0,J02)
      YLX(J0,I01) = YLX(I0,J01)
      YLX(J0,I02) = YLX(I0,J02)
      TABLX(I0,J11) = 0.0
      TABLX(I0,J12) = 0.0
      TABLX(J0,I11) = 0.0
      TABLX(J0,I12) = 0.0
      YLX(I0,J11) = 0.0
      YLX(I0,J12) = 0.0
      YLX(J0,I11) = 0.0
      YLX(J0,I12) = 0.0
570   CONTINUE
      IF (IS.EQ.0) GO TO 575
      READ (KE) ((ULX(K),K=1,MF2),((Z1(IA,K),Z2(IA,K)),IA=1,3),K=1,MF)
      A1LX1 = 0.0
      A1LX2 = 0.0
      IF (DF.EQ.0 ) GO TO 574
      DO 573 K=1,3
      K1 = 4-K
      A1LX1 = DF*(A1LX1+Z1(K1,K0))
573   A1LX2 = DF*(A1LX2+Z2(K1,K0))
574   TABLX(I0,I11) = A1LX1+ULX(K01)
      TABLX(I0,I12) = A1LX2+ULX(K02)
```

```
      YLX(IO,I11) = TABLX(IO,I11)
      YLX(IO,I12) = TABLX(IO,I12)
575   CONTINUE
576   CONTINUE
      IF (JDG.NE.6) GO TO 578
      DO 577 I=1,N20
      IROW(I) = I
      ICOL(I) = I
577   CONTINUE
      WRITE (KW,1001)
1001  FORMAT (23H1MODE SHAPE DIAGNOSTICS//19H MODIFIED IMPEDANCE)
      CALL MOUTC(YLX,N20,N20,M6,JR,JR,IROW,ICOL,LINE,KW)
C                CALCULATE CHARACTERISTIC VECTOR
C   INVERT MODIFIED IMPEDANCE
578   N2 = NB+IS*NB
      ID=1
      CALL PINVC(ID,KW,NB,N40,N40,IS,IROT,YLX,JDG)
      IF (JDG.NE.6) GO TO 580
      DO 579 I=1,N20
      IROW(I) = I
      ICOL(I) = I
579   CONTINUE
      WRITE (KW,1002)
1002  FORMAT (//30H INVERSE OF MODIFIED IMPEDANCE)
      CALL MOUTC(YLX,N20,N20,M6,JR,JR,IROW,ICOL,LINE,KW)
      WRITE (KW,1091)
C   SEARCH FOR LARGEST ELEMENT
580   KMAX = 0
      DO 606 K=1,N2
      DO 606 I=1,N2
      I2 = 2*I
      I1 = I2-1
      IF (K9(I).EQ.2) GO TO 606
      TRY = YLX(K,I1)**2+YLX(K,I2)**2
      IF (KMAX.EQ.1) GO TO 602
      KMAX = 1
      GO TO 604
602   IF (TRY.LE.AMAX) GO TO 606
604   AMAX = TRY
      IX = I
      KX = K
606   CONTINUE
C          NORMALIZATION
      IX2 = 2*IX
      IX1 = IX2-1
      DLX(1) = YLX(KX,IX1)
      DLX(2) = YLX(KX,IX2)
      DLO = DLX(1)**2+DLX(2)**2
      LB = (2*IROT-3)*NB
      MB = (IROT-1)*NB
      DO 720 I=1,N2
      IO = I
      IF (IS.EQ.1) GO TO 718
      IO = IO+MB
```

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      I1 = I0-LB
      I12 = 2*I1
      I11 = I12-1
      WLX(I11) = 0.0
      WLX(I12) = 0.0
718   I02 = 2*I0
      I01 = I02-1
      WLX(I01) = (YLX(I0,IX1)*DLX(1)+YLX(I0,IX2)*DLX(2))/DLO
      WLX(I02) = (YLX(I0,IX2)*DLX(1)-YLX(I0,IX1)*DLX(2))/DLO
720   CONTINUE
C     RESPONSE/MODE-SHAPE TABULATION
      IOR = 0
      JOR = 0
      IFLAG = 0
      IF (IRUN.EQ.2) SPD = Q2(M)
      RPM = 60.0*SPD
722   DO 725 I=1,NB
      I2 = 2*I
      I1 = I2-1
      L = I+NB
      L2 = 2*L
      L1 = L2-1
725   CALL ORBIT(IOR,BIG(I),SMALL(I),SLANT(I),PHASE(I),WLX(I1),WLX(I2),
     +                                                  WLX(L1),WLX(L2))
      IF (IFLAG.EQ.0) GO TO 727
      WRITE (KW,1011)
1011  FORMAT (///18H RESONANT RESPONSE)
      IF(IT.EQ.0) GO TO 1016
      CALL XCITAB(KW,IT,NB,M8,K8,PLX)
      WRITE (KW,1012)
1012  FORMAT (//15H RESPONSE ORBIT/)
      GO TO 728
1016  CALL XCITAB(KW,IT,NB,M8,K8,ALX)
      WRITE (KW,1012)
      GO TO 728
727   IF (IRUN.EQ.4.OR.M.EQ.1) WRITE (KW,1091)
      WRITE (KW,1015) RPM,Q2(M),B(M),AM9(M)
1015  FORMAT(19H0ROTATIONAL SPEED =,1PE11.4,4H RPM/17H FREQUENCY
     +,2H =,E11.4,4H  HZ/19H CRIT DAMP RATIO  =,E11.4/
     +19H PRECISION INDEX  =,E11.4)
      WRITE (KW,1110)
1110  FORMAT (19H NATURAL MODE SHAPE)
728   CALL ORBTAB(KW,NB,M8,K8,SPD,Q2(M),BIG,SMALL,SLANT,PHASE,POSIT,JOR)
      IF (IFLAG.GT.0.OR.IRUN.EQ.5) GO TO 800
C     RESONANT RESPONSE
      IFLAG = 1
C          FORCE INTERPOLATION
      DO 756 I=1,N20
      I2 = 2*I
      I1 = I2-1
      IF (IRUN.EQ.2.AND.I.GT.NB) GO TO 755
      REWIND KS
      DO 752 K=1,NF
      K2 = 2*K
```

```
      K1 = K2-1
      READ(KS) SPD,FRE,((YLX(II,J1),J1=1,N202),II=1,N20)
      READ (KS) (WLX(II),II=1,N202)
      ULX(K1) = WLX(I1)
752   ULX(K2) = WLX(I2)
      DO 753 K=1,NF
      K1 = 2*K-1
753   Y(K) = ULX(K1)
      XINT(1) = Q2(M)
      IER = 0
      CALL SPLINE (ISP,NF,RET,XINT,F,Y,Z1,KW,IER)
      IF (IER.NE.0) GO TO 806
      REP = RET(1)
      DO 754 K=1,NF
      K2 = 2*K
754   Y(K) = ULX(K2)
      IER = 0
      CALL SPLINE (ISP,NF,RET,XINT,F,Y,Z2,KW,IER)
      IF (IER.NE.0) GO TO 806
      PLX(I1) = REP
      PLX(I2) = RET(1)
      GO TO 756
755   PLX(I1) = 0.0
      PLX(I2) = 0.0
756   CONTINUE
C           ENSURE WHIRL REPRESENTATION
      DO 765 I=1,NB
      I2 = 2*I
      I1 = I2-1
      L = I+NB
      L2 = 2*L
      L1 = L2-1
      IF(IT.EQ.1) GO TO 760
      ALX(I1) = PLX(I1)
      ALX(I2) = PLX(I2)
      ALX(L1) = PLX(L1)
      ALX(L2) = PLX(L2)
      GO TO 765
760   ALX(I1) = 0.5*PLX(I1)
      ALX(I2) = 0.5*PLX(I2)
      ALX(L1) = ALX(I1)
      ALX(L2) = ALX(I2)
      DLX(1) =-0.5*PLX(L2)
      DLX(2) = 0.5*PLX(L1)
      ALX(I1) = ALX(I1)+DLX(1)
      ALX(I2) = ALX(I2)+DLX(2)
      ALX(L1) = ALX(L1)-DLX(1)
      ALX(L2) = ALX(L2)-DLX(2)
765   CONTINUE
C           RESPONSE CALCULATION
      IF (JDG.NE.6) GO TO 769
      DO 767 I=1,N20
      IROW(I) = I
      ICOL(I) = I
```

```
 767   CONTINUE
       WRITE (KW,1003)
 1003  FORMAT (21H1RESPONSE DIAGNOSTICS//23H INTERPOLATED IMPEDANCE)
       CALL MOUTC(TABLX,N20,N20,M6,JR,JR,IROW,ICOL,LINE,KW)
 C   INVERT IMPEDANCE MATRIX
 769   ID = 1
       CALL PINVC(ID,KW,NB,N40,N40,IS,IROT,TABLX,JDG)
       IF (JDG.NE.6) GO TO 762
       DO 768 I=1,N20
       IROW(I) = I
       ICOL(I) = I
 768   CONTINUE
       WRITE (KW,1004)
 1C04  FORMAT (//34H INVERSE OF INTERPOLATED IMPEDANCE)
       CALL MOUTC(TABLX,N20,N20,M6,JR,JR,IROW,ICOL,LINE,KW)
       WRITE (KW,1091)
 762   DO 770 I=1,NB
       I2 = 2*I
       I1 = I2-1
       L = I+NB
       L2 = 2*L
       L1 = L2-1
       WLX(I1) = 0.0
       WLX(I2) = 0.0
       WLX(L1) = 0.0
       WLX(L2) = 0.0
       DO 770 J=1,NB
       J2 = 2*J
       J1 = J2-1
       K = J+NB
       K2 = 2*K
       K1 = K2-1
       WLX(I1)=WLX(I1)+TABLX(I,J1)*ALX(J1)-TABLX(I,J2)*ALX(J2)
       WLX(I2)=WLX(I2)+TABLX(I,J2)*ALX(J1)+TABLX(I,J1)*ALX(J2)
       IF (IS.EQ.0) GO TO 766
       WLX(L1)=WLX(L1)+TABLX(L,J1)*ALX(J1)-TABLX(L,J2)*ALX(J2)
       WLX(L2)=WLX(L2)+TABLX(L,J2)*ALX(J1)+TABLX(L,J1)*ALX(J2)
       IF (IRUN.EQ.2) GO TO 770
       WLX(I1)=WLX(I1)+TABLX(I,K1)*ALX(K1)-TABLX(I,K2)*ALX(K2)
       WLX(I2)=WLX(I2)+TABLX(I,K2)*ALX(K1)+TABLX(I,K1)*ALX(K2)
 766   WLX(L1)=WLX(L1)+TABLX(L,K1)*ALX(K1)-TABLX(L,K2)*ALX(K2)
       WLX(L2)=WLX(L2)+TABLX(L,K2)*ALX(K1)+TABLX(L,K1)*ALX(K2)
 770   CONTINUE
       GO TO 722
 800   CONTINUE
 805   IF (IROT.EQ.2.OR.IRUN.EQ.2) GO TO 810
       IROT = 2
       GO TO 100
 806   WRITE (KW,1200)
 1200  FORMAT (/49H ERROR ENCOUNTERED DURING EXECUTION OF SUBROUTINE,
      +8H SPLINE.)
       CALL EXIT
 810   CONTINUE
       RETURN
       END
```

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